

Tracking Control of Surface Vessels via Adaptive Type-2 Fuzzy Logic Control

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Abstract—This paper presents an indirect adaptive type-2 fuzzy logic controller (FLC) as well as a direct adaptive type-2 FLC for tracking control of surface vessels under time-varying hydrodynamic disturbances. The combination of approximation-based adaptive control technique and type-2 fuzzy logic system (FLS) allows us to handle time-varying disturbances without exact information on them. The stability of the design is proved through Lyapunov analysis where globally asymptotical convergence of the tracking errors is guaranteed. Although designed from different points of view, both indirect and direct adaptive type-2 FLC yield same and passive closed-loop systems. Comparative simulations with their adaptive type-1 counterparts are carried out. The proposed techniques are found to be effective, robust, and reduce tracking errors.

Index Terms—Adaptive control, type-2 fuzzy logic system, marine vehicle control, function approximation, tracking.

I. INTRODUCTION

Essential to any controller design process is knowledge of the dynamic characteristics of the plant or physical system to be controlled. Accurate modeling of the plant can increase the probability that a satisfactory controller, designed based on mathematical plant model, achieves similar performance in reality. For this reason, more and more realistic dynamic models for surface vessels have been proposed [1]. One of the latest vessel models was presented in [2], [3]. Due to the time-varying hydrodynamic disturbances in this model, few controllers are designed for it. In [4], an indirect adaptive type-2 fuzzy logic controller (FLC) is proposed for regulation control of the vessels. As a complementary work to [4], this paper considers the tracking control of the vessels. In marine vessels tracking and guidance applications, the weakness of traditional model-based adaptive control is the requirement that uncertainties are parametric, time-invariant, and linear in the parameters [5]. Hence, approximation-based adaptive control technique [6], [7], which do not require parametric or functional certainty, is adopted to compensate for functional uncertainties and disturbances from environment. Such approximators normally have a standard regressor function whose configuration is independent of the dynamic characteristics of the vessel model.

It has been proved in [8]–[11] that type-1 fuzzy logic system (FLS) are universal approximators, i.e., for any given continuous nonlinear function, there exists a type-1 FLS such

that it can uniformly approximate the function to any desired accuracy. As an extension of type-1 FLS, type-2 FLS has been proposed and drawn much attention [12]–[15]. The main differences between type-1 and type-2 FLS lie in for type-2 FLS at least one of the antecedent or consequent fuzzy sets is a type-2 fuzzy set, whose membership value is a type-1 fuzzy set. Thus a type-reducer is needed to convert the type-2 fuzzy output sets into type-1 sets so that they can be processed by the defuzzifier to produce a crisp output. The type-2 FLS has already found its applications to several engineering problems [16]–[19]. This paper serves as another engineering application to investigate the approximation property of type-2 FLS.

By combining approximation-based adaptive control technique and type-2 FLS, an indirect as well as a direct adaptive type-2 FLC are proposed for tracking control of newly developed vessel model [2], [3]. It is proved that the tracking errors under these two controller globally asymptotically converge to zero and the resultant closed-loop systems are passive.

The rest of the paper is organized as follows. Section II describes the singleton interval type-2 (IT2) FLS and constructs it in the linear in the parameters form. Next, the modeling of marine vessels is introduced in Section III. Then the design and stability analysis of the two adaptive type-2 FLCs is delineated in Section IV. After that, Section V describes the simulation results of the closed-loop system. Comparisons with their type-1 counterparts are also shown in this section. Finally, conclusions are drawn in Section VI.

II. DESCRIPTION OF SINGLETON IT2 FLS

In this paper, a singleton IT2 FLS whose general configuration is depicted in Fig. 1 is considered. It contains five components which are fuzzifier, rule base, fuzzy inference engine, type-reducer, and defuzzifier. Here, a multi-input, single-output fuzzy system is described, as a multi-output system can generally be separated into several single-output systems.

1) *Fuzzification*: The fuzzifier maps a crisp point $\mathbf{x} = (x_1, \dots, x_n) \in \mathbf{U} \subset R^n$ into a IT2 fuzzy set $\tilde{A}_{\mathbf{x}}$ in $\mathbf{U}$. In the singleton fuzzification, the input fuzzy set $\tilde{A}_{\mathbf{x}}$ has only a single point of nonzero membership as follows.

$$\mu_{\tilde{A}_{\mathbf{x}}}(\mathbf{x}) = \begin{cases} 1; & \mathbf{x} = \mathbf{x}' \\ 0; & \mathbf{x} \neq \mathbf{x}' \end{cases} \quad (1)$$

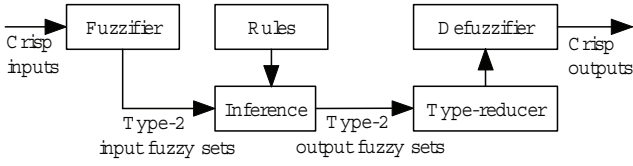


Fig. 1. A singleton interval type-2 FLS.

2) *Rule Base*: The fuzzy rule base consists of a group of fuzzy **IF-THEN** rules, which can be extracted from numerical data or provided by experts. Consider a IT2 FLS having n inputs and one output, then the l th rule in the rule base can be written as

$$R^l : \text{IF } x_1 \text{ is } \tilde{F}_1^l \text{ and } \dots \text{ and } x_n \text{ is } \tilde{F}_n^l \text{ THEN } y \text{ is } \tilde{G}^l \quad (2)$$

where $l = 1, 2, \dots, M$, x_i ($i = 1, 2, \dots, n$) and y are the inputs and output to the IT2 FLS respectively, $\tilde{F}_i^l$ and $\tilde{G}^l$ are labels of antecedent and consequent fuzzy sets in $\mathbf{U}_i$ and R , respectively. This rule represents a type-2 relation between the input space $\mathbf{U}$ and output space R .

3) *Fuzzy Inference Engine*: The fuzzy inference engine combines fuzzy **IF-THEN** rules and provides a mapping from input IT2 fuzzy sets in $\mathbf{U}$ to output IT2 fuzzy set in R . Each rule is interpreted as a fuzzy implication. For the singleton IT2 FLS, using the extended sup-star compositional rule of inference, the output consequent set corresponding to rule R^l can be expressed as

$$\mu_{\tilde{B}^l}(y) = \mu_{\tilde{G}^l}(y) \cap \left[\bigcap_{i=1}^n \mu_{\tilde{F}_i^l}(x'_i) \right]. \quad (3)$$

where the firing set $\bigcap_{i=1}^n \mu_{\tilde{F}_i^l}(x'_i) \equiv F^l(x')$ is an interval type-1 fuzzy set, i.e.,

$$F^l(x') = \left[\underline{f}^l(x'), \bar{f}^l(x') \right] \quad (4)$$

$$\underline{f}^l(x') = \underline{\mu}_{\tilde{F}_1^l}(x'_1) \star \dots \star \underline{\mu}_{\tilde{F}_n^l}(x'_n) \quad (5)$$

$$\bar{f}^l(x') = \bar{\mu}_{\tilde{F}_1^l}(x'_1) \star \dots \star \bar{\mu}_{\tilde{F}_n^l}(x'_n). \quad (6)$$

Here $\underline{\mu}_{\tilde{F}_i^l}(x'_i)$ and $\bar{\mu}_{\tilde{F}_i^l}(x'_i)$ are the lower and upper membership grade of IT2 fuzzy set $\tilde{F}_i^l$, respectively, symbol $\star$ denotes the t-norm corresponding to the conjunction "and" in (2). The fired output consequent set $\mu_{\tilde{B}^l}(y)$ for the l th rule can be written as

$$\mu_{\tilde{B}^l}(y) = \int_{b^l \in [\underline{f}^l \star \underline{\mu}_{\tilde{G}^l}(y), \bar{f}^l \star \bar{\mu}_{\tilde{G}^l}(y)]} 1/b^l \quad (7)$$

where $\underline{\mu}_{\tilde{G}^l}$ and $\bar{\mu}_{\tilde{G}^l}$ are the lower and upper membership grades of consequent fuzzy set, $\tilde{G}^l$.

4) *Type-Reduction and Defuzzification*: By Passing through the type-reduction operator, the IT2 fuzzy sets, which are outputs of the inference engine, are type-reduced into a type-1 fuzzy set. This type-1 fuzzy set is then defuzzified to obtain crisp output. So far, there are many kinds of type-reduction method available, such as centroid, center of sums, height and center of sets type-reduction. In this paper, center of sets type-reduction is used, as it has reasonable computational

complexity. Assume maximum t-conorm, product t-norm, and product implication are used. Then in the type-reduction, the type-reduced set is an interval type-1 fuzzy set $[y'_l, y'_r]$, whose left end point y'_l and right end point y'_r can be computed using the Karnik-Mendel iterative procedure [13]. In this procedure, assume singleton consequent sets are used, the consequent parameters have to be reordered in ascending order first. Let $\mathbf{y} = (y^1, \dots, y^M)^T$ represent the original rule-ordered consequent values, and let $\hat{\mathbf{y}} = (\hat{y}^1, \dots, \hat{y}^M)^T$ denote the reordered sequences, where $\hat{y}^1 \leq \hat{y}^2 \leq \dots \leq \hat{y}^M$. The formula illustrating relationship between $\mathbf{y}$ and $\hat{\mathbf{y}}$ is

$$\hat{\mathbf{y}} = \mathbf{Q}\mathbf{y} \quad (8)$$

where $\mathbf{Q}$ is $M \times M$ permutation matrix with elementary vectors as columns, and these vectors are permuted to move elements in $\mathbf{y}$ to new reordered locations in ascending order in the transformed vector $\hat{\mathbf{y}}$. The original rule firing strength $\underline{\mathbf{f}} = (\underline{f}^1, \underline{f}^2, \dots, \underline{f}^M)^T$ and $\bar{\mathbf{f}} = (\bar{f}^1, \bar{f}^2, \dots, \bar{f}^M)^T$ are accordingly reordered. To compute the end points, the new orders for $\underline{\mathbf{f}}$ and $\bar{\mathbf{f}}$ are $\mathbf{Q}\underline{\mathbf{f}}$ and $\mathbf{Q}\bar{\mathbf{f}}$ respectively. According to [20], [21], the left end point y'_l can be computed as

$$\begin{aligned} y'_l &= \frac{\sum_{i=1}^L (\mathbf{Q}\bar{\mathbf{f}})_i \hat{y}^i + \sum_{i=L+1}^M (\mathbf{Q}\underline{\mathbf{f}})_i \hat{y}^i}{\sum_{i=1}^L (\mathbf{Q}\bar{\mathbf{f}})_i + \sum_{i=L+1}^M (\mathbf{Q}\underline{\mathbf{f}})_i} \\ &= \frac{\bar{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_1^T \mathbf{E}_1 \mathbf{Q} \mathbf{y} + \underline{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_2^T \mathbf{E}_2 \mathbf{Q} \mathbf{y}}{\mathbf{p}_l^T \mathbf{Q} \bar{\mathbf{f}} + \mathbf{g}_l^T \mathbf{Q} \underline{\mathbf{f}}} \end{aligned} \quad (9)$$

where L denotes the left crossover point,

$$\mathbf{p}_l = \underbrace{(1, 1, \dots, 1, 0, \dots, 0)}_L \in R^{M \times 1}$$

$$\mathbf{g}_l = (0, \dots, 0, \underbrace{1, 1, \dots, 1}_{M-L})^T \in R^{M \times 1}$$

$$\mathbf{E}_1 = (\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_L, \mathbf{0}, \dots, \mathbf{0})^T \in R^{L \times M}$$

$$\mathbf{E}_2 = (\mathbf{0}, \dots, \mathbf{0}, \mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_{M-L})^T \in R^{(M-L) \times M}$$

and where $\mathbf{e}_i \in R^L$ and $\mathbf{e}_i \in R^{M-L}$ are elementary vectors. Similarly, the output y'_r can be expressed as

$$\begin{aligned} y'_r &= \frac{\sum_{i=1}^R (\mathbf{Q}\underline{\mathbf{f}})_i \hat{y}^i + \sum_{i=R+1}^M (\mathbf{Q}\bar{\mathbf{f}})_i \hat{y}^i}{\sum_{i=1}^R (\mathbf{Q}\underline{\mathbf{f}})_i + \sum_{i=R+1}^M (\mathbf{Q}\bar{\mathbf{f}})_i} \\ &= \frac{\underline{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_3^T \mathbf{E}_3 \mathbf{Q} \mathbf{y} + \bar{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_4^T \mathbf{E}_4 \mathbf{Q} \mathbf{y}}{\mathbf{p}_r^T \mathbf{Q} \underline{\mathbf{f}} + \mathbf{g}_r^T \mathbf{Q} \bar{\mathbf{f}}} \end{aligned} \quad (10)$$

where R denotes the right crossover point,

$$\mathbf{p}_r = \underbrace{(1, 1, \dots, 1, 0, \dots, 0)}_R \in R^{M \times 1}$$

$$\mathbf{g}_r = (0, \dots, 0, \underbrace{1, 1, \dots, 1}_{M-R})^T \in R^{M \times 1}$$

$$\mathbf{E}_3 = (\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_R, \mathbf{0}, \dots, \mathbf{0})^T \in R^{R \times M}$$

$$\mathbf{E}_4 = (\mathbf{0}, \dots, \mathbf{0}, \mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_{M-R})^T \in R^{(M-R) \times M}$$

and where $\mathbf{e}_i \in R^R$ and $\mathbf{e}_i \in R^{M-R}$ are elementary vectors. Finally, the interval type-1 fuzzy set is defuzzified to generate

the system output by means of computing the average of y'_l and y'_r . Hence the defuzzified output is

$$y' = \frac{y'_l + y'_r}{2}. \quad (11)$$

View the consequent parameters $\theta = \mathbf{y} = (y^1, \dots, y^M)^T$ as adjustable parameters, then (11) can be written as

$$y' = \phi^T \theta \quad (12)$$

where

$$\phi^T = \frac{\bar{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_1^T \mathbf{E}_1 \mathbf{Q} + \underline{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_2^T \mathbf{E}_2 \mathbf{Q}}{2(\mathbf{p}_l^T \mathbf{Q} \bar{\mathbf{f}} + \mathbf{g}_l^T \mathbf{Q} \underline{\mathbf{f}})} + \frac{\bar{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_3^T \mathbf{E}_3 \mathbf{Q} + \underline{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_4^T \mathbf{E}_4 \mathbf{Q}}{2(\mathbf{p}_r^T \mathbf{Q} \bar{\mathbf{f}} + \mathbf{g}_r^T \mathbf{Q} \underline{\mathbf{f}})} \quad (13)$$

is a regressive vector. Now as illustrated in (12), the singleton IT2 FLS using maximum t-conorm, product t-norm, product implication, and center of sets type-reduction can be expressed as linear in the parameters.

III. MODELING OF MARINE VESSELS

The model of surface vessels may be classified into two categories [22], namely process plant model and control plant model. The process plant model, which simulates the real plant dynamics as closely as possible including process disturbance, sensor outputs and control inputs, is used for numerical analysis and simulation of the performance and stability of the closed-loop system. The control plant model, which is a simplified form of the complicated process plant model, is used for controller design and theoretical stability analysis (e.g., in the sense of Lyapunov).

A. Process Plant Model

1) *Kinematics*: The motions and state variables of the surface vessels are conventionally defined and measured with respect to two coordinate frames as shown in Fig. 2, namely earth-fixed frame and body-fixed frame. The earth-fixed frame,

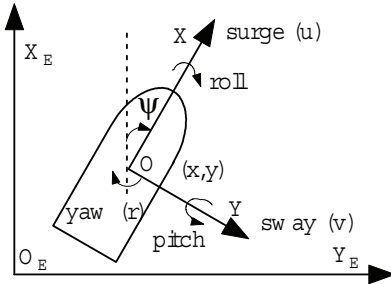


Fig. 2. 2D earth-fixed and body-fixed coordinate frames.

denoted as $X_E Y_E Z_E$, is assumed fixed on the earth surface with X_E axis pointing north, the Y_E axis pointing east, and the Z_E axis down of the earth tangent plane. It is mainly used for global guidance and navigation. The body-fixed frame, denoted as XYZ , is fixed to the hull of the vessel, with X axis positive toward the bow, the Y axis positive towards

starboard, and the Z axis positive downward. The origin of this frame is normally located at the vessel's center of gravity. The motion of a surface vessel has 6 degrees of freedom (DOF), which are surge, sway, heave, roll, pitch, and yaw. Let $\eta' = [x, y, z, \phi, \theta, \psi]^T$ be the vector representing the position and orientation of the vessel with respect to an earth-fixed frame, and let $\nu' = [u, v, w, p, q, r]^T$ denote the translation and rotation velocities of the vessel decomposed in the body-fixed frame. Then, the transformation between the earth-fixed and body-fixed velocity vectors is

$$\dot{\eta}' = \mathbf{J}'(\eta') \nu'. \quad (14)$$

where $\mathbf{J}'(\eta')$ is a state dependent transformation matrix, whose detailed definition can be found in [1].

2) *Dynamics*: The 6 DOF model formulation introduced here is based on [2], [3]. The equation of motion, which is in body-fixed frame, for a surface vessel whose speed is not low, is given by

$$\mathbf{M}' \dot{\nu}' + \mathbf{C}'_{RB} \nu' + \mathbf{C}'_A \nu'_r + \bar{\mathbf{B}}' \nu'_r + \mathbf{G}' \eta' + \mu' = \tau' + \tau'_h \quad (15)$$

where $\mathbf{M}' = \mathbf{M}'_{RB} + \bar{\mathbf{A}}' \in R^{6 \times 6}$ is the sum of the system inertia matrix and added mass matrix, $\mathbf{C}'_{RB} \in R^{6 \times 6}$ is the Coriolis-Centripetal matrix, $\mathbf{C}'_A \in R^{6 \times 6}$ is the constant infinite frequency added mass matrix, $\bar{\mathbf{B}}' \in R^{6 \times 6}$ is the constant infinite frequency potential damping matrix, $\mathbf{G}' \in R^{6 \times 6}$ is the restoring matrix, $\mu' \in R^6$ is computed using a state space model to represent the fluid memory effect,

$$\dot{\chi} = \mathbf{A}_r \chi + \mathbf{B}_r \nu'_r \quad (16)$$

$$\mu' = \mathbf{C}_r \chi + \mathbf{D}_r \nu'_r \quad (17)$$

and where $(\mathbf{A}_r, \mathbf{B}_r, \mathbf{C}_r, \mathbf{D}_r)$ are constant matrices of appropriate dimensions. $\tau' \in R^6$ is control force vector produced by thruster system, $\tau'_h = \tau'_{\text{CFD}} + \tau'_{\text{wave}} \in R^6$ is the sum of the cross-flow drag and surge resistance vector and wave load vector, which is computed according to Response Amplitude Operator tables, and $\nu'_r = \nu' - \nu'_c \in R^6$ is the relative velocity vector between vessel velocity vector ν' and sea current velocity vector ν'_c . All the detailed definition and computation of the above matrices and vectors can be found in [1], [2] and references therein.

B. Control Plant Model

The process plant model described in previous subsection is simplified on the basis of the following assumptions to facilitate the controller design and analytical stability analysis.

Assumption 1: The motions along heave, roll, and pitch directions are ignored for surface vessels. As a conventional vessel is metacentric stable, which means that there are restoring force and moments along those directions, this assumption is appropriate. As an application of this assumption, the dimension of the restoring matrix $\mathbf{G}' \in R^{6 \times 6}$ in (15) is reduced to generate $\mathbf{G} \in R^{3 \times 3} = \mathbf{0}$.

Assumption 2: The state variables of the vessel, i.e. displacements and velocities along surge, sway, and yaw, are measured by its own on-board devices such as gyro system, GPS, and accelerometers. Due to the development of modern

integration system of inertial sensors and satellite navigation system, this assumption is also appropriate.

Assumption 3: The influence of sea current is ignored. This assumption is mainly for the convenience of theoretical analysis.

Assumption 4: The time-varying hydrodynamic disturbances, i.e. μ' and τ_h' in (15), could be estimated by singleton IT2 FLSs in the form of

$$\begin{aligned}\tau_d' &= -\Phi'(\nu')\Theta'. \\ &= \tau_h' - \mu'\end{aligned}\quad (18)$$

where

$$\Theta' = (\theta_1'^T, \theta_2'^T, \dots, \theta_6'^T)^T \in R^{6M \times 1} \quad (19)$$

is a weighting vector and

$$\Phi'(\nu') = \text{diag}[\phi_1'^T, \phi_2'^T, \dots, \phi_6'^T] \in R^{6 \times 6M} \quad (20)$$

is a regressive matrix, M is the total number of rules for each sub-FLS. Thus far, this assumption has not been well established, for there is still a lack of rigorous proof showing singleton IT2 FLS could be universal approximator, but the propriety of this assumption will be checked by subsequent simulations.

1) Control Plant Model for Indirect Adaptive Control:

Applying Assumptions 1-4 to (14) and (15), the following control plant model for indirect adaptive type-2 FLC design is obtained.

$$\dot{\eta} = \mathbf{J}(\eta)\nu \quad (21)$$

$$\mathbf{M}\dot{\nu} + \mathbf{D}\nu = \tau - \Phi(\nu)\Theta \quad (22)$$

where $\eta = [x, y, \psi]^T$ is the displacement vector along surge, sway, and yaw, whereas $\nu = [u, v, r]^T$ is velocity vector, $\mathbf{J}(\eta) \in R^{3 \times 3}$ is the rotation matrix, note that $\mathbf{J}\mathbf{J}^T = \mathbf{I}$ ($\mathbf{I}$ is the identity matrix), $\mathbf{M} \in R^{3 \times 3}$ is the system inertia matrix including added mass, $\mathbf{D} = \mathbf{C}_{RB} + \mathbf{C}_A + \bar{\mathbf{B}} \in R^{3 \times 3}$ is the damping matrix, and where $\mathbf{C}_{RB} \in R^{3 \times 3}$, $\mathbf{C}_A \in R^{3 \times 3}$, and $\bar{\mathbf{B}} \in R^{3 \times 3}$ are three dimensional version of their corresponding matrices.

$$\Theta = (\theta_1^T, \theta_2^T, \theta_3^T)^T \in R^{3M \times 1} \quad (23)$$

is the weighting vector and

$$\Phi(\nu) = \text{diag}[\phi_1^T(\nu), \phi_2^T(\nu), \phi_3^T(\nu)] \in R^{3 \times 3M} \quad (24)$$

is the regressive matrix. Thus, here three sub-FLSs are used to approximate the hydrodynamic disturbances.

2) Control Plant Model for Direct Adaptive Control:

Application of Assumptions 1-3 to (14) and (15) yields the following control plant model for direct adaptive type-2 FLC design.

$$\dot{\eta} = \mathbf{J}(\eta)\nu \quad (25)$$

$$\mathbf{M}\dot{\nu} + \mathbf{D}\nu + \mu = \tau + \tau_h \quad (26)$$

where $\mu \in R^3$ and $\tau_h \in R^3$ are three dimensional version of μ' and τ_h' respectively.

IV. ADAPTIVE FUZZY LOGIC CONTROLLER DESIGN

The control objective in this paper is to manipulate a surface vessel to track the desired trajectory $\eta_d = [x_d, y_d, \psi_d]^T$, whose second derivative is continuous, as close as possible. In this section, both indirect and direct adaptive fuzzy logic controller are proposed for this control objective. However, the closed-loop systems under these two control schemes turn out to be same. The tracking error under these two control techniques is proved to globally asymptotically converge to zero. Rigorous analysis shows the closed-loop system is passive.

A. Indirect Adaptive Fuzzy Control

In indirect adaptive fuzzy control, the disturbances of the plant model are estimated by singleton IT2 FLSs as illustrated in (18) and the controller is designed assuming that the estimated disturbances represent the true time-varying disturbances. The control and adaptive law of the indirect adaptive type-2 FLC for plant (21), (22) are as follows.

$$\tau = \mathbf{M}\dot{\nu}_t + \mathbf{D}\nu_t - \lambda_1 s + \Phi(\nu)\hat{\Theta} \quad (27)$$

$$\dot{\hat{\Theta}} = \lambda_2 \Phi^T(\nu)s \quad (28)$$

where $\nu_t = \nu_d - \lambda_3 \mathbf{J}^T(\eta)e = \mathbf{J}^T(\eta)\dot{\eta}_d - \lambda_3 \mathbf{J}^T(\eta)e = \mathbf{J}^T(\eta)(\dot{\eta}_d - \lambda_3 e)$, $\tilde{\Theta} = \Theta - \hat{\Theta} \in R^{3M}$ is the error vector between true Θ in (22) and adapted $\hat{\Theta}$, λ_i ($i = 1, 2, 3$) are positive constant, and $s = \nu - \nu_t = e_\nu + \lambda_3 \mathbf{J}^T(\eta)e = \mathbf{J}^T(\eta)\dot{e} + \lambda_3 \mathbf{J}^T(\eta)e = \mathbf{J}^T(\eta)(\dot{e} + \lambda_3 e)$, and where $e = \eta - \eta_d \in R^3$ is the tracking error vector of vessel displacement, whereas $e_\nu = \nu - \nu_d \in R^3$ is the tracking error vector of vessel velocity.

B. Direct Adaptive Fuzzy Control

In direct adaptive fuzzy control, the parameters of the controller are directly adjusted to reduce the tracking error. The direct adaptive type-2 FLC is designed based on the following assumption.

Assumption 5: The function of $\mathbf{M}\dot{\nu}_t + \mathbf{D}\nu_t - \tau_h + \mu - \lambda_1 s$ could be estimated by singleton IT2 FLSs, i.e.,

$$\mathbf{M}\dot{\nu}_t + \mathbf{D}\nu_t - \tau_h + \mu - \lambda_1 s = \Phi(\nu)\Theta. \quad (29)$$

The control and adaptive law of the indirect adaptive type-2 FLC for plant (25), (26) are as follows.

$$\begin{aligned}\tau &= \Phi(\nu)\hat{\Theta} \\ &= \mathbf{M}\dot{\nu}_t + \mathbf{D}\nu_t - \tau_h + \mu - \lambda_1 s - \Phi(\nu)\tilde{\Theta}\end{aligned} \quad (30)$$

$$\dot{\hat{\Theta}} = \lambda_2 \Phi^T(\nu)s \quad (31)$$

C. Stability Analysis

Application of indirect and direct adaptive control and adaptive law (27), (28) and (30), (31) respectively to the plant (21), (22) and (25), (26) yields the same closed-loop system

$$\mathbf{M}\dot{s} + \mathbf{D}s = -\lambda_1 s - \Phi(\nu)\tilde{\Theta} \quad (32)$$

$$\dot{\tilde{\Theta}} = \lambda_2 \Phi^T(\nu)s \quad (33)$$

$$\dot{e} = -\lambda_3 e + \mathbf{J}(\eta)s. \quad (34)$$

Theorem 1: The states (s, e) of the closed-loop system (32)-(34) globally asymptotically converge to $(0, 0)$.

Proof: Consider the following Lyapunov function candidate, which is positive definite.

$$V = \frac{1}{2} \mathbf{s}^T \mathbf{M} \mathbf{s} + \lambda_1 \lambda_3 \mathbf{e}^T \mathbf{e} + \frac{1}{2\lambda_2} \tilde{\Theta}^T \tilde{\Theta} \quad (35)$$

Differentiation of V along the trajectory of the closed-loop system (32)-(34) yields

$$\begin{aligned} \dot{V} &= \mathbf{s}^T (-\mathbf{D} \mathbf{s} - \lambda_1 \mathbf{s} - \Phi(\boldsymbol{\nu}) \tilde{\Theta}) + \tilde{\Theta}^T \Phi^T(\boldsymbol{\nu}) \mathbf{s} \\ &\quad + 2\lambda_1 \lambda_3 \mathbf{e}^T (-\lambda_3 \mathbf{e} + \mathbf{J}(\boldsymbol{\eta}) \mathbf{s}) \\ &= -\frac{1}{2} \mathbf{s}^T (\mathbf{D} + \mathbf{D}^T) \mathbf{s} - \lambda_1 \dot{\mathbf{e}}^T \dot{\mathbf{e}} - \lambda_1 \lambda_3^2 \mathbf{e}^T \mathbf{e} \\ &\leq 0 \end{aligned}$$

which means the derivative of the Lyapunov function candidate is semi-negative definite. Moreover, the largest invariant set contained in the set $\dot{V} \equiv 0$ is contained in the set $(s, e) = (0, 0)$.

Now, according to Krasovskii-La Salle Invariant Set Theorem, we may conclude the point (s, e) globally asymptotically converges to $(0, 0)$ as time tends to infinite, and $\dot{\tilde{\Theta}} = 0$, which means $\tilde{\Theta}$ is bounded. ■

Remark 1: Here, the theorem is developed for the case where the gains λ_i ($i = 1, 2, 3$) are scalar, the case where the gains are matrices can be extended to in a similar approach.

D. Passivity Interpretation

According to energy-related considerations, by means of an input-output description passivity theory provides a framework for the design and analysis of control systems. And the input-output description further allows for a modular method of control systems design and analysis [23]. As shown in Fig. 3, the closed-loop system (32)-(34) can be interpreted as the negative feedback interconnection of three subsystems with respective inputs u_1, u_2, u_3 and outputs y_1, y_2 , and y_3 , with $y_1 = u_2 = u_3$ and $u_1 = -y_2 - y_3$, and the three subsystems as follows.

Subsystem 1:

$$\begin{cases} \mathbf{M} \dot{\mathbf{s}} + \mathbf{D} \mathbf{s} = u_1 \\ y_1 = \mathbf{s} \end{cases} \quad (36)$$

Subsystem 2:

$$\begin{cases} \dot{\tilde{\Theta}} = \lambda_2 \Phi^T(\boldsymbol{\nu}) u_2 \\ y_2 = \Phi(\boldsymbol{\nu}) \tilde{\Theta} \end{cases} \quad (37)$$

Subsystem 3:

$$\begin{cases} \dot{\mathbf{e}} = -\lambda_3 \mathbf{e} + \mathbf{J}(\boldsymbol{\eta}) u_3 \\ y_3 = \lambda_1 u_3 \end{cases} \quad (38)$$

Theorem 2: The closed-loop system (32)-(34) is passive.

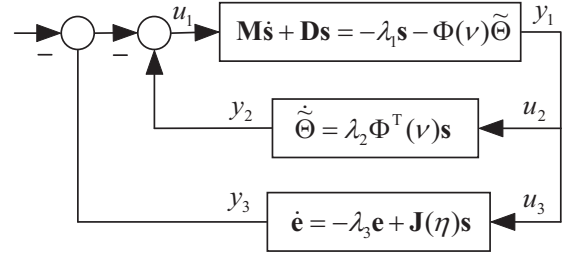


Fig. 3. Closed-loop equivalent representation.

Proof: For subsystem 1, its supply rate is

$$\begin{aligned} &\int_0^t u_1^T y_1 dr \\ &= \int_0^t (\mathbf{M} \dot{\mathbf{s}} + \mathbf{D} \mathbf{s})^T \mathbf{s} dr \\ &= \int_0^t \mathbf{s}^T \mathbf{M} \dot{\mathbf{s}} dr + \int_0^t \mathbf{s}^T \mathbf{D} \mathbf{s} dr \\ &= \frac{1}{2} \mathbf{s}^T(t) \mathbf{M} \mathbf{s}(t) - \frac{1}{2} \mathbf{s}^T(0) \mathbf{M} \mathbf{s}(0) + \int_0^t \mathbf{s}^T \mathbf{D} \mathbf{s} dr \end{aligned}$$

Thus, the subsystem 1 is output strictly passive with supply rate $\int_0^t u_1^T y_1 dr$ and storage function $\frac{1}{2} \mathbf{s}^T \mathbf{M} \mathbf{s}$.

For subsystem 2 its supply rate is

$$\begin{aligned} &\int_0^t u_2^T y_2 dr = \int_0^t \tilde{\Theta}^T \Phi^T(\boldsymbol{\nu}) u_2 dr \\ &= \int_0^t \tilde{\Theta}^T \frac{1}{\lambda_2} \dot{\tilde{\Theta}} dr \\ &= \frac{1}{2\lambda_2} \tilde{\Theta}^T(t) \tilde{\Theta}(t) - \frac{1}{2\lambda_2} \tilde{\Theta}^T(0) \tilde{\Theta}(0) \end{aligned}$$

which means the subsystem 2 is passive with supply rate $\int_0^t u_2^T y_2 dr$ and storage function $\frac{1}{2\lambda_2} \tilde{\Theta}^T \tilde{\Theta}$.

For subsystem 3, its supply rate is

$$\begin{aligned} &\int_0^t u_3^T y_3 dr = \int_0^t u_3^T \lambda_1 u_3 dr \\ &= \int_0^t \frac{1}{\lambda_1} y_3^T y_3 dr \\ &= \frac{\lambda_1}{2} \int_0^t u_3^T u_3 dr + \int_0^t \frac{1}{2\lambda_1} y_3^T y_3 dr \end{aligned}$$

which means the subsystem 3 is very strictly passive with supply rate $\int_0^t u_3^T y_3 dr$.

According to the fact that the negative feedback interconnection of two passive systems is passive, the closed-loop system (32)-(34), which can be interpreted as negative feedback interconnection of subsystem 1 and subsystem 2, then together with subsystem 3, is passive. ■

V. SIMULATION STUDIES

Marine System Simulator, which is developed in Norwegian University of Science and Technology [24], is employed as the platform for the simulation studies. The International Towing

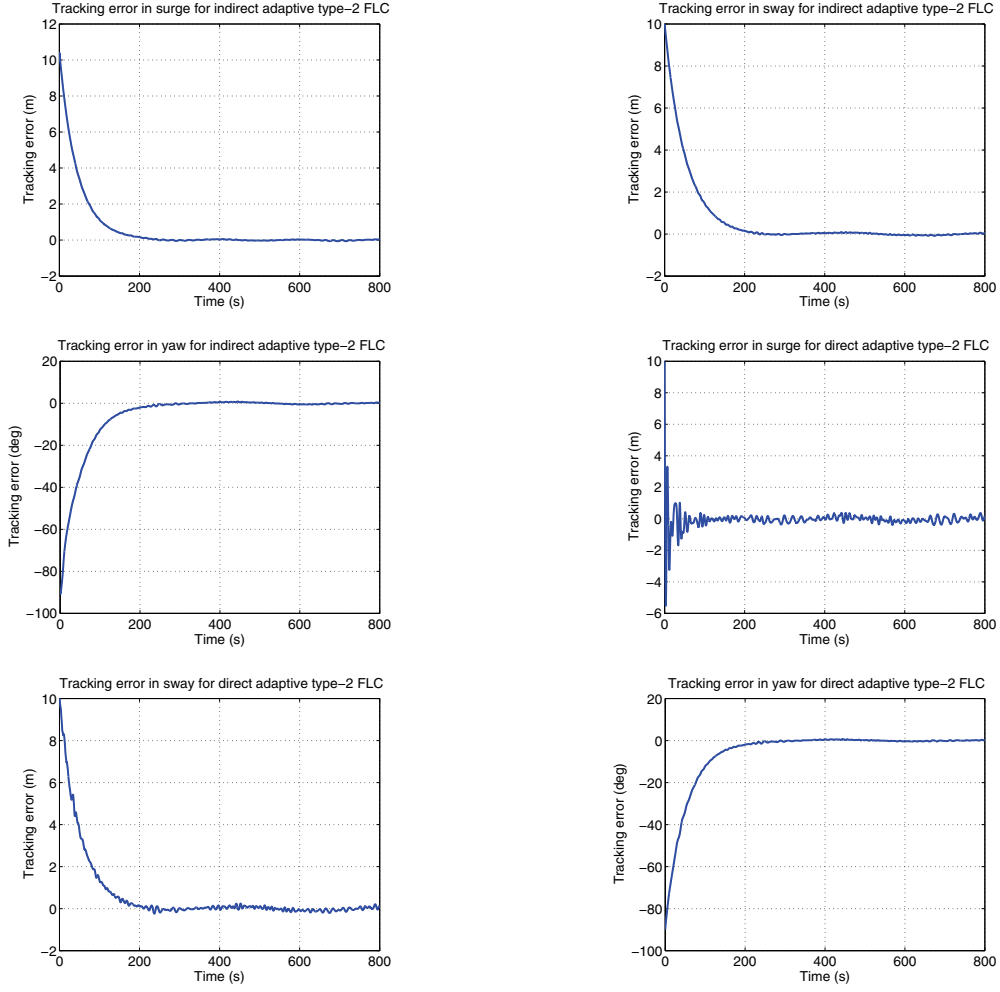


Fig. 4. Plots 1-3 show the tracking errors for indirect adaptive type-2 FLC. Plots 4-6 show the tracking errors for direct adaptive type-2 FLC.

Tank Conference (ITTC) wave spectrum with significant wave height $H_s = 3$ m and peak frequency $\omega_0 = 0.56$ rad/s is used to imitate rough sea with large waves. Sea current with speed $V_c = 0.2$ m/s and direction $\beta_c = 30^\circ$ is also included in the simulations, although ignored during the controller design. A container ship named as S-175 is used as case study. The control objective is to control the vessel to track the desired trajectory $\eta_d = [x_d, y_d, \psi_d]^T$, where $x_d = 8t$, $y_d = 200 \cos(0.005\pi t)$, and $\psi_d = \arctan(y_d/x_d)$. The initial states are set as $[\eta_0^T, \nu_0^T] = [10, 210, 0, 0, 0, 0]$. All the IT2 FLSs in indirect adaptive fuzzy controller (27), (28) and direct adaptive fuzzy controller (30), (31) are configured as follows. As there are three degrees of freedom considered for the motion of the vessel, three singleton IT2 sub-FLSs are employed for each controller, which is demonstrated in (23) and (24). For each sub-system, three fuzzy membership functions labeled as **N**, **Z**, and **P** are employed to partition each of the input domain. As there are three inputs, there are totally 27 rules and 27 adjustable consequent parameters. Gaussian function with uncertain standard deviation is chosen to be the primary membership functions of the antecedent IT2 fuzzy sets

and in the following form.

$$\mu_{\tilde{F}_i^j}(x_j) = \exp\left[-\frac{1}{2}\left(\frac{x_j - m_i}{\sigma}\right)^2\right] \quad \sigma \in [\underline{\sigma}, \overline{\sigma}] \quad (39)$$

where $j = 1, 2, 3$ is the index of three inputs and $i = 1, 2, 3$ is the index of three fuzzy membership functions for j th input. For each input, the means of the Gaussian functions are $m_1 = -1$, $m_2 = 0$, and $m_3 = 1$, the standard deviations of the lower membership functions are $\underline{\sigma} = 0.6$, and the standard deviations of the upper membership functions are $\overline{\sigma} = 0.8$.

A. Closed-loop Performance

Although the indirect and direct adaptive fuzzy controllers (27), (28) and (30), (31) are designed based on the control plant models (21), (22) and (25), (26) respectively, they are tested with the process plant model (14), (15) in the simulations.

1) *Indirect Adaptive Control:* Set $\lambda_1 = 4.3 \times 10^8$, $\lambda_2 = 1 \times 10^8$, and $\lambda_3 = 0.02$ in (27) and (28). The tracking errors of the vessel to the desired trajectory under the indirect adaptive fuzzy controller are shown in plot 1-3 of Fig. 4, from which it can be observed that the tracking errors asymptotically

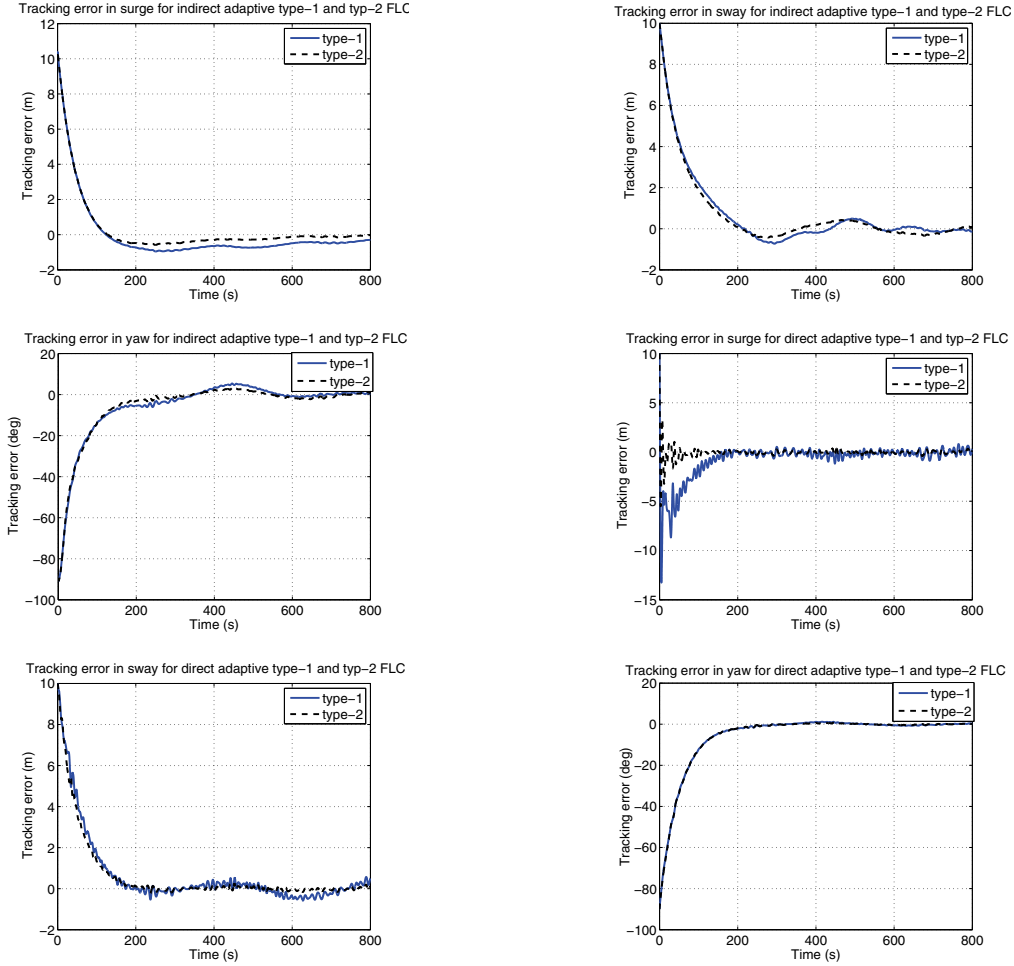


Fig. 5. Plots 1-3 show the tracking errors for indirect type-1 and type-2 FLC. Plots 4-6 show the tracking errors for direct type-1 and type-2 FLC.

converge to zero despite the time-varying hydrodynamic disturbances and the sub-FLSs effectively approximate the complex disturbances.

2) *Direct Adaptive Control*: Set $\lambda_2 = 1 \times 10^8$ and $\lambda_3 = 0.02$ in (30) and (31). The tracking errors under the direct adaptive fuzzy controller are shown in plot 4-6 of Fig. 4. The tracking performance is satisfactory in the presence of the time-varying disturbances, and the sub-FLSs effectively generate the control forces and moment. Fluctuations in the tracking errors are observed, that is because the inputs to the sub-FLSs are insufficient to make the sub-FLSs accurately approximate the complex function (29).

The control performance under indirect adaptive fuzzy control is noted to be better than that under direct adaptive fuzzy control. One may conclude without hesitation that the indirect adaptive fuzzy controller is better than the direct one, however, the direct adaptive fuzzy controller has an advantage over the indirect one, i.e., the direct adaptive fuzzy controller does not need the knowledge of inertial matrix $\mathbf{M}$ and damping matrix $\mathbf{D}$, thus can be used under the situation where such knowledge is not available.

B. Comparison with Adaptive Type-1 Fuzzy Controllers

Replace the components in (24) and correspondingly in indirect adaptive controller (27), (28) and direct adaptive controller (30), (31) with fuzzy basis function vector [7], [8],

$$\phi^T = (\phi^1(\mathbf{x}), \dots, \phi^M(\mathbf{x})) \quad (40)$$

where fuzzy basis function is defined as

$$\phi^l(\mathbf{x}) = \frac{\prod_{i=1}^n \mu_{F_i^l}(x_i)}{\sum_{l=1}^M (\prod_{i=1}^n \mu_{F_i^l}(x_i))}, \quad (41)$$

the controllers (27), (28) and (30), (31) become adaptive type-1 FLCs. In this subsection, the adaptive type-2 FLCs are compared with their type-1 counterparts. For each sub-FLS in the type-1 cases, five fuzzy membership functions labeled as **NL**, **NS**, **Z**, **PS**, and **PL** are employed to partition each of the input domain. As there are three inputs, there are 125 rules and 125 adjustable consequent parameters in contrast to 27 rules and parameters for type-2 cases. Gaussian functions are chosen to be the membership functions of the antecedent sets for each input, they can be written as

$$\mu_{F_i^j}(x_j) = \exp\left[-\frac{1}{2}\left(\frac{x_j - m_i}{\sigma}\right)^2\right], \quad (42)$$

where $j = 1, 2, 3$ is the index of three inputs and $i = 1, \dots, 5$ is the index of five fuzzy membership functions for j th input. The means are fixed as $(m_1, \dots, m_5) = (-1, -0.5, 0, 0.5, 1)$, whereas the standard deviations for all the Gaussian functions are set as $\sigma = 0.4$.

1) *Indirect Adaptive Control*: Set $\lambda_1 = 4.3 \times 10^8$, $\lambda_2 = 1 \times 10^7$, and $\lambda_3 = 0.02$ in (27) and (28) for both indirect adaptive type-1 and type-2 FLCs, the tracking errors are shown in plot 1-3 of Fig. 5.

2) *Direct Adaptive Control*: Set $\lambda_2 = 1 \times 10^8$ and $\lambda_3 = 0.02$ in (30) and (31) for both direct adaptive type-1 and type-2 FLCs, the tracking errors are depicted in plot 4-6 of Fig. 5.

It can be observed from Fig. 5 that the adaptive type-2 FLCs perform comparably with, if not better than, their type-1 counterparts, even though type-1 cases use more than four times the number of the rules. In order to quantify the performance differences between adaptive type-1 and type-2 FLCs, the integral of time-weighted absolute errors (ITAE) for all the above four cases in the following form are calculated.

$$P_x = \int_0^T t|e_x|dt \quad (43)$$

$$P_y = \int_0^T t|e_y|dt \quad (44)$$

$$P_\psi = \int_0^T t|e_\psi|dt \quad (45)$$

where T is the total simulation time and e_x , e_y , and e_ψ are three components of tracking error vector e . According to the results shown in Table I, the values of the ITAE for the adaptive type-2 FLCs are smaller than those for type-1 cases along all the three degrees of freedom. As regards the computational burden, type-reductions for type-2 cases do take some time, but the time is very little as only a few rules are fired at the same time and Karnik-Mendel algorithm has been proved to be super-exponentially convergent [25]. Furthermore, it has been reported the type-2 FLS can be implemented in real-time applications [26]. Thus, type-reduction should not be an issue and will not cause any stability problem. In conclusion, It seems that the type-2 FLS has better approximation property with less rules than its type-1 counterpart in this application.

TABLE I
ITAE FOR ADAPTIVE TYPE-1 AND TYPE-2 FLCs

Cases	Indirect T1	Indirect T2	Direct T1	Direct T2
P_x	192746	81796	162892	53917
P_y	85064	66428	78648	43013
P_ψ	14154	9592	5680	4660

VI. CONCLUSIONS

In this paper, indirect and direct adaptive type-2 FLCs have been designed for tracking control of surface vessels in the presence of time-varying hydrodynamic disturbances. It has been proved the tracking errors under both of the proposed controls globally asymptotically converge to zero. Although designed through different approaches, the closed-loop systems for both indirect and direct type-2 FLCs are same and passive. Simulation results have demonstrated that the two controllers are effective and robust. Comparison with their

type-1 counterparts suggests the type-2 FLS has comparable approximation property.

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