

Modeling of Fuzzy Integral Based Nonlinear Multi-regressions Systems with QPSO-GS

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Abstract—A nonlinear multi-regression based on fuzzy integral (NAFI) model that include outliers under inherent interaction among feature attributes is considered in this paper. The modeling of the proposed model is also performed via a modified algorithm base on particle swarm optimization with quantum-behavior (MQPSO) and the high breakdown value estimator, least trimmed squares (LTS). That is, we successfully integrate mechanisms of the genetic algorithm and the simulated annealing into the QPSO algorithm to estimate parameters of the NAFI model; meanwhile, the LTS estimator is also introduced to filter out outliers. From simulation results, the proposed MQPSO algorithm with LTS estimator (named QPSO-GS) readily corrects the deviation caused by outliers and swiftly achieves convergences on estimating the parameters of the proposed NAFI model with outliers.

Keywords- *Fuzzy integral; nonlinear multi-regression based on fuzzy integral; particle swarm optimization with quantum-behavior; least trimmed squares.*

I. INTRODUCTION

In the modern world, to accomplish an optimal decision in aeronautics, economics, industry, science, manufacturing, traffic control, and many other military and civilian applications extremely depend on useful and crucial information which is drawn from messages or data via transformation, classification and/or other procedure. Therefore, multi-sensor systems providing these messages or data are becoming increasingly important in meeting the goal of optimal decision-making. Besides, a feasible model to elaborate on information fusion and a soft computing technique to perform the heavy computations are also critical.

Within the consideration to a feasible model, traditionally, the most common forms are the weighted average model and the linear regression model. These models are all linear and must assume that there is no interaction among feature attributes (i.e. inputs). However, in many real-world systems, the inherent interaction among feature attributes must be considered circumspectly and these kinds of systems are essentially non-additive systems. Hence, a nonlinear multi-regression based on fuzzy integral (NAFI) model with respect to non-additive set function is a powerful consideration to cope with these kinds of systems. In general, Choquet integral is the most frequent form of the fuzzy integral and there are a few

literatures have been proposed [1-4]. Liu et al. [1] proposed a NAFI model derived from one of the following three kinds of fuzzy supports, that is, the bespoke fuzzy support, the sample relative fuzzy support and the response correlative fuzzy support. This model deals with the interaction among feature attributes based on the correlation in statistics. Wang et al. proposed the original [2] and weighted [3-4] NAFI model to deal with the information with numerical and categorical feature attributes, respectively. In fact, the weighted NAFI model is the generalized form of the original one. In these two models, the interaction among the feature attributes toward the objective attributes (i.e. outputs) is described as non-additive set functions and is practically deferent from the co-relationship in the statistics. The problem that exactly finds out these parameters is an essential optimization problem and the basic idea is making the residuals as small as possible. Residuals here are defined as the difference between what is actually observed and what is estimated. To minimize residuals, the well-known tool, Least Square (LS) method is introduced and always makes a remarkable estimation in the circumstance that all attributes are uncontaminated with outliers. Unfortunately, these feature and objective attributes are usually subject to outliers in the real world applications. That is, outliers may occur due to various reasons, such as erroneous measurements or the data with a heavy-tailed distribution functions. Whenever the outliers exist, they make a serious deviation of what is estimated all the time. Within outlier detection literatures [5-7], the least trimmed squares (LTS) estimator which possesses not only high breakdown value but also several advantages over the other methods is the most popular form to eliminate effects caused by outliers. In this study, therefore, we focus our efforts on the LTS estimator to eliminate the inference from outliers.

Subsequent to confirming the feasible model and previous analysis, to efficiently and swiftly estimate parameters satisfying appointed criteria from observations, a timesaving soft computing technique is necessary for the information fusion system with contaminated attributes. In literatures, there are many outstanding soft computing techniques qualify for this work; they are neural network [8], genetic algorithm (GA) [9], hybrid learning [10], etc. Particle swarm optimization with quantum-behavior (QPSO) which is an improved type on the traditional particle swarm optimization (PSO) would be one of the powerful choices [11-12]. The QPSO algorithm behaves

high performance in single mode systems, because of the property of swift convergence. However, particles usually fall into local extreme state in multimode optimization systems and then take on the premature phenomenon. In order to overcome these defects, we proposed a QPSO algorithm with elitist crossover mechanism of the GA (named MQPSO) in our previous work [13] and performed a superior performance. In this paper, we improve the MQPSO algorithm to manipulate systems with outliers which make serious deviations. That is, the mechanism of the LTS estimator is introduced to eliminate deviation caused by outliers and enhance robustness of the MQPSO, and it is named QPSO-GS. The significant improvement is that the QPSO-GS algorithm combines the concepts of the GA and the simulated annealing (SA) within the QPSO algorithm to achieve global search and overcome prematurity in optimal process, respectively; meanwhile, the LTS estimator is also performed to eliminate the inference from outliers. In order to verify the proposed QPSO-GS algorithm, a numerical example is also performed in this study. From the results of experiment, the proposed QPSO-GS algorithm is able to make a reasonable estimation on parameters of the NAFI model and then, quite precise objective attributes are also acquired.

The rest of paper is organized as follows: in Section II, the NAFI model is proposed. Section III, the least trimmed square estimator and the QPSO algorithm are briefly described. Next, we propose the QPSO-GS algorithm in detail. Section V, the numerical simulation results are demonstrated and then the paper is concluded in Section VI.

II. CHARACTERIZATION OF THE NAFI MODEL

In traditional linear aggregations, the most frequent model to describe the relation between feature attributes X and objective attribute Y is the Lebesgue-like integral [14]. However, in many real-world systems, the inherent interaction among feature attributes must be considered circumspectly. Hence, to reasonably describe the inherent interaction among feature attributes, Wang and Klir [15-16] proposed a regular non-additive set function μ defined on the power set of feature attributes and named fuzzy measure (FM). The formal definition of the FM can be express as

$$\mu(\emptyset) = 0, \mu(X) = 1, \emptyset \in P(X), \quad (1)$$

$$A \in P(X), B \in P(X), \text{ and } A \subset B \Rightarrow \mu(A) \leq \mu(B). \quad (2)$$

Besides, a fuzzy integral is also introduced to aggregate the feature attributes. That is, whenever we deal with information fusion systems where information possesses inherent interaction, the fuzzy integral with respect to the FM is the most reasonable tools. In practical applications, there are many kinds of fuzzy integrals such as Choquet integral [17], Sugeno integral [18] and Wang integral [19], and so on. Considering simplicity and accuracy, the Choquet integral is the most frequent form and then, the Choquet integral with respect to the FM is defined as follows

$$\int f d\mu = \int_0^\infty \mu(F_\alpha) d\alpha, \quad (3)$$

where f is non-negative measureable function with n -dimensions on X , and $F_\alpha = \{x | f(x) \geq \alpha, x \in X\}$, $\alpha \in [0, \infty)$, is called the α -cut set of function f . Since X is a finite set, in this work, and the value of measureable function f can be sorted as

$$\min_{1 \leq i \leq n} f(x_i) = f(x_1^*) \leq \dots \leq f(x_n^*) = \max_{1 \leq i \leq n} f(x_i), \quad (4)$$

where $\{x_1^*, x_2^*, \dots, x_n^*\}$ is a permutation of $\{x_1, x_2, \dots, x_n\}$.

Then, the discrete type of Choquet integral with respect to the FM defined above can be express as

$$\int f d\mu = \sum_{i=1}^n (f(x_i^*) - f(x_{i-1}^*)) \cdot \mu(\{x_i^*, \dots, x_n^*\}). \quad (5)$$

In addition, to deal with observations with categorical attributes, i.e., these feature attributes have only finitely many states, and numerical attributes, simultaneously, the NAFI model which indicates the relation between hybrid attributes and objective attributes can be expressed by the following formula [4]

$$y = c + q \cdot \int (\omega \cdot f) d\mu + er, \quad (6)$$

where c and q are constants, $\int f d\mu$ is the Choquet integral of function f with respect to the FM μ , vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)$ is an n -dimensional weighting vector which is used for coping with categorical attributes, i.e. vector ω is used for balancing the units among various attributes and satisfies the following constrain

$$0 < \omega_i \leq 1 \text{ and } \max_{1 \leq i \leq n} \omega_i = 1, i = 1, 2, \dots, n. \quad (7)$$

In the NAFI model, totally, there are $2^n + n$ unknown parameters and increase exponentially with the dimension of feature attributes. In order to complete the NAFI model, these parameters have to be determined in advance. That is so called the learning state or the training state of the NAFI model. In this state, associating (6) with available observations constitute an over-determined system with Choquet integral. Thus, the analytic solution of the model parameters cannot be exact figured out. Furthermore, constants c and q are essential of difference from the other parameters which are governed by the Choquet integral. Therefore, the dual optimization procedures must be performed, simultaneously; meanwhile, the performance index of optimization J (called fitness function) is also introduced and expressed as

$$J = \text{Min} \left\{ \sum_{j=1}^k \left(y_j - c - q \cdot \int (\omega \cdot f_j) d\mu \right)^2 \right\}, \quad (8)$$

where k is the length of available observations for the training state. Because the kernel of the performance index of optimization is the LS estimator, it usually suffers from outliers in real world systems. Fortunately, there are many high breakdown value regression estimators have been proposed to

eliminate the effect of outliers [6-7]. For the reason of simplicity and efficiency, the least median of squares (LMS) and the LTS are the most popular regression estimators in scientific applications. Furthermore, the LTS estimator possesses not only the same breakdown value as LMS, but also several merits: for instance, its objective function is smoother; its statistical efficiency is better, and so on. Therefore, we focus the treatment of outliers on LTS method and then the fitness function J is revised as the form

$$J = \text{Min} \left\{ \sum_{j=1}^h \left(y_j^* - c - q \cdot \int (\omega \cdot f_j^*) d\mu \right)^2 \right\}, \quad (9)$$

where y_j^* and f_j^* are a permutation of observations under the best model parameters and h is a trimmed parameter of the LTS estimator. The block diagram of the proposed structure for the training state and information fusion systems are shown in Fig. 1. Where the block named QPSO-GS receives the differences of objective attributes between observations and estimations when the terminative criterion is not satisfied yet, and then the parameters of the NAFI model are updated base on these differences; meanwhile, in the LTS block used for filtering out these atypical observations, the trimmed parameter h is also revised by the global optimal parameters so far.

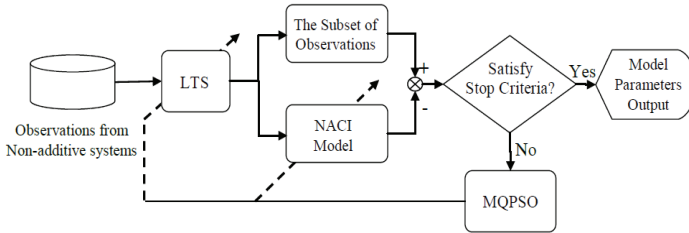


Figure 1. Block diagram of the proposed structure for the parameters estimation of the NACI model via MQPSO and LTS.

III. THE LTS ESTIMATOR AND THE QPSO ALGORITHM

In robust regression analysis [6], the LTS estimator is formulated as

$$\text{Min} \left\{ \sum_{i=1}^h r^2(d_i) \right\}, r^2(d_1) \leq \dots \leq r^2(d_h) \leq \dots \leq r^2(d_k), \quad (10)$$

where d_i is the i_{th} observation, $r^2(d_i)$ is the i_{th} squared residual, k is the length of observations and h is the number of data points which are not trimmed from the data set. The maximum tolerance of the LTS estimators to outliers (named Maximum Breakdown Point) for any equi-variant regression estimator satisfies

$$\text{Breakdown Point} \leq \frac{1}{k} \left(\left((k - \zeta)/2 + 1 \right) \right), \quad (11)$$

where ζ is the dimension of variables. Intuitively, 50% is the upper bound that can be expected and it is actually attained for $h = (k + \zeta + 1)/2$. In theorem, the solution of (10) always exists. Of course, one can achieve the optimal solution by performing $C(k, h)$ ordinary least square problems for all

subsets of $\{1, 2, \dots, k\}$ with h elements and selecting the best one among all candidates. Obviously, it is laborious and impractical for real world systems with large observations. In order to cope with a great deal of observations, the FAST-LTS method was been proposed [7]. The major distinguishing features are the initial h -subset, the C-step and the nested extensions. By and large, the initial h -subset is a preselecting mechanism to confirm that a clean h -subset $\{d_1, d_2, \dots, d_h\}$ drawn from all observations can be attained and the C-step is a recursive procedure and used for increasing accuracy of the estimated model parameters. For a small to moderate data size k , these two procedures work well and do not take much time. When observations are large enough, the procedure named nested extension is introduced to avoid the heavy computation.

After drawing observations without contaminations, a proper soft computing technique is essential and can help us to efficiently estimate parameters of the NAFI model. The QPSO algorithm is one of these soft computing techniques, and possesses a significant ability of global and local search. In the QPSO algorithm, particles move in a quantum multi-dimensional space, the state of particles is usually depicted by normalized wave function $\Psi(\rho, t)$, i.e., the probability amplitude of position where particles are presence; and the wave function satisfies the constrain

$$\int_{\text{whole space}} |\Psi(\rho, t)|^2 d\rho = 1, \quad (12)$$

where ρ is the n -dimensional coordinates. That is, a single particle with mass m is subjected to the influence of a potential field $V(\rho, t)$ in the quantum space and the wave function is governed by the Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t} \Psi(\rho, t) = -\frac{\hbar^2}{2m} \nabla^2 \Psi(\rho, t) + V(\rho, t) \Psi(\rho, t), \quad (13)$$

where $\hbar$ is the Planck constant and ∇^2 is the Laplacian operator. In the environment with potential field, the particles are then attracted to the center of potential field through the optimization process, and such attraction leads to the global optimum. Base on the assumption that the attractive potential field is time-independent (called stationary state); the solution of the time-independent Schrödinger equation has the form [20]

$$\Psi(\rho, t) = \varphi(\rho) \cdot \exp(-i\omega t), \quad (14)$$

where ω has the dimensions of an angular frequency. In theorem, any type of potential well can describe this system which is bounded and attracted by a potential field, but the simplest one is the Delta Potential Well and the potential field is given by

$$V(\rho) = -\gamma \delta(\rho), \quad (15)$$

where γ is a positive number proportional to the “depth” of the potential well. For the sake of simplicity, the solution of time-independent Schrödinger equation for this system in one dimensional space is considered and expressed as

$$Q(z) = |\varphi(z)|^2 = \frac{1}{L} e^{-\frac{2|z|}{L}}, L = \frac{\hbar^2}{m\gamma}, \quad (16)$$

where $Q(z)$ is the probability density function for measuring a particle's state and L is the characteristic length of Delta Potential Well which specifies the search scope of a particle. In order to obtain the precise position of particles, Monte Carlo Method is used for simulating the procedure that the quantum state collapses to the classic state, and the particle's position can be expressed as

$$\varphi_i = pf^{cnt}(i) \pm \frac{L}{2} \cdot \ln\left(\frac{1}{u}\right), i = 1, 2, \dots, NP, \quad (17)$$

where NP is the number of particles in a population, u is random number uniformly distribute on $[0,1]$ and pf^{cnt} is the center of potential field which is proposed by Clerc and Kennedy [21] and defined as

$$pf^{cnt}(i) = \frac{(c_1 \cdot p_i^{loc} + c_2 \cdot p^{gol})}{(c_1 + c_2)}, i = 1, 2, \dots, NP, \quad (18)$$

where c_1, c_2 are constriction coefficients and p_i^{loc}, p^{gol} are the best position of the i_{th} particle and the global best position found so far. In order to improve performance of the QPSO algorithm, Sun et al [12] employ a Mean Best Position ($mbest$) to evaluate parameter L . However, to extend the global search of the QPSO algorithm, the $mbest$ is modified and then these two parameters can be expressed as the following form

$$mbest = \left[\sum_{i=1}^{NP} \frac{\phi_{i,1}}{NP}, \sum_{i=1}^{NP} \frac{\phi_{i,2}}{NP}, \dots, \sum_{i=1}^{NP} \frac{\phi_{i,n}}{NP} \right], \quad (19)$$

$$L = 2 \cdot \beta |mbest - \varphi_i|, \quad (20)$$

where β is a creative coefficient which is used to adjust the convergent speed of individual particle and the performance of the QPSO algorithm.

IV. THE QPSO-GS ALGORITHM

Within empirical applications, however, the QPSO algorithm usually represents stagnating phenomenon for searching the global optimal solution in multi-mode problems and systems; meanwhile, it is also strongly influenced by the creative coefficient β . In order to improve these defects, a nonlinearly updating mechanism of the creative coefficient β and instantaneously monitoring the convergence of optimization procedure are introduced. The nonlinearly revising mechanism which is similar to the SA is expressed as the form

$$\beta = \beta_{ini} - \Delta\beta \cdot (1 + \exp(-(\Delta fit)))^{-1}, \quad (21)$$

where $\Delta\beta$ is step length of β , Δfit is the changing rate of optimal estimation so far and β_{ini} is the initial value of β .

The other improvement of the proposed QPSO-GS algorithm is the mechanism to overcome prematurity. Inspired by these mechanisms of mutation and elite crossover in the GA, an index of conquering stagnation (named ECM which is an abbreviation of elite crossover and mutation) is used for monitoring the status of optimization procedure. That is, during the optimization procedure, the QPSO-GS algorithm preserves each different p^{gol} ; meanwhile, the index of conquering stagnation, ECM is set to zero whenever p^{gol} is updated. Of course, the ECM increases by one whenever p^{gol} is unchanged. Before finishing current iteration, the QPSO-GS algorithm makes a judgment whether ECM exceeds the specific criteria. If it is true, the QPSO-GS algorithm lets the new population be these collected p^{gol} instead of original population (all/or these worse particles) and sets the ECM to zero, instantaneously.

Because the kernel of estimating fitness is the LS estimator, the MQPSO algorithm presents superior performances for estimating parameters for observations without outliers in our previous work. However, the MQPSO algorithm always makes a serious deviation in the contaminated circumstance. Hence, the LTS estimator is introduced to sieve out the observations without contamination and the proposed QPSO-GS algorithm is detailed below:

Step 1: Perform the LTS estimator to sieve out these h observations without contamination.

Step 2: Randomly initialize the population of particles with dimension $2^n - 2 + n$ and then, evaluate their fitness values by (9).

Step 3: Sort particles according to their fitness values and then initialize p_i^{loc}, p^{gol} .

Step 4: Calculate pf^{cnt} , $mbest$ and L by (18), (19) and (20), respectively.

Step 5: Select (22) or (23) with randomly probability to update φ_i .

$$\varphi_{i+1} = pf^{cnt}(i) + \beta \cdot \|mbest - \varphi_i\| \cdot \ln\left(\frac{1}{u}\right), \quad (22)$$

$$\varphi_{i+1} = pf^{cnt}(i) - \beta \cdot \|mbest - \varphi_i\| \cdot \ln\left(\frac{1}{u}\right). \quad (23)$$

Step 6: Evaluate the fitness values of all particles by (9).

Step 7: According to evaluation in Step 6, update p_i^{loc} .

Step 8: Check over whether the maximum iteration is reached or the terminative criterion is satisfied? If yes, go to Step 11, else perform next Step.

Step 9: Check over whether p^{gol} is updated? If p^{gol} is updated, sets ECM to 0 and perform the LTS

procedure, then go to Step 4. If p^{gol} is unchanged, increase ECM by 1 and perform next Step.

Step 10: Check over whether the maximum ECM is reached?

If yes, let these collected p^{gol} instead of ϕ_{i+1} and go to Step 4, else keep ϕ_{i+1} and go to Step 4.

Step 11: Check over whether p^{gol} should be updated and then output results.

V. NUMERICAL SIMULATION AND RESULTS

To demonstrate and verify the effectiveness and feasibility of the proposed systems are shown in Figure 1, an artificial observation profile which simulates 4 continuous sampling data with outliers is made to estimate the model parameters by associating the proposed QPSO-GS algorithm based on the NAFI model. All simulations are implemented in the Matlab environment and conducted on Intel Core 2 Duo CPU P8400, 4GB Ram capacity PC.

Example 1: The original model parameters are set as: $c=4$, $q=1.5$, $\mu = \{0.22, 0.3, 0.45, 0.28, 0.42, 0.49, 0.6, 0.25, 0.45, 0.52, 0.62, 0.74, 0.82, 0.88, 1\}$, $w = \{0.3, 0.26, 1, 0.48\}$. Then, we randomly create 400 4-dimensional feature attributes with 10% random contamination (marked with shadows) to produce training data as shown in Table I, where y_{true} and y_{cont} are the original and contaminated objective attributes, respectively. After performing the proposed QPSO-GS algorithm for many times, the average results of estimating model parameters are shown in Tables II, where y_{esti} is the estimated objective attributes, p_{esti} and p_{true} are the contaminated and original model parameters, respectively. In addition, we also show plots of the training data and estimated results in Figures 2~3. In this Example, the terminative criteria of program are maximum iterations which reach 1500 times or the mean square error which is less than 10^{-4} . From Table II and Figures 2~3, these estimated results almost the same as original objective attributes and these estimated model parameters are really precise. That is, the QPSO-GS algorithm is able to efficiently filter out outliers from observations. That is, the proposed QPSO-GS readily corrects the deviation caused by outliers and swiftly achieves convergences on estimating the parameters of the proposed NAFI model with outliers.

VI. CONCLUSIONS

In this paper, the NAFI model association with the QPSO-GS algorithm is considered and developed to deal with a non-additive system with outliers. Whenever atypical observations are presence, the estimation method of parameters based on the LS estimator is no longer feasible. Therefore, there is an excellently alternative ways to replace the LS estimator with the LTS estimator. That is, we successfully integrate mechanisms of the GA and the SA into the QPSO algorithm to estimate parameters of the NAFI model with outliers; meanwhile, the LTS estimator is also introduced to filter out outliers. From the simulation results, the proposed QPSO-GS algorithm can precisely estimate parameters of the NAFI model for observations contaminated with outliers; meanwhile, it still

maintains high coincidence between the estimated and original objective attributes.

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TABLE I. TANNING DATA WITH AND WITHOUT CONTAMINATIONS USED FOR VERIFYING THE PROPOSED NACI MODEL AND LTS-MQPSO ALGORITHM

	x_1	x_2	x_3	x_4	y_{true}	y_{cont}
01	0.70	0.73	0.65	0.32	4.4605	4.4605
02	0.01	0.33	0.30	0.46	4.2969	4.2969
03	0.56	0.50	0.28	0.27	4.2657	2.6707
04	0.94	0.39	0.47	0.65	4.4739	4.4739
05	0.09	0.73	0.78	0.83	4.6472	4.6472
06	0.59	0.10	0.13	0.47	4.2169	4.2169
07	0.32	0.56	0.99	0.49	4.6259	4.6259
08	0.35	0.78	0.29	0.81	4.4204	5.7115
09	0.62	0.49	0.78	0.68	4.6095	4.6095
10	0.28	0.64	0.49	0.83	4.5308	4.5308
⋮	⋮	⋮	⋮	⋮	⋮	⋮
391	0.05	0.80	0.93	0.16	4.5037	4.5037
392	0.85	0.77	0.51	0.77	4.5539	4.5539
393	0.26	0.99	0.72	0.96	4.6884	3.5601
394	0.46	0.51	0.35	0.53	4.3749	4.3749
395	0.86	0.08	0.27	0.46	4.3057	4.3057
396	0.52	0.74	0.90	0.93	4.7545	4.7545
397	0.22	0.50	0.79	0.22	4.4464	4.4464
398	0.43	0.97	0.51	0.59	4.4858	4.4858
399	0.86	0.03	0.63	0.82	4.5693	4.5693
400	0.03	0.37	0.14	0.47	4.2093	4.2093

TABLE II. THE AVERAGE RESULTS OF MODEL PARAMETERS ESTIMATED BY THE PROPOSED LTS-MQPSO ALGORITHM

	p_{esti}	p_{true}	Data	y_{esti}	y_{cont}	y_{true}
μ_1	0.2243	0.22	d(1)	4.4605	4.4605	4.4605
μ_2	0.2988	0.30	d(2)	4.2969	4.2969	4.2969
$\mu_{1,2}$	0.4532	0.45	d(3)	4.2657	2.6707	4.2657
μ_3	0.2795	0.28	d(4)	4.4740	4.4739	4.4739
$\mu_{1,3}$	0.4216	0.42	d(2)	4.6472	4.6472	4.6472
$\mu_{2,3}$	0.4890	0.49	d(6)	4.2167	4.2169	4.2169
$\mu_{1,2,3}$	0.6010	0.60	d(7)	4.6259	4.6259	4.6259
μ_4	0.2495	0.25	d(8)	4.4204	5.7115	4.4204
$\mu_{1,4}$	0.4528	0.45	d(9)	4.6096	4.6095	4.6095
$\mu_{2,4}$	0.5186	0.52	d(10)	4.5308	4.5308	4.5308
$\mu_{3,4}$	0.6213	0.62	⋮	⋮	⋮	⋮
$\mu_{1,2,4}$	0.7385	0.74	d(391)	4.5036	4.5037	4.5037
$\mu_{1,3,4}$	0.8195	0.82	d(392)	4.5541	4.5539	4.5539
$\mu_{2,3,4}$	0.8787	0.88	d(393)	4.6885	4.6884	3.5601
$\mu_{1,2,3,4}$	1.0000	1.00	d(394)	4.3750	4.3749	4.3749
ω_1	0.2963	0.30	d(395)	4.3055	4.3057	4.3057
ω_2	0.2598	0.26	d(396)	4.7545	4.7545	4.7545
ω_3	1.0000	1.00	d(397)	4.4464	4.4464	4.4464
ω_4	0.4803	0.48	d(398)	4.4858	4.4858	4.4858
c	3.99999	4.00	d(399)	4.5692	4.5693	4.5693
q	1.50236	1.50	d(400)	4.2093	4.2093	4.2093

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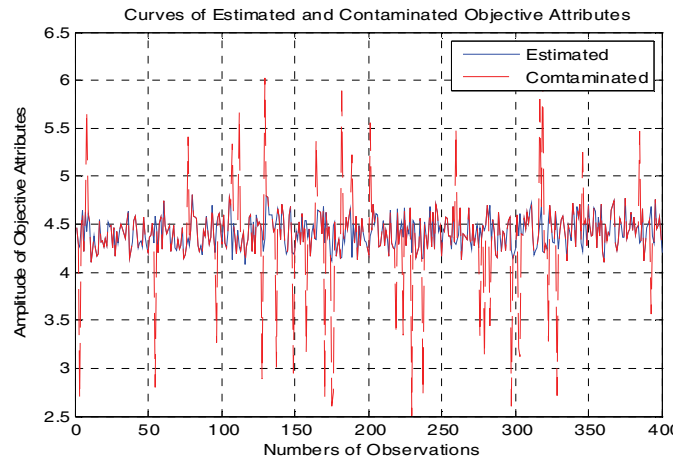


Figure 2. Shown the results for the contaminated objective attributes and the estimated objective attributes.

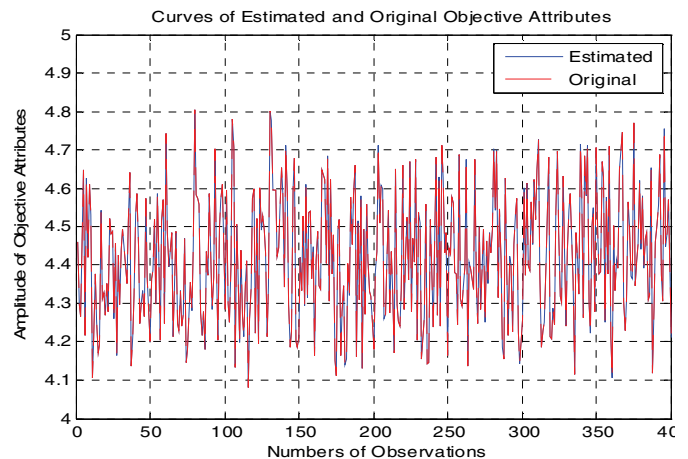


Figure 3. Shown the results for the original objective attributes and the estimated objective attributes.