

Decentralized Fuzzy Fault Tolerant Control for Multiple Satellites Attitude Synchronization

Junquan Li^{*}, K. D. Kumar[†],
^{*} [†] Department of Aerospace Engineering

Ryerson University
Toronto, Canada

Email: junquan.li@ryerson.ca, kdkumar@ryerson.ca

Abstract—This paper presents a decentralized adaptive approximation design to achieve attitude tracking control for decentralized formation flying in presence of control input saturation, model uncertainties, external disturbances and reaction wheel faults. A nonsingular fast terminal sliding mode control is designed for finite time distributed cooperative attitude synchronization. In the proposed control scheme, a fuzzy logic system (FLS) is introduced to approximate unknown individual satellite attitude dynamics on-line due to the actuators fault. In order to achieve the capability of fault management without the involvement of ground station operators, the proposed control laws do not require an explicit fault detection and isolation mechanism. In the attitude control system of each satellite four reaction wheels are placed in a pyramid configuration, numerical simulation results including actuator dynamics and initial conditions' uncertainties show that the proposed strategy with FLS can compensate for the fault and the system continues to operate satisfactorily with wheel voltage or wheel speed faults and the closed loop distributed tracking control system is stochastically stable. Several simulation examples compared with the existing decentralized fault tolerant controller are presented for illustrating the effectiveness of the proposed fault tolerant control methodology.

Keywords: *Fuzzy Logic System; Fast Terminal Sliding Mode Control; Decentralized Fault Tolerant Control*

I. INTRODUCTION

Decentralized multi-agent cooperative control, including consensus, formation control and flocking has been studied extensively. The benefits such as scalability, robustness and easy maintenance can be obtained by embodying decentralized cooperative control [1]. The motivation of decentralized controller is to avoid the complete redesign of the control system when new subsystems are added and also guarantee the stability of the whole system. One limitation of those above approaches is that they did not provide finite-time control of a group of spacecraft, which could provide faster formation rearrangement capability during attitude coordination. In the reference [2], it is proved that nonsingular fast terminal sliding mode control can get rid of the singularity problem completely. Neural networks and fuzzy logic systems have been proposed as the tools to be capable of approximating any well defined functions over a compact set to arbitrary accuracy [3][4]. The fuzzy logic system is a very powerful technique in the discipline of systems with large uncertainties and strong nonlinearities. Therefore, a nonsingular terminal sliding mode control with fuzzy logic system is proposed

in this paper as a fault-tolerant control strategy. Inspired by the work in [2] and [5], a nonsingular fast terminal fuzzy sliding mode control (NFFTSMC) is proposed for distributed cooperative attitude synchronization and precise 3-axis attitude tracking control of picosatellites in the presence of control input saturation, model uncertainties, and external disturbances. Also, the existing attitude synchronization or coordination literature addresses important issues relating to attitude control of multiple spacecraft, such as adapting the control system to modeling uncertainties, robust capability to attenuate external disturbances, communication delay, and actuator saturation constraint accommodation. Most traditional attitude fault-tolerant control reconfiguration techniques rely on failure detection and isolation. Reconfigurable attitude control system design for spacecraft based on FDI approaches and system redesign is fairly complex and depends strongly on the efficacy of the fault detection algorithm. Also, explicit failure identification and system redesign may require substantial on-board computational resources that works against the prevailing cost reduction measures undertaken by most commercial and military satellite markets. The problem of fault tolerance for attitude synchronization systems is still an open problem and there are a lot of theoretical challenges. An alternative approach to reconfigurable ACS design is based on adaptive or approximation control theory where the controller parameters are constantly updated based on adaptive or other estimates of the altered dynamics after failure. Neural network for adaptive approximation has been successfully applied to the design of nonlinear fault tolerant controllers [6]. In this paper, a fault-tolerant control strategy using adaptive nonsingular fast terminal fuzzy sliding mode control is proposed for multiple picosatellite attitude synchronization which is represented by quaternion. In order to achieving the capability of fault management without involvement of ground station operators, the proposed control laws do not require an explicit fault detection and isolation mechanism. Therefore, failure time instants, patterns, and values of actuator failures are unknown to the designer. When a reaction wheel failure or fault occurs, the control signals are autonomously redistributed to the remaining healthy wheels using an adaptive gain that is updated on-line based on the attitude tracking error. Each satellite with four reaction wheels is in a pyramid configuration. The numerical simulation results and a predicted theoretical

framework show that the proposed strategy can compensate for faults, the system continues to operate satisfactorily with initial conditions' uncertainties or wheel speeds faults, and the closed loop distributed tracking control system is stochastically stable.

A. Satellite With Four Reaction Wheels Attitude Dynamics and Kinematics Model

The individual satellites in the formation are modeled as rigid bodies [7]. The proposed system consists of four *rigid body satellites* in a circular orbit around the Earth. An *Earth centered inertial* (ECI) frame, denoted by $\mathbb{L}-X_I Y_I Z_I$, has its origin located at the center of the Earth, with Z_I -axis passing through the celestial North pole, X_I -axis directed towards the vernal equinox, and Y_I -axis completes the right-handed triad. Next, we define a local vertical local horizontal (LVLH) orbital reference frame $\mathcal{L}-x_o y_o z_o$ fixed at the center of the satellite with the x_o -axis along the direction of motion, the y_o -axis opposite to the direction of the angular velocity (normal to the orbit plane), and the z_o -axis pointing towards the Earth. The nodal line represents the reference line in orbit for the measurement of the true anomaly (eccentric orbit) or reference angle θ (circular orbit). The corresponding principal body-fixed coordinate axes of the satellite are denoted by $j-xyz$ with its origin located at the center of mass of the satellite. The notation [8] used for representing the relative velocity vector, $\mathbf{v}$, of frame $\mathcal{L}$ relative to frame $\mathbb{L}$, expressed in the coordinates of j is: $\mathbf{v}_{LL}^B$. The cross-product of vectors $a \times b$ is defined as $[a^\times]b$ where $[a^\times]$ is a skew-symmetric operator.

Consider the i 'th rigid satellite with a 4-wheel cluster installed to provide internal torques. The attitude dynamics of a rigid satellite controlled by reaction wheels can in general be described the following nonlinear differential equation [9]:

$$J_i \dot{\omega}_{iBI}^B = -\omega_{iBI}^B \times (J_{si} \omega_{iBI}^B + A_i J_{wi} \Omega_i) + A_i \tau_{rwi} + \tau_{ie} \quad (1)$$

The torques generated by the reaction wheels τ_{rwi} are given by

$$\tau_{rwi} = J_{wi}(\dot{\Omega}_i + A_i^T \dot{\omega}_{iBI}^B) \quad (2)$$

where ω_{iBI}^B is the angular velocity of the satellite relative to the inertial frame $\mathbb{L}$, expressed in the j frame, and $\tau_{ie} \in \mathbb{R}^{3 \times 1}$ represents the external torque acting on the system. A_i is the 3×4 matrix whose columns represent the influence of each reaction wheel on the angular acceleration of the satellite. We define a matrix $J_i = J_{si} - A_i J_{wi} A_i^T$, where $J_{si} \in \mathbb{R}^{3 \times 3}$ is the moment of inertia of the satellite, including the wheels, and $J_{wi} \in \mathbb{R}^{4 \times 4} = \text{diag}([J_{wi1}, J_{wi2}, J_{wi3}, J_{wi4}])$ denotes the axial moment of inertia of the reaction wheels. $\Omega_i \in \mathbb{R}^{4 \times 1}$ denotes the axial angular velocity of the reaction wheels.

The external torques (τ_{ie}) are assumed to include gravity gradient torque (τ_{gi}) acting on the satellite. The gravity gradient torque is given by [10]

$$\tau_{ie} = 3\omega_0^2 [\hat{c}_3^\times] J_{si} \hat{c}_3, \quad \hat{c}_3 = C_{iL}^B [0 \ 0 \ 1]^T \quad (3)$$

In this paper, we adopt the unit quaternion[7] to describe the attitude of the satellite. The unit quaternion $\bar{q}_i$ is defined

by

$$\bar{q}_i = \begin{bmatrix} \bar{e} \sin(\Phi/2) \\ \cos(\Phi/2) \end{bmatrix} = \begin{bmatrix} q_{vi} \\ q_{4i} \end{bmatrix} \quad (4)$$

where Φ denotes the principal angle, and $\bar{e} = [e_1, e_2, e_3]^T$ denotes the principal axis associated with Euler's Theorem ($e_1^2 + e_2^2 + e_3^2 = 1$). $q_{4i} \in \mathbb{R}$ and $q_{vi} \in \mathbb{R}^{3 \times 1} = [q_{i1}, q_{i2}, q_{i3}]^T$ denote the Euler parameters that represent the orientation of the satellite body frame, j , with respect to the orbital frame, $\mathcal{L}$, and satisfy the constraint $q_{vi}^T q_{vi} + q_{4i}^2 = 1$. The nonlinear differential equations governing the kinematics of the satellite in terms of Euler parameters can be expressed as

$$\begin{bmatrix} \dot{q}_{vi} \\ \dot{q}_{4i} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q_{4i} I + [q_{vi}^\times] \\ -q_{vi}^T \end{bmatrix} \omega_{iBL}^B \quad (5)$$

where $I \in \mathbb{R}^{3 \times 3}$ represents the identity matrix, and $[q_{vi}^\times]$ denotes a skew-symmetric matrix which is given by

$$[q_{vi}^\times] = \begin{bmatrix} 0 & -q_{i3} & q_{i2} \\ q_{i3} & 0 & -q_{i1} \\ -q_{i2} & q_{i1} & 0 \end{bmatrix} \quad (6)$$

The angular velocity of the body-fixed reference frame, j , with respect to the inertial frame, $\mathbb{L}$, can be expressed as

$$\omega_{iBI}^B = \omega_{iBL}^B + \omega_{iLI}^B \quad (7)$$

The direction cosine matrix C_{iL}^B that describes the orientation of the satellite body-fixed frame, j , with respect to the LVLH reference frame, $\mathcal{L}$ defined in terms of Euler parameters is given by

$$C_{iL}^B = (q_{4i}^2 - q_{vi}^T q_{vi})I + 2q_{vi} q_{vi}^T - 2q_{4i} [q_{vi}^\times] \quad (8)$$

The orbital angular velocity expressed in the j frame, ω_{iLI}^B , can be obtained by rotating the angular velocity in $\mathcal{L}$ using the transformation matrix C_{iL}^B as follows:

$$\omega_{iLI}^B = C_{iL}^B \begin{bmatrix} 0 & -\omega_{i0} & 0 \end{bmatrix}^T \quad (9)$$

where $\omega_{i0} = \dot{\theta}$ is the magnitude of orbital angular velocity of the LVLH frame. For a circular orbit, $\dot{\theta} = \sqrt{\mu_e/R_c^3}$, where μ represents the gravitational parameter of the Earth, and R_c defines the distance of the spacecraft measured from the center of the Earth.

Reaction wheels are widely used to perform precise satellites attitude maneuvers because they allow continuous and smooth control. They are capable of changing its internal angular momentum vector as each wheel spins about a body-fixed axis with variable spin speed [10]. Torques are produced on the satellites by accelerating or decelerating the reaction wheels. Let the torque demanded by the satellite be denoted as τ_{rwi} . The input voltage required to control the actuator dynamics of the reaction wheel can be shown as

$$e_{ai} = K_{bi} \Omega_i - R_{bi} K_{ti}^{-1} (A_i^T \tau_{rwi}) \quad (10)$$

where $K_{ti} \in \mathbb{R}^{4 \times 4} = \text{diag}([k_{t1}, k_{t2}, k_{t3}, k_{t4}])$ is the motor torque constant. $K_{bi} \in \mathbb{R}^{4 \times 4} = \text{diag}([k_{b1}, k_{b2}, k_{b3}, k_{b4}])$ is the back-EMF constant, $R_{bi} \in \mathbb{R}^{4 \times 4} = \text{diag}([r_{b1}, r_{b2}, r_{b3}, r_{b4}])$ is the armature resistance, and we ignore the friction in the reaction wheels.

B. Station Keeping Attitude Tracking and Formation Keeping Attitude Tracking

In the case of tracking a desired rotational motion, the target attitude of the i 'th satellite in the desired reference frame j_o with respect to the LVLH frame $\mathcal{L}$ is described by the orientation $(q_{dvi}, q_{d4i}) \in \mathbb{R}^3 \times \mathbb{R}$ that satisfies the constraint $q_{dvi}^T q_{dvi} + q_{d4i}^2 = 1$. Let $\omega_{di} \in \mathbb{R}^3$ denote the angular velocity of j_o with respect to $\mathcal{L}$, expressed in the frame j_o . To address the station-keeping attitude tracking problem, we define the quaternion absolute tracking error $(q_{ie}, q_{i4e}) \in \mathbb{R}^3 \times \mathbb{R}$ as the relative orientation between the body-fixed frame j and the desired reference frame j_o , which is computed as

$$\begin{aligned} q_{ie} &= q_{d4i} q_{vi} - q_{4i} q_{dvi} + [q_{vi}^\times] q_{dvi} \\ q_{i4e} &= q_{d4i} q_{4i} + q_{dvi}^T q_{vi} \end{aligned} \quad (11)$$

where $q_{ie}^T q_{ie} + q_{i4e}^2 = 1$. The corresponding rotation matrix $C_{ie} = C_i(q_{ie}, q_{i4e}) \in SO(3)$ is given by

$$C_{ie} = (q_{i4e}^2 - q_{ie}^T q_{ie})I + 2q_{ie} q_{ie}^T - 2q_{i4e} [q_{ie}^\times] \quad (12)$$

where $C_{ie}^T C_{ie} = I$, $\|C_{ie}\| = 1$, $\det(C_{ie}) = 1$, and $\dot{C}_{ie} = -[\omega_{ie}^\times] C_{ie}$. Next, we define the absolute angular velocity tracking error $\omega_{ie} \in \mathbb{R}^3$ of j with respect to $\mathcal{L}$ as follows:

$$\omega_{ie} = \omega_{iBL}^B - C_{ie} \omega_{id} \quad (13)$$

From Eqs. (5), (11), and (13), the equations that govern the absolute attitude (station-keeping) error dynamics and kinematics are given by

$$\dot{q}_{ie} = \frac{1}{2}(q_{i4e}I + [q_{ie}^\times])\omega_{ie} \quad (14)$$

$$\dot{q}_{i4e} = -\frac{1}{2}q_{ie}^T \omega_{ie} \quad (15)$$

Assumption 1: We assume that there exists known, finite constant, $c_1 > 0$ such that $\|q_{di}\|^2 + \|\dot{q}_{di}\|^2 + \|\ddot{q}_{di}\|^2 \leq c_1$ for all $t \geq 0$.

We choose $\bar{q}_{ij}$ and ω_{ij} to represent the quaternion mapping of the i 'th satellite body-fixed coordinate with respect to the j 'th satellite body-fixed coordinate and the angular velocity of the i 'th satellite with respect to the j 'th satellite [11]

$$\bar{q}_{ij} = [q_{ij}^T q_4]^T = \bar{q}_i \otimes \bar{q}_j^{-1} \quad (16)$$

$$q_{vij} = q_{vi} - C_{ij} q_{vj} \quad (17)$$

$$\omega_{ij} = \omega_i - C_{ij} \omega_j \quad (18)$$

where C_{ij} is the rotation matrix from the i 'th body reference frame to the j 'th body reference frame, which is a proper orthogonal matrix. q_{vij} is defined as the formation flying attitude error.

$$C_{ij} = C_{ij}(\bar{q}_i \otimes \bar{q}_j^{-1}) = C_i(\bar{q}_i) C_j(\bar{q}_j)^T \quad (19)$$

According to Eq. (8),

$$C_i(\bar{q}_i) = (q_{4i}^2 - q_{vi}^T q_{vi})I + 2q_{vi} q_{vi}^T - 2q_{4i} [q_{vi}^\times] \quad (20)$$

The quaternion (q_{vij}) differential equations are obtained by the reference.

$$\dot{q}_{vij} = \frac{1}{2}(q_{ij4} \omega_{ij} - \omega_{ij} \otimes q_{ij}) \quad (21)$$

C. Consensus Tracking

A weighted graph $G = (V, E, \bar{A})$ consists of a node set V , an edge set E and a weighted adjacency matrix $\bar{A}$. It is natural to describe the topology of the information flow between satellites in the formation by a graph. The weighted adjacency matrix $A = a_{ij} \in \mathbb{R}^{n \times n}$ is the weighted adjacency matrix of graph G with nonnegative elements. Node v_i represents the i 'th satellite, and an edge in G is denoted by an unordered pair (ij) . $ij \in E \Leftrightarrow ji \in E$. The adjacency element a_{ij} denotes the communication quality between the i 'th satellite and the j 'th satellite. It is assumed that $a_{ij} = a_{ji}$ and $\bar{A}$ is a symmetric matrix.

Let $D = \text{diag}(d_1, d_2, \dots, d_n)$ be the degree matrix of G . The Laplacian matrix of L of the weighted graph G is defined by

$$L = D - \bar{A} \quad (22)$$

It is assumed that the multi-satellites formation flying studied in this paper has a connected graph topology G . The theorem [12] gives the existing results of the Laplacian matrix and graph theory.

Theorem 1: If $G = (V, E, \bar{A})$ is an undirected connected graph, then the graph Laplacian matrix L is a symmetric matrix with n real eigenvalues in an ascending order as:

$$0 = \lambda_1, \lambda_2 \leq \lambda_3 \leq \dots \leq \lambda_n \leq 2d_M \quad (23)$$

where $d_M = \max_{1 \leq i \leq n} \{d_i\}$ is the maximum degree of a graph, and λ_2 is called the algebraic connectivity of a graph.

Let $a_{i1} \in \mathbb{R}^3$ and $a_{i2} \in \mathbb{R}^3$ be the consensus attitude error including the station-keeping and formation-keeping errors of the i 'th satellite in the formation, which are defined as:

$$a_{i1} = k_p \sum_{j=1, j \neq i}^n a_{ij} q_{vij} + k_i q_{ie} \quad (24)$$

$$a_{i2} = k_p \sum_{j=1, j \neq i}^n a_{ij} \dot{q}_{vij} + k_i \dot{q}_{ie} \quad (25)$$

where $i = 1, 2, \dots, n$. a_{ij} is the element of the weighted adjacency matrix A . k_p and k_i are positive constants determined by the controller designer, respectively.

The consensus error is used to evaluate the motion synchronization performance. Eqs. (24), (25) have to be written as follows:

$$a_{i1} = k_p \sum_{j=1}^n l_{ij} q_{je} + k_i q_{ie} \quad (26)$$

$$a_{i2} = k_p \sum_{j=1}^n l_{ij} \dot{q}_{je} + k_i \dot{q}_{ie} \quad (27)$$

Define $A_1 = (a_{11}^T, a_{21}^T, \dots, a_{n1}^T)^T$, $A_2 = (a_{12}^T, a_{22}^T, \dots, a_{n2}^T)^T$, $E_1 = (q_{e11}^T, q_{e21}^T, \dots, q_{en1}^T)^T$, $E_2 = (\dot{q}_{e11}^T, \dot{q}_{e21}^T, \dots, \dot{q}_{en1}^T)^T$, and $K = \text{diag}(k_1, k_2, \dots, k_n)$. The Eqs. (26), (27) can be expressed in terms of the aforementioned quantities as follows:

$$A_1 = (k_p L \otimes I_3 + K \otimes I_3) E_1 \quad (28)$$

$$A_2 = (k_p L \otimes I_3 + K \otimes I_3) E_2 \quad (29)$$

Lemma 1: The matrix $M_1 = (k_p L \otimes I_3 + K \otimes I_3)$ is symmetric and positive definite.

$$\lambda_m \|x\|^2 \leq x^T ((k_p L \otimes I_3 + K \otimes I_3)) x \leq \lambda_M \|x\|^2, \forall x \in \mathbb{R}^{mn} \quad (30)$$

where $\lambda_m = \min_{1 \leq i \leq n} (k_i)$ and $\lambda_M = k_p \lambda_n + \max_{1 \leq i \leq n} (k_i)$ are positive constants.

The objective of control design becomes to design appropriate control input that guarantees the convergence of both the station-keeping q_{ie} and formation-keeping q_{vij} tracking error and consensus synchronization error a_{i1} and a_{i2} , even in the presence of unknown constant parameters, periodic disturbances, and actuator faults. In short, one needs to achieve $q_{ie}, q_{vij} \rightarrow 0$ and $a_{i1} \rightarrow 0$ as $t \rightarrow \infty$.

D. Finite Time Stability

The finite time convergence means that the system will converge on the sliding surface. On the sliding surface, the system is totally determined by a reduced-order system determined by the designer. Without the finite time convergence, the system moves in a vicinity of the sliding surface, where the system may converge slowly or even may not converge, determined by the sliding surface, the whole system dynamics including disturbances and uncertainties. Besides, with the finite time convergence, the matched disturbances and/or uncertainties can be rejected completely.

Lemma 2: [13] Assume there is a continuous positive definite function $V(t)$ satisfying the following differential inequality:

$$\dot{V} \leq -\gamma_1 V^{\bar{\rho}} \quad \forall t \geq t_0, V(t_0) \geq 0, \quad (31)$$

where $\gamma_1 > 0$ and $0 < \bar{\rho} < 1$ are positive constants. Then, for any given t_0 , $V(t)$ satisfies the following inequality:

$$V^{1-\bar{\rho}}(t) \leq V^{1-\bar{\rho}}(t_0) - \gamma_1(1-\bar{\rho})(t-t_0), t_0 \leq t \leq t_1 \quad (32)$$

and $V(t) \equiv 0, \forall t \geq t_1$ with $t_1 = t_0 + \frac{V^{1-\bar{\rho}}(t_0)}{\gamma_1(1-\bar{\rho})}$.

E. Fuzzy Logic System

Fuzzy logic system is a collection of IF-THEN fuzzy rules as:

$$R^k: \text{ IF } x_1 \text{ is } A_1^k, \text{ and } \dots, x_n \text{ is } A_n^k, \quad (33) \\ \text{ THEN } y \text{ is } B^k.$$

The output of the fuzzy system (using singleton fuzzification, product inference and center average defuzzification) can be written as

$$y_{out} = \frac{\sum_{l=1}^P \theta_F^l(X) \prod_{i=1}^N \mu_{A_i^l}(x_i)}{\sum_{l=1}^P \prod_{i=1}^N \mu_{A_i^l}(x_i)} = \theta^T \xi \quad (34)$$

where P is the total fuzzy rules number. The membership functions $\mu_{A_1^l}(x_1), \dots, \mu_{A_N^l}(x_n)$ (N is the number of membership functions) are Gaussian functions, ξ is fuzzy basis

function $\left(\xi^l(x) = \frac{\prod_{i=1}^N \mu_{A_i^l}(x_i)}{\sum_{l=1}^P \prod_{i=1}^N \mu_{A_i^l}(x_i)} \right)$. In this paper, $N = 7$. Therefore, the membership functions can be chosen as linguistic terms Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (ZE), Positive Small (PS), Positive Medium (PM) and Positive Big (PB), that are assigned to the inputs (x_1 to x_n) and the output ($\hat{F}$).

Therefore, the continuous nonlinear function $F_i(x)$ of each satellite can be approximated by the fuzzy logic system [14].

$$\hat{F}_i(x) = \theta_i^T \xi_i(x) \quad (35)$$

where ξ_F is the vector of the fuzzy basis function and is supposed to be fixed, while the parameters $\theta_{F_i}^T$ are the variable which will be designed by adaptive laws. It is assumed that there exists an optimal fuzzy logic system to learn the nonlinear terms $F_i(x)$ such that

$$F_i(x) - \hat{F}_i^*(x) = \varepsilon \quad (36)$$

where ε is the approximation error and is bounded. Approximation errors can be reduced by increasing the number of fuzzy rules.

In order to decrease the fuzzy rules, we use the nonsingular fast terminal sliding surface S instead of the state x for the adaptive parameters. Then, the adaptive parameters θ_F are updated by the following adaptive laws:

$$\dot{\theta}_i = \alpha_i M_2 \sigma_2 p / q \dot{A}_1^{p/q-1} S \xi_i \quad (37)$$

$$\xi_i(s_i) = \frac{\prod_{i=1}^N \mu_{A_i^l}(s_i)}{\sum_{l=1}^P \prod_{i=1}^N \mu_{A_i^l}(s_i)} \quad (38)$$

where α_i is a positive constant.

In order to guarantee that the estimated Fuzzy Logic System parameters remain within some known bounded sets, the smooth projection is considered in this paper. Let θ_i be the estimation of the optimal weight matrix θ_i^* , and define a smooth projection of θ_i as:

$$\pi_i(\theta_i) = \theta_{i\pi} = \pi_{i,jk}(\theta_{i,jk}), j = 1, 2, \dots, m, k = 1, 2, \dots, N_1 \quad (39)$$

Each projection operator $\pi_{i,jk}$ is a real-value smooth nondecreasing function defined by

$$\pi_{i,jk}(\theta_{i,jk}) = \theta_{i,jk} \quad (40)$$

Define $\tilde{\theta}_i = \theta_i^* - \theta_i$, $\tilde{\theta}_{i\pi} = \theta_i^* - \theta_{i\pi}$, and

$$V_{\theta_i} = \frac{1}{\delta_i} \sum_{j=1}^m \sum_{k=1}^{N_1} \int_0^{\tilde{\theta}_i} (\theta_{i,jk}^* - \pi_{i,jk}(\theta_{i,jk}^* - \omega_{i,jk})) d\omega_{i,jk} \quad (41)$$

where, V_{θ_i} is positive definite with respect to $\tilde{\theta}_{i,jk}$ for $\theta_{i,jk}^* \in [\theta_{imin}, \theta_{imax}]$. Furthermore,

$$\dot{V}_{\theta_i} = -\frac{1}{\delta_i} \sum_{j=1}^m \sum_{k=1}^{N_1} \tilde{\theta}_{i\pi,jk} \dot{\theta}_{i,jk}^*, i = 1, 2, \dots, n \quad (42)$$

Assumption 2: ε is bounded such that $\|\varepsilon_i\| \leq \varepsilon_{Mi}$ ($i = 1, 2, \dots, n$), where ε_{Mi} is a positive constant.

Assumption 3: The optimal weight matrix $\theta_i^* (i = 1, 2, \dots, n)$ lies in a known bounded set ϱ_{Wi} defined by

$$\theta_i^* \in \varrho_{Wi} = \{\theta_i^* : \theta_{imin}^* \leq \theta_{i,jk}^* \leq \theta_{imax}^*\} \quad (43)$$

where θ_{imin}^* and θ_{imax}^* are known constants, respectively.

F. Mathematical Model for Controller Design

To employ the proposed control design methodology described in this paper, we need to construct a model to represent the nonlinear equations Eq. (1).

Denoting $p = 2[q_{4i}I + (q_{vi}^\times)]^{-1}$, after taking the time derivative of Eq. (5), we obtain the following expression of $\dot{\omega}_{iBL}^B$:

$$\dot{\omega}_{iBL}^B = \dot{p}\dot{q}_{vi} + p\ddot{q}_{vi} \quad (44)$$

where $T = [q_{4i}I + (q_{vi}^\times)]^{-1}$.

Making use of Eq. (5) and Eq. (44), the quaternion equation can be written as:

$$\ddot{q}_{vi} = 2(T^{-1})^T (\dot{\omega}_{iBL}^B - \dot{\omega}_{iLI}^B) - 4(T^{-1})^T (\dot{T}^{-1})\dot{q}_{vi} \quad (45)$$

Since T is a skew-symmetric matrix, we derive $\dot{T}^{-1} = -T^{-1}\dot{T}T^{-1}$.

Substituting the Eqs. (1) and (9) into the Eq. (45), we obtain:

$$\begin{aligned} \ddot{q}_{vi} = 2(T^{-1})^T [J_i^{-1}(-\omega_{iBL}^B \times (J_{si}\omega_{iBL}^B + A_iJ_{wi}\Omega_i) \\ + A_i\tau_{rwi} + \tau_{ie}) - \dot{\omega}_{iLI}^B] - 4(T^{-1})^T (\dot{T}^{-1})\dot{q}_{vi} \end{aligned} \quad (46)$$

Using $\dot{C}_L^B = -[\omega_{BL}^B]^\times C_L^B$, we have:

$$\dot{\omega}_{iLI}^B = -\omega_{iLI}^B \times \omega_{iLI}^B \quad (47)$$

The dynamics equation with actuator fault can be written as:

$$\ddot{q}_{vi} = f_i(q_{ie}, \dot{q}_{ie}) + (\tau_i + \Delta\tau_i) + \nu_i \quad (48)$$

A dynamic model of the satellite's relative attitude tracking with multiplicative reaction wheel faults (e.g. reduced control torque) is represented by a diagonal matrix E .

$$\begin{aligned} f_i(q_{ie}, \dot{q}_{ie}) = 2(T^{-1})^T [J_i^{-1}(-\omega_{iBL}^B \times (J_{si}\omega_{iBL}^B + A_iJ_{wi}\Omega_i)) \\ - 4(T^{-1})^T (\dot{T}^{-1})\dot{q}_{vi} + 2(T^{-1})^T J_i^{-1}\omega_{iLI}^B \times \omega_{iLI}^B] \end{aligned} \quad (49)$$

$$\tau_i = E[2(T^{-1})^T J_i^{-1}A_i\tau_{rwi}] \quad (50)$$

$$\nu_i = 2(T^{-1})^T J_i^{-1}\tau_{ie} \quad (51)$$

Assumption 4: The disturbance ν_i is bounded such that $\|\nu_i\| \leq \nu_{Mi} (i = 1, 2, \dots, n)$, where ν_{Mi} is a positive constant.

Assumption 5: The fault $\Delta\tau_i$ is bounded such that $\|\Delta\tau_i\| \leq \Delta\tau_{Mi} (i = 1, 2, \dots, n)$, where τ_{Mi} is a positive constant.

II. DESIGN OF NONSINGULAR FAST FUZZY TERMINAL SLIDING MODE CONTROL

In this section we present the theoretical basis for developing NFFTSMC control algorithms for the four rigid body satellites attitude consensus tracking. The controller is decentralized as each controller does not require the decision from the other controllers. For each satellite, the NFFTSMC control overall design can be divided into two main steps.

Step 1 involves the construction of a sliding surface containing absolute attitude and relative attitude tracking errors to ensure that once the system is restricted to the sliding surface, the satellite can be expected to be in the desired trajectory. Step 2 entails the derivation of parameter adaptation laws and fuzzy logic feedback control gains that can drive the relative state trajectory to the sliding surface, and maintain it in the manifold.

A. Nonsingular Fast Terminal Sliding Mode

The nonsingular fast terminal sliding mode can be defined by the following nonlinear differential equation:

$$s = x + \sigma_1 \text{sign}(x) + \sigma_2 \dot{x}^{\frac{p}{q}} \quad (52)$$

where $x \in \mathbb{R}$ and σ_1, σ_2 are positive constants. The positive odd integers p, q are chosen such that $1 < q/p < 2$.

We define the fast terminal sliding mode $s_i \in \mathbb{R}^m$ for the i 'th satellite in the formation as:

$$s_i = a_{i2} + \beta_i a_{i1} = \Lambda_i + \beta_i \quad (53)$$

$$\Lambda_i(a_{i1}) = a_{i1} + \sigma_{i1} \text{sign}(a_{i1})$$

$$\beta_i(a_{i2}) = \sigma_{i2} a_{i2}^{\frac{p}{q}} \quad (54)$$

where $i = 1, 2, \dots, n$. $\Lambda_i(a_{i1})$ and $\beta_i(a_{i2}) \in \mathbb{R}^m$. σ_{i1} and σ_{i2} are positive constants for the designer.

Defining $\beta = (\beta_1^T, \beta_2^T, \dots, \beta_n^T)^T$ and $s = (s_1^T, s_2^T, \dots, s_n^T)^T$, we have

$$s = \Lambda + \beta \quad (55)$$

The time derivative of the sliding mode Eq. (55) is given by

$$\dot{s} = \bar{\Lambda} + \bar{\beta} \quad (56)$$

B. Nonsingular Fast Fuzzy Terminal Sliding Mode Controller Design

The controller design for the i 'th satellite in the formation is now given by

$$\tau_{rwi} = \bar{\tau}_i = \ddot{q}_d - K_{i1}s_i - K_{i2}s_i^\rho - \frac{q}{p\sigma_2}a_{i2}^{2-p/q} - \theta_i^T \xi_i - \varphi_i \quad (57)$$

where K_{i1} and K_{i2} are designed parameters. with φ_i defined by [15]

$$\varphi_i \equiv \kappa_i \tanh\left(\frac{3K_u \kappa_i s_i}{\epsilon}\right) \quad (58)$$

The controller consists three components: the nonlinear compensator $\ddot{q}_d$ is applied to compensate for the desired satellite dynamics, motivated by the fact that the attitude of each satellite in the formation reaches the desired trajectory of the formation, then station-keeping errors and formation-keeping errors will be zero. The nonlinear feedback $K_{i1}s_i + K_{i2}s_i^\rho + \frac{q}{p\sigma_2}a_{i2}^{2-p/q}$ is used to drive the satellite attitude to the sliding surface in finite time. The robust term $\varphi_i + \theta_i^T \xi_i$ is used to counteract the external disturbance and unknown system uncertainties due to the actuator faults. κ_i is a positive constant

satisfying $\varphi_i \geq d_{Mi}$. ϵ is a positive scalar. K_{i1} and K_{i2} are positive definite, diagonal and constant matrices. ρ is p/q . The proposed controller can provide finite time stability in both the reaching phase and the sliding phase, which is guaranteed by the Lyapunov theory described in the next subsection.

C. Stability Analysis

Theorem 2: Consider the system Eq. (48) and Assumption 1 ~ 5, the nonsingular fast fuzzy terminal sliding control laws defined by Eqs. (57-58), with adaptive control laws given by Eq. (37) guarantees the state of the system starting from any initial point will move toward the switching surface and reach the sliding surface $s = 0$ in a finite time. Furthermore, the aggregated attitude tracking errors $a_{i1,j}$ and $a_{i2,j}$ ($i = 1, 2, \dots, n, j=1,2,3$) will converge to the regions in finite time, respectively, and the station-keeping attitude tracking errors will converge to the regions in finite time, respectively.

Proof of Theorem 2: The dynamics equations for the absolute errors of the i 'th satellite can be written as:

$$q_{ie} = e_{1i} \quad \dot{q}_{ie} = e_{2i} \quad (59)$$

$$\dot{e}_{i1} = e_{i2} \quad \dot{e}_{i2} = -\ddot{x}_{id} + f_i(q_{ie}, \dot{q}_{ie}) + \tau_i + \Delta\tau_i + \nu_i \quad (60)$$

Define $E_1 = (e_{11}^T, e_{21}^T, \dots, e_{n1}^T)$, $E_2 = (e_{12}^T, e_{22}^T, \dots, e_{n2}^T)$, $Q_d = (\ddot{q}_1^T, \ddot{q}_2^T, \dots, \ddot{q}_n^T)$, $F = (f_1^T, f_2^T, \dots, f_n^T)$, $\bar{\tau} = (\bar{\tau}_1^T, \bar{\tau}_2^T, \dots, \bar{\tau}_n^T)$, $\Delta\tau = (\Delta\tau_{i1}^T, \Delta\tau_{i2}^T, \dots, \Delta\tau_{in}^T)$ and $\nu = (\nu_1^T, \nu_2^T, \dots, \nu_n^T)$. The Eqs. (59) can be expressed in terms of the aforementioned quantities as follows:

$$\begin{aligned} \dot{E}_1 &= E_2 \\ \dot{E}_2 &= -Q_d + F + \tau + \Delta\tau + \nu \end{aligned} \quad (61)$$

Denote $M_1 = (k_p L + K) \otimes I_3$. Eqs. (28) and (29) can be expressed in terms of the aforementioned quantities as follows:

$$\begin{aligned} A_1 &= M_1 E_1 \\ A_2 &= M_1 E_2 \end{aligned} \quad (62)$$

The dynamics equations for A_1 and A_2 are defined as:

$$\begin{aligned} \dot{A}_1 &= A_2 \\ M_2 \dot{A}_2 &= -Q_d + F + \tau + \Delta\tau + \nu \end{aligned} \quad (63)$$

respectively, where $M_2 = M_1^{-1}$.

Using Eqs. (56) and (63), we obtained

$$\begin{aligned} M_2 \dot{s} &= M_2 \bar{\Lambda} + M_2 \bar{\beta} = M_2 \dot{A}_1 \\ &+ M_2 \sigma_2 p / q \dot{A}_1^{p/q-1} (-Q_d + F + \tau + \Delta\tau + \nu) \end{aligned} \quad (64)$$

Using the approximation property of Fuzzy Logic Systems, the unknown function f_i ($i = 1, 2, \dots, n$) can be approximated over the compact set Ω_d by Eqs. (35) and (37). Substituting

the control law Eq. (57) into the error dynamics Eq. (64) yields the following dynamic equation for sliding surface s .

$$\begin{aligned} M_2 \dot{s} &= M_2 \bar{\Lambda} + M_2 \bar{\beta} \\ &= M_2 \dot{A}_1 + M_2 \sigma_2 p / q \dot{A}_1^{p/q-1} (-Q_d + F - F_d + F_d \\ &\quad - \theta^T \xi - K_1 s - K_2 s^\rho \frac{q}{p \sigma_2} \dot{A}_1^{2-p/q} - \varphi + \nu + \Delta\tau) \\ &= M_2 \sigma_2 p / q \dot{A}_1^{p/q-1} (\varsigma + \tilde{\theta}^T \xi - K_1 s - K_2 s^\rho \\ &\quad - \varphi + \nu + \varepsilon + \Delta\tau) \end{aligned} \quad (65)$$

where $\varsigma = -Q_d + F - F_d$ and $K_1 = (K_{11}^T, K_{21}^T, \dots, K_{n1}^T)$, $K_2 = (K_{12}^T, K_{22}^T, \dots, K_{n2}^T)$, $\varphi = (\varphi_1^T, \varphi_2^T, \dots, \varphi_n^T)$, $\varepsilon = (\varepsilon_1^T, \varepsilon_2^T, \dots, \varepsilon_n^T)$, $\theta^* = \text{diag}(\theta_1^{*T}, \theta_2^{*T}, \dots, \theta_n^{*T})$, $\theta = \text{diag}(\theta_{1\pi}^T, \theta_{2\pi}^T, \dots, \theta_{n\pi}^T)$, $\theta_\pi^* = \theta^* - \theta_\pi$, and $\xi = (\xi_1^T, \xi_2^T, \dots, \xi_n^T)$.

Consider the following Lyapunov function candidate

$$V = \frac{1}{2} s^T M_2 s + \sum_{i=1}^n V_{\theta_i} \quad (66)$$

Thus, the time derivative of V is obtained as

$$\dot{V} = s^T M_2 \dot{s} + \sum_{i=1}^n \dot{V}_{\theta_i} \quad (67)$$

Due to the fact that

$$s^T M_2 \sigma_2 p / q \dot{A}_1^{p/q-1} \tilde{\theta}^T \xi - \frac{1}{\delta_i} \sum_{j=1}^m \sum_{k=1}^{N_1} \tilde{\theta}_{F_{i\pi,jk}} \dot{\theta}_{i,jk}^* = 0 \quad (68)$$

Substituting the Eqs. (42) and (65) to the Eq. (67), we obtain:

$$\begin{aligned} \dot{V} &= M_2 \sigma_2 p / q \dot{A}_1^{p/q-1} (s^T \varsigma + s^T \tilde{\theta}^T \xi - s^T K_1 s - s^T K_2 s^\rho - s^T \varphi \\ &\quad + s^T (\nu + \varepsilon + \Delta\tau) - \frac{1}{\delta_i} \sum_{j=1}^m \sum_{k=1}^{N_1} \tilde{\theta}_{F_{i\pi,jk}} \dot{\theta}_{i,jk}^*) \\ &= M_2 \sigma_2 p / q \dot{A}_1^{p/q-1} (s^T \varsigma - s^T K_1 s - s^T K_2 s^\rho - s^T \varphi \\ &\quad + s^T (\nu + \varepsilon + \Delta\tau)) \end{aligned} \quad (69)$$

Using the following inequalities

$$0 \leq -s^T \varphi + s^T (\nu + \varepsilon + \Delta\tau) \leq n\epsilon \quad (70)$$

$$0 \leq s^T \varsigma \leq c_5 s^T s + \frac{\varsigma^2 M}{4c_5} \quad (71)$$

where c_5 is a positive constant satisfying $c_5 < \lambda_{\min}(K_1)$ with $\lambda_{\min}(K_1)$ being the minimum eigenvalue of the matrix K_1 , Eq. (69) becomes

$$\begin{aligned} \dot{V} &\leq M_2 \sigma_2 p / q \dot{A}_1^{p/q-1} (-\lambda_{\min}(K_1) - c_5) s^T s + c_4 - s^T K_2 s^\rho \\ &\leq -M_2 \sigma_2 p / q \dot{A}_1^{p/q-1} |s|^2 \end{aligned} \quad (72)$$

where $c_4 = n\epsilon + \frac{s^2 M}{4c_5}$ is a positive constant.

$$\begin{aligned}
-s^T K_2 s^\rho &\leq -\lambda_{\min}(K_2) \sum_{i=1}^n \sum_{j=1}^m s_{ij}^{\rho+1} \\
&\leq -\lambda_{\min}(K_2) \left(\frac{2}{\lambda_{\max}} \right)^{(\rho+1)/2} \left(\sum_{i=1}^n \sum_{j=1}^m \frac{\lambda_{\max}}{2} s_{ij}^2 \right)^{(\rho+1)/2} \\
&\leq -\lambda_{\min}(K_2) \left(\frac{2}{\lambda_{\max}} \right)^{(\rho+1)/2} (0.5 s^T M_2 s)^{(\rho+1)/2} \\
&= -\gamma_2 V^{\rho+1}/2
\end{aligned} \tag{73}$$

where $\gamma_2 = \lambda_{\min}(K_2)(2/\lambda_{\max}(M_2))^{(\rho+1)/2}$. The Eq. (72) can be expressed as:

$$\dot{V} \leq -\gamma_1 V - \gamma_2 V^{(\rho+1)/2} + c_4 \tag{74}$$

where $\gamma_1 = 2(\lambda_{\min}(K_1) - c_5)/\lambda_{\max}(M_2)$.

Then Eq. (74) can be written as the following two forms:

$$\dot{V} \leq -\gamma_1 V - (\gamma_2 - \frac{c_4}{V^{(\rho+1)/2}}) V^{(\rho+1)/2} \tag{75}$$

From Eq. (75), if $\gamma_1 - c_4/V > 0$, then the finite-time stability is still guaranteed. According to the reference [16], the sliding surface s will reach the region $\|s\| \leq \Delta$ in finite time, which means $s_{ij} \leq \Delta$ $i = 1, 2, \dots, n, j = 1, 2, \dots, m, i.e..$ All signals in the closed-loop system are uniformly ultimately bounded.

III. RESULTS AND DISCUSSION

To study the effectiveness and performance of the proposed controller, the detailed response is numerically simulated using the set of governing system equations of motion Eq. (48) in conjunction with the control laws Eqs. (57-58).

The information exchange among satellite is represented by a graph with the weighted adjacency matrix $\bar{A}$ defined by

$$\bar{A} = \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ 0.5 & 0 & 0.6 & 0 \\ 0 & 0.6 & 0 & 0.7 \\ 0 & 0 & 0.7 & 0 \end{bmatrix}$$

Seven Gaussian membership functions are defined for each

variable $\tilde{A}_i, i = 1, 2, \dots, 7$ as:

$$\begin{aligned}
\mu_{\tilde{A}_1}(s_i) &= \frac{1}{1 + \exp(5(s_i + 0.9 \times a))}, \\
\mu_{\tilde{A}_2}(s_i) &= \exp\left(-\left(\frac{s_i + 0.6 \times a}{b}\right)^2\right), \\
\mu_{\tilde{A}_3}(s_i) &= \exp\left(-\left(\frac{s_i + 0.3 \times a}{b}\right)^2\right), \\
\mu_{\tilde{A}_4}(s_i) &= \exp\left(-\left(\frac{s_i}{b}\right)^2\right), \\
\mu_{\tilde{A}_5}(s_i) &= \exp\left(-\left(\frac{s_i - 0.3 \times a}{b}\right)^2\right), \\
\mu_{\tilde{A}_6}(s_i) &= \exp\left(-\left(\frac{s_i - 0.6 \times a}{b}\right)^2\right), \\
\mu_{\tilde{A}_7}(s_i) &= \frac{1}{1 + \exp(5(s_i - 0.9 \times a))}
\end{aligned} \tag{76}$$

where a and b are different constant numbers designed according to s_i . In this paper, $a = b = 1$.

In this section, we present the simulation results for the following case I: Desired Attitude Trajectory is $q_d = [0, 0.05\cos(0.02t), 0.05\sin(0.02t)]^T$. For reaction wheel pyramid configuration, with desired trajectory q_d , after $t = 50s$, the wheels of satellites *three* and *four* only supply 40 and 50 of the designed speed. After $t = 70s$ another fault of satellite *one* occurs: three wheels of satellite one only supply 70 of their design speed and the second wheel's input voltage is zero. There is no fault within satellite *two*.

To compare the control performances of the nonsingular fast fuzzy terminal sliding control laws defined by Eqs. (57-58), a nonsingular fast terminal sliding mode controller $\hat{\tau}_{rwi}$ is designed in Eq.77 as following:

$$\hat{\tau}_{rwi} = \ddot{q}_d - K_{i1}s_i - K_{i2}s_i^\rho - \frac{q}{p\sigma_2}a_{i2}^{2-p/q} - \varphi_i \tag{77}$$

Figs. 1 shows the station-keeping attitude tracking errors by using quaternion $e_i (i = 1, 2, 3, 4)$, respectively. The formation-keeping attitude error case is given in Fig. 2. It is found that the proposed controller can still drive the station-keeping attitude tracking errors and formation-keeping to the neighborhood around zero in finite time even when the faults happened. Therefore, the proposed controller can still provide finite-time control for a group of satellite under actuation faults as in the previous case. Without the Fuzzy Logic System, the station keeping and formation keeping tracking errors can not drive the system back to the neighborhood around zero in finite time.

IV. CONCLUSIONS

The primary contribution of this paper is the development and analysis of a decentralized, attitude-coordinated control law that provides finite-time stability and convergence of attitude synchronization. The simulation results clearly establish

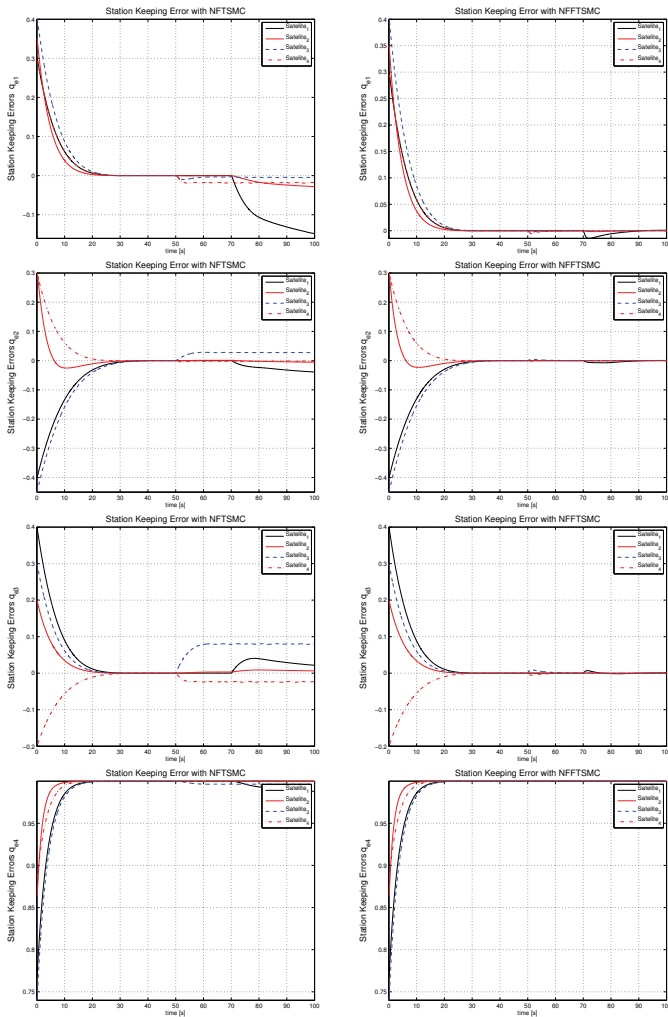


Fig. 1. Station-keeping Tracking Errors using Quaternion Simulation Result without Fuzzy Logic System and with Fuzzy Logic System

the robustness of the proposed adaptive control methodologies using nonsingular fast terminal sliding mode and fuzzy logic systems in tracking the attitude of formation flying satellites in the presence of actuator faults and time-varying disturbances.

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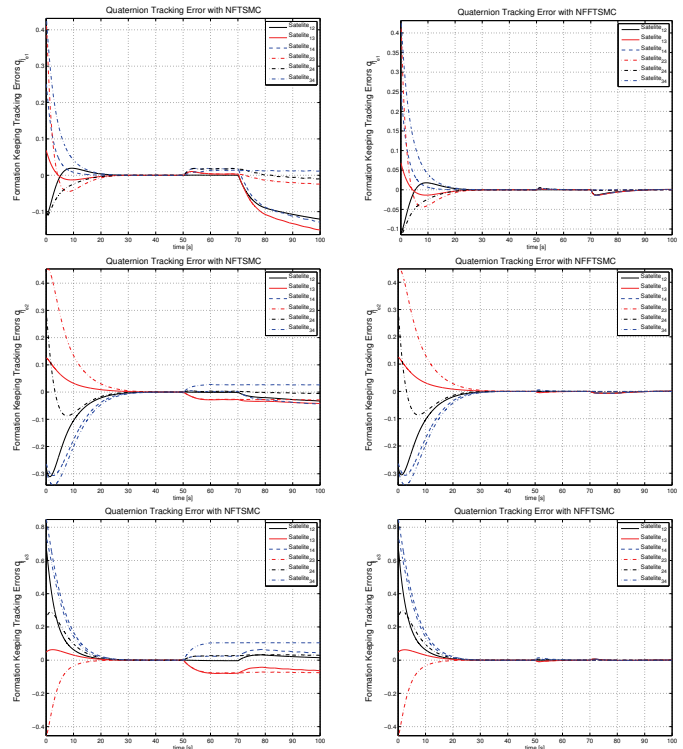


Fig. 2. Formation-keeping Tracking Errors Simulation Result without Fuzzy Logic System and with Fuzzy Logic System