

Terminal Sliding Mode Controlled CVCF Inverters

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Abstract—A control technique called Terminal Sliding Mode Control (TSMC) to generate high-quality sinusoidal output voltage for a constant-voltage constant-frequency (CVCF) inverter is presented in this paper. Conventional Sliding Mode Control (SMC) has used extensively due to its robustness and simplicity, but the convergence of the system states to the equilibrium point is usually asymptotic. TSMC not only has the robustness of the SMC but also guarantees the finite reaching time to the equilibrium point. The presented TSMC adjustable parameters are used to decrease the system state error to zero in finite time. With the better robustness of the proposed controller, the output voltage of the CVCF inverter will have the features of low distortion, and insensitively to parameter variations and external disturbances. Simulation results are given to conform that the proposed controller can achieve satisfactory response under linear and nonlinear loads.

Keywords- Terminal Sliding Mode Control (TSMC), constant-voltage constant-frequency (CVCF) inverter, finite time, distortion, nonlinear loads.

I. INTRODUCTION

Constant-voltage constant-frequency inverters are being broadly adopted in many AC power conditioning systems, such as uninterruptible power supply (UPS) systems, photovoltaic (PV) energy systems, and programmable AC source (PAS) systems [1], [2]. The CVCF inverters are required to maintain the high-quality AC output voltage of low harmonic distortion (THD) and fast dynamic response even feeding distorting loads (especially highly nonlinear loads such as diode rectifiers with electrolytic capacitors) that can be achieved by employing feedback control technique [3], [4]. The variable structure system with sliding mode was initially proposed in the 1950's [5], [6]. The most attractive advantage of a sliding mode is its insensitively to modeling uncertainly, plant parameter variation, and/or external disturbance [7-12]. The sliding mode control system utilizes a switching control laws to drive the system to reach and remain on the intersection of a set of prescribed switching surface. This indicates that in the design procedure of a sliding mode control, the following steps must be satisfied:

1. There exists a sliding cure whenever the trajectory of the state error will be forced toward the sliding cure.
2. When the state error hits the sliding cure, and then the trajectory of the state error must sliding along the cure to

the origin.

However, as the above mentioned, the most commonly used switching surfaces are linear hyperplanes [7-12]. The linear hyperplanes guarantee the asymptotic stability of the sliding mode. The system state will converge to the origin in infinite time. Similarly, in more recent literature, the good inverter performance cannot be achieved on account of the infinite system-state convergence time [13-15]. The tracking control scheme based on fixed-frequency SMC technique is applied to the design of Buck-based inverter. In this case, the resulting output voltage also makes a concession between steady state and transient response [13]. A discrete-time SMC method with dual-loop design for UPS inverter is developed by [14]; the system trajectory could not hit the desired sliding surface accurately, and thus a slight distortion exists in the output voltage waveform with a rectified load. In the work [15], a sliding mode controlled PWM inverter is presented, but incurs steady-state error problems.

Thus, in this paper, a terminal sliding mode control through nonlinear hyperplanes is proposed and the system parameter is designed. TSMC not only has the advantages of the conventional SMC but also takes the system to converge the origin point in finite time [16-23]. By using the proposed controller, the high-quality sinusoidal output voltage for a CVCF inverter can be generated and allow for the system state error converge to zero in finite time.

II. MODELING OF THE CVCF INVERTER

The system structure of CVCF inverter is shown in Fig. 1. The inverter output voltage is generally desired to be maintained as the same as possible to a sinusoidal reference waveform. Thus, design of the inverters can be regarded as a typical tracking problem. Let $v_o(t)$ denote the load voltage, $v_o^*(t)$ is the reference waveform, and $v_{in}(t)$ represents the inverter output voltage that takes three values: $(+V_{dc}, -V_{dc}$ or 0).

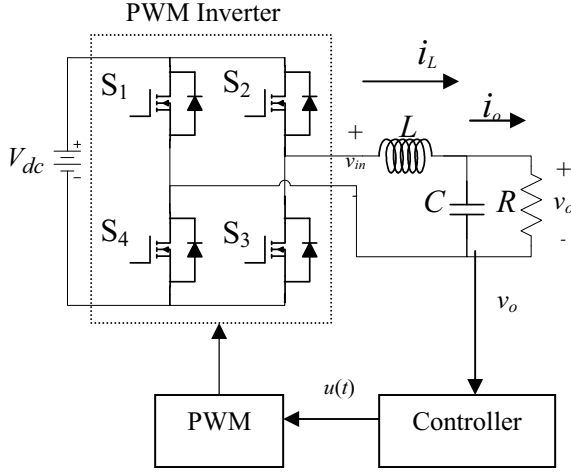


Fig. 1 Block diagram of CVCF inverter

From Fig. 1, we can get two equations:

$$i_L = C \frac{dv_o}{dt} + \frac{v_o}{R} \quad (1)$$

$$L \frac{di_L}{dt} + v_o = v_{in} \quad (2)$$

Substituting (1) into (2) has

$$\frac{d^2 v_o}{dt^2} + \frac{1}{RC} \frac{dv_o}{dt} + \frac{1}{LC} v_o = \frac{1}{LC} v_{in} \quad (3)$$

Let

$$\begin{aligned} x_1 &= v_o \\ x_2 &= \dot{x}_1 = \dot{v}_o \\ \dot{x}_2 &= \ddot{x}_1 = \ddot{v}_o \\ u &= v_{in} \end{aligned} \quad (4)$$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix} u \quad (5)$$

If the desired output voltage is $x_1^* = v_o^*$, and new state variables are defined as:

$$\begin{aligned} e_1 &= x_1 - x_1^* = v_o - v_o^* \\ e_2 &= \dot{e}_1 = \dot{x}_1 - \dot{x}_1^* = \dot{v}_o - \dot{v}_o^* \\ \dot{e}_2 &= \ddot{e}_1 = \ddot{x}_1 - \ddot{x}_1^* = \ddot{v}_o - \ddot{v}_o^* \\ &= -\frac{1}{LC} v_o - \frac{1}{RC} \dot{v}_o + \frac{1}{LC} u - \ddot{v}_o^* \\ &= -\frac{1}{LC} e_1 - \frac{1}{RC} e_2 + \frac{1}{LC} u - \frac{1}{LC} v_o^* - \frac{1}{RC} \dot{v}_o^* - \ddot{v}_o^* \end{aligned} \quad (6)$$

Then, the system dynamics can be represented as follows:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ a_1 \end{bmatrix} u + \begin{bmatrix} 0 \\ -a_1 v_o^* - a_2 \dot{v}_o^* - \ddot{v}_o^* \end{bmatrix} \quad (7)$$

where a_1 is equal to $1/LC$, a_2 is equal to $1/RC$.

III. CONTROL DESIGN

Define the function as follows:

$$S_0 = e_1 \quad (8)$$

From (8), the terminal sliding surface can be constructed as

$$\sigma = \dot{S}_0 + \beta_1 S_0^{\gamma_1} \quad (9)$$

where $\gamma_1 < 1$. Since γ_1 is less than one, the dynamics will be finite-time convergence; If γ_1 is equal to one, (9) becomes conventional sliding surface, and has an asymptotic convergence time.

Thus, the differentiation of the (9) can be obtained as

$$\begin{aligned} \dot{\sigma} &= \ddot{S}_0 + \beta_1 \gamma_1 S_0^{\gamma_1-1} \dot{S}_0 \\ &= -\sum_{i=1}^2 a_i e_i + bu - f(t) + \beta_1 \gamma_1 S_0^{\gamma_1-1} \dot{S}_0 \\ &= -\sum_{i=1}^2 a_i e_i + bu - f(t) + \beta_1 \frac{d}{dt} S_0^{\gamma_1} \end{aligned} \quad (10)$$

Let

$$\begin{aligned} a_i &= a_{i0} + \Delta a_i, \quad i = 1, 2 \\ b &= b_0 + \Delta b \end{aligned} \quad (11)$$

$$f(t) = a_1 v_o^* + a_2 \dot{v}_o^* + \ddot{v}_o^*$$

where a_{i0} is nominal value of a_i , b_0 is nominal value of b , Δa_i is the variation of a_i , and $f(t)$ is the disturbance.

For system (7), if the control u is designed as

$$u = u_{eq} + u_{tsm} \quad (12)$$

where u_{eq} is the equivalent control input, and is defined as

$$\sigma|_{u=u_{eq}, a_i=a_{i0}, a_2=a_{20}, b=b_0, f(t)=0} = 0 \quad (13)$$

$$\therefore u_{eq} = \left(\sum_{i=1}^2 a_{i0} e_i - \beta_1 \gamma_1 S_0^{\gamma_1-1} \dot{S}_0 \right) / b_0 \quad (14)$$

When the system dynamics in sliding motion, $\sigma = 0$, we obtain

$$\dot{S}_0 = -\beta_1 S_0^{\gamma_1} \quad (15)$$

Substitution of equation (15) into (14) yields

$$u_{eq} = \left(\sum_{i=1}^2 a_{i0} e_i + \beta_1^2 \gamma_1 S_0^{2\gamma_1-1} \right) / b_0 \quad (16)$$

The u_{tsm} is used to eliminate the influence due to the plant parameter variations in Δa_2 to guarantee the existence of a terminal sliding mode as follows:

$$u_{tsm} = \sum_{i=1}^2 \Omega_i e_i + \sum_{i=1}^2 H_i S_0^{2\gamma_1-1} + N \quad (17)$$

The condition for the existence of a sliding motion is $\sigma \dot{\sigma} < 0$, and then

$$\begin{aligned} \sigma \dot{\sigma}|_{u=u_{eq}+u_{tsm}} &= \left\{ -\sum_{i=1}^2 a_i e_i + bu - f(t) + \beta_1^2 \gamma_1 S_0^{2\gamma_1-1} \right\} \sigma \\ &= \left\{ -\sum_{i=1}^2 a_i e_i + [(b_0 + \Delta b)(u_{eq} + u_{tsm})] - f(t) + \beta_1^2 \gamma_1 S_0^{2\gamma_1-1} \right\} \sigma \\ &= \left\{ -\sum_{i=1}^2 \Delta a_i e_i - \Delta a_2 e_2 + \left[\frac{\Delta b}{b_0} \left(\sum_{i=1}^2 a_{i0} e_i \right) + \left[b \left(\sum_{i=1}^2 \Omega_i e_i \right) \right] \right. \right. \\ &\quad \left. \left. + (bH_1 + \frac{\Delta b}{b_0} \beta_1^2 \gamma_1) S_0^{2\gamma_1-1} + (\beta_1^2 \gamma_1 S_0^{2\gamma_1-1} + bK_2 S_0^{2\gamma_1-1}) \right. \right. \\ &\quad \left. \left. + [bN - f(t)] \right\} \sigma < 0 \end{aligned} \quad (18)$$

The conditions for satisfying (18) are

$$\Omega_1 = \begin{cases} \rho_1 < (\Delta a_1 - a_{10}\Delta b/b_0)/b & \text{if } e_1\sigma > 0 \\ \xi_1 > (\Delta a_1 - a_{10}\Delta b/b_0)/b & \text{if } e_1\sigma < 0 \end{cases} \quad (19)$$

$$\Omega_2 = \begin{cases} \rho_2 < (\Delta a_2 - a_{20}\Delta b/b_0)/b & \text{if } e_2\sigma > 0 \\ \xi_2 > (\Delta a_2 - a_{20}\Delta b/b_0)/b & \text{if } e_2\sigma < 0 \end{cases} \quad (20)$$

$$H_1 = \begin{cases} h_1 < (-\beta_1^2 \gamma_1 \Delta b/b_0)/b & \text{if } S_0^{2\gamma_1-1} \sigma > 0 \\ \bar{h}_1 > (-\beta_1^2 \gamma_1 \Delta b/b_0)/b & \text{if } S_0^{2\gamma_1-1} \sigma < 0 \end{cases} \quad (21)$$

$$H_2 = \begin{cases} h_2 < -\beta_1^2 \gamma_1 / b & \text{if } S_0^{2\gamma_1-1} \sigma > 0 \\ \bar{h}_2 > -\beta_1^2 \gamma_1 / b & \text{if } S_0^{2\gamma_1-1} \sigma < 0 \end{cases} \quad (22)$$

$$N = \begin{cases} \zeta < f(t)/b & \text{if } \sigma > 0 \\ \eta > f(t)/b & \text{if } \sigma < 0 \end{cases} \quad (23)$$

Remarks:

1. By adjusting the parameters (19) to (23), the system state error can be decrease to zero in finite time.
2. Notice that the control law (12) implies the sign function across the surface σ caused by the derived inequalities form (19) to (23), and thus chattering problem exists. A solution to this problem is to smooth the discontinuous control law in a thin boundary layer. The discontinuous function can be replaced by the following continuous function:

$$\frac{\sigma}{|\sigma| + \varepsilon}, \quad \varepsilon = \begin{cases} \varepsilon_0 & |\sigma| \geq \delta \\ \varepsilon_0 & |\sigma| < \delta \end{cases} \quad (24)$$

where ε_0 and δ are positive constants.

IV. SIMULATION RESULTS

To demonstrate the feasibility of the single-phase CVCF inverter using the proposed controller, the closed-loop system has been tested by computer simulations. Some key parameters are listed in Table I.

Table I Parameters of the CVCF inverter system

Switching frequency	$f_{sw} = 20 \text{ kHz}$
DC source voltage	$V_{dc} = 200 \text{ V}$
Rated output voltage	$v_o = 110 \text{ V}$
Rated output frequency	$f_o = 60 \text{ Hz}$
Rated load	$R = 10 \Omega$
Filter inductor	$L = 0.5 \text{ mH}$
Filter capacitor	$C = 50 \mu\text{F}$

Fig. 2 shows the simulated steady-state output voltage of the inverter with the proposed controller at full resistive load and Fig. 3 shows that with the conventional SMC at the same load condition. Both output voltages show good regulation with low THD sinusoidal waveform. Figure 4 shows the transient response with the load changing from no load to full load at $t=0.0208\text{sec}$. It indicates that the transient response of the

proposed control is very fast taking only a few sampling intervals to reach steady-state; On the contrary, the waveforms shown in Fig. 5 with conventional SMC has a significant voltage dip. A final test, as shown in Fig. 6 is done applying a nonlinear load consisting of a diode bridge, a capacitor (200 μF) and a resistive load (10 Ω). The %THD computed in the output voltage is 3.11%; The AC voltage is still satisfactory and a good behavior of the inverter is demonstrated also in this critical condition. However, under the same nonlinear loading, Fig. 7 obtained using the conventional SMC exhibits high %THD output voltage (%THD of output voltage is equal to 8.96%). The comparison between TSMC and conventional SMC under linear load and nonlinear load are also given in Table II.

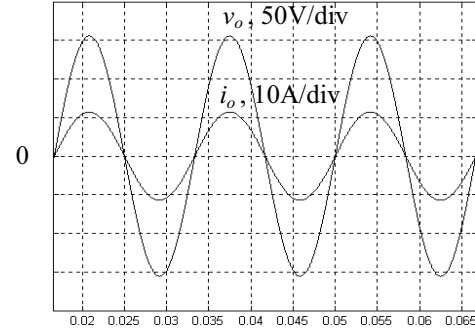


Fig. 2 Simulation results of output voltage and load current under full resistive load with the proposed controller.

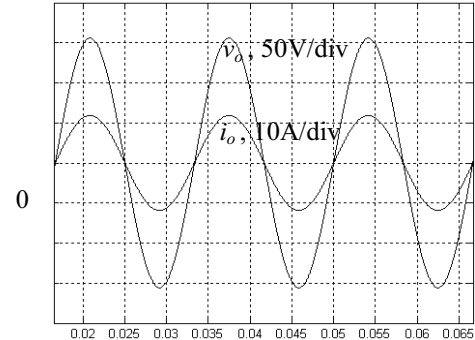


Fig. 3 Simulation results of output voltage and load current under full resistive load with the conventional SMC.

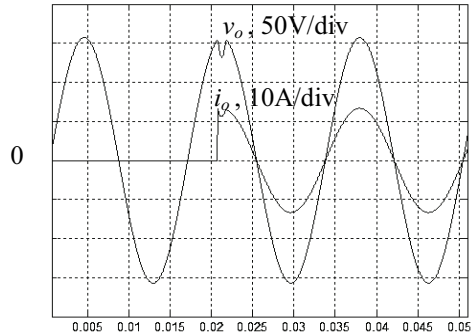


Fig. 4 Simulation results of output voltage and load current under load changing from no load to full load with the proposed controller.

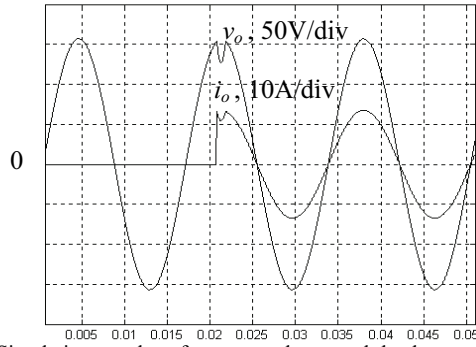


Fig. 5 Simulation results of output voltage and load current under load changing from no load to full load with the conventional SMC.

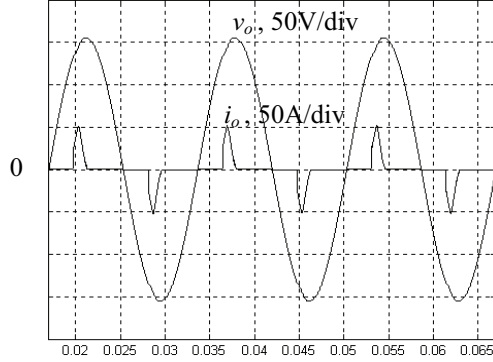


Fig. 6 Simulation results of output voltage and load current under nonlinear load with the proposed controller.

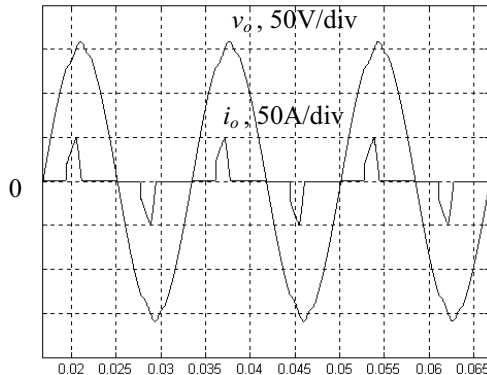


Fig. 7 Simulation results of output voltage and load current under nonlinear load with the conventional SMC.

Table II Comparison of fundamental amplitudes and %THD under linear loads and rectifier loads

		Linear Load	Rectifier Load
		10Ω	
TSMC	Fundamental Amplitude	109.5V	108.1V
	%THD	0.19%	3.11%
SMC	Fundamental Amplitude	109.3V	105.9V
	%THD	0.20%	8.96%

V. CONCLUSIONS

An approach for the control of CVCF inverters is developed on the basis of TSMC with nonlinear switching

surface. This method not only possesses the characteristic of SMC but also has the finite time convergence of the system state. Even under nonlinear rectified load, the proposed controller still provides excellent performance. To verify the correct operation of the proposed control method, computer simulation is fulfilled, and good agreement is reported. It is apparent from the simulation results that the proposed CVCF inverter system is robust under different kinds of load conditions

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