

An Interactive Fuzzy Satisficing Method for Multiobjective Stochastic Defensive Location Problems

Takeshi Uno

Institute of Socio-Arts and Sciences,
The University of Tokushima
1-1, Minamijosanjima-cho,
Tokushima, 770-8502, JAPAN
Email: uno@ias.tokushima-u.ac.jp

Kosuke Kato

Faculty of Applied Information Science,
Hiroshima Institute of Technology
2-1-1, Miyake, Saeki-ku,
Hiroshima, 731-5193, JAPAN
Email: k.katoh.me@it-hiroshima.ac.jp

Abstract—This paper extends the defensive location problem (DLP), proposed by Uno and Katagiri, considering the situations that the defender locates defensive facilities for preventing invaders from reaching some important vertices in a network. Their DLPs assumed that invaders, with a given energy, exist on a given vertex in it. On the other hand, this paper considers a new DLP that invaders' sites and energies are given uncertainly. By representing their existing vertices and energies as random variables with scenarios, the new DLP can be formulated as a multiobjective bilevel stochastic programming problem. In order to find a satisficing solution of the decision maker for the multiobjective DLP, an interactive fuzzy satisficing method with tabu search algorithm based on strategic oscillation is proposed. The efficiency of the proposed method is shown by applying it to numerical examples of the DLPs.

Index Terms—Facility location, Stackelberg equilibrium, stochastic programming, multiobjective programming, tabu search.

I. INTRODUCTION

Defensive location problem (DLP), proposed by Uno and Katagiri [12], is one of the bilevel location problems in a competitive environment. Hakimi [4] considered a bilevel location problem of commercial facilities, e.g. shops and supermarkets, on a network. His problem deals with a type of competitiveness between two decision makers (DMs) locating their own facilities, with the aim of maximizing her/his profit obtaining from their customers. For the detail of Hakimi's problem and its applications, the readers can refer to the book of Miller et al. [8]. As an extension of Hakimi's location problem, Shiode and Drezner [11] considered a problem with random demands, Uno et al [13] considered a problem with fuzzy random demands, and Yang et al [14] considered a bilevel flow interception problem. Meanwhile, O'Hanley and Church [9] considered a bilevel maximum covering problem, which is a noncompetitive type of location problem.

On the other hand, the DLP deals with another type of competitiveness between two types of decision makers, called a defender and invader. Uno and Katagiri [12] considered the situation that the defender locates defensive facilities for preventing the invaders from reaching some important sites,

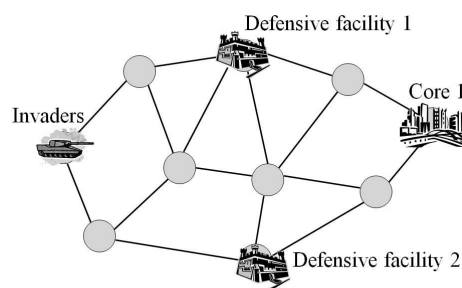


Fig. 1. An example of DLPs

called cores. Examples of their DLPs are “a government of a country locates self-defense bases to prevent her/his aggressors from reaching the capital of the country,” “a coach of a soccer/football team locates its players to defend goal from its opponent team,” etc. By representing the location region as a network and the core is a vertex in the network and the defender and the invaders as an upper and a lower DM, their DLP can be formulated as a bilevel programming problem. An example of DLPs is illustrated in Fig. 1.

Although the DLP can be expected for many situations that there are both a defender and invaders, their DLP assumed that the site of invaders is on a given vertex and their energy is a given value. This means that their DLP cannot deal with the situations that there are several vertices with several types of invaders given uncertainly, e.g., the defender locates fire stations for preventing that wildfire, which can be regarded as invaders, marching toward important sites. Then, in this paper, we consider a new DLP that invaders' sites and energies are given uncertainly. By representing their existing vertices and energies as random variables with scenarios, the new DLP can be formulated as a multiobjective bilevel stochastic programming problem. We define each objective function of the DLP as minimizing the degree of invasion for each core

under a probability given by the defender.

In order to find a satisficing solution for the defender, we apply an interactive fuzzy satisficing method in order to find a satisficing solution for multiobjective linear fractional programming problems. As other applications of the method, Katagiri et al [6] considered multiobjective fuzzy random linear programming problems whose objectives are possibility and probability, and Kato et al [7] considered fractile criterion optimization for multiobjective stochastic integer programming problems. For applying the method to the multiobjective DLP, we need to solve minimax problems for finding M-Pareto optimal solutions, which are candidates of satisficing solutions. We propose its solution method based upon tabu search algorithm proposed by Glover [3]; for the details of algorithm, the readers can refer to the book of Reeves [10]. Since there are many M-Pareto optimal solutions, in order to solve the problem efficiently, we introduce the algorithm to the idea of strategic oscillation, suggested by Hanafi and Freville [5], by utilizing characteristics of DLPs. We show an efficiency of the method by applying it to examples of the multiobjective DLPs.

The remaining structure of this article is organized as follows: In Section 2, we formulate an extended DLP as a multiobjective bilevel stochastic programming problem. We propose an interactive fuzzy satisficing method for finding a satisficing solution of the defender in Section 3. For solving minimax problems in the method, we propose a tabu search algorithm with strategic oscillation in Section 4. In Section 5, we show the efficiency of the interactive fuzzy satisficing method by applying it to examples of the DLPs. Finally, concluding comments and future extensions are summarized in Section 6.

II. FORMULATION OF STOCHASTIC DLP

We represent a location region as network $N = (V, E)$, where $V = \{v_1, \dots, v_n\}$ is a set of n vertices and E is a set of edges. For edge $e \in E$, let $d(e)$ be its weight, meaning the distance between vertices with e . In the network N , there are k core vertices, denoted by $c_1, \dots, c_k \in V$. that the defender needs to defend from the invader.

In the new DLP, first the defender locates her/his defensive facilities before a scenario is determined., and then the invaders choose sets of paths with knowing a determined scenario. The defender can locate at most one defensive facility on each vertex in V . For i -th vertex, let $q_i \in \{0, 1\}$ denote the variable such that $q_i = 1$ if a defensive facility is located on v_i and $q_i = 0$ otherwise. Then the decision vector of the DLP for the defender can be represented as $\mathbf{q} := (q_1, \dots, q_n)^T$.

On the other hand, sites of the invaders are given randomly with scenarios, whose set are denoted by S . For each scenario $s \in S$, the set of the invaders' sites and their energies are given as $\Xi(s) \subseteq V$ and $\gamma(\xi), \xi \in \Xi(s)$, respectively, Let p_h^ξ be the path ξ to h -th core c_h . Then, for scenario s and h -th core, the decision vector of the DLP for the invader can be represented as $\mathbf{p}_h(s) = \{p_h^\xi | \xi \in \Xi(s)\}$.

We define the degree of invasion for h -th core as the following function:

$$f_h(\mathbf{q}, \mathbf{p}_h(s) | s) := 0 \vee \max_{\xi \in \Xi(s)} \{\gamma(\xi) - D(p_h^\xi) - \beta \cdot W(\mathbf{q}, p_h^\xi)\}, \quad (1)$$

where

$$D(\mathbf{p}_h(s)) := \sum_{e \in p_h^\xi} d(e), \quad (2)$$

β is a given positive number meaning the capacity that a defensive facility can reduce the energy of invaders on ξ , and $W(\mathbf{q}, p_h^\xi)$ is the number of facilities in p_h^ξ . Equation (1) is the objective function of the invader for scenario s and h -th core. Hence the invaders have k objective function for each scenario.

On the other hand, Since the defender needs to decide her/his location before a scenario is determined, an objective function value of the defender for h -th core can be represented as a random variable whose value for each scenario s is $f_h(\mathbf{q}, \mathbf{p}_h(s) | s)$. By integrating decision vectors of the invaders for all scenarios, we represent paths to h -th core as a random path as $\bar{\mathbf{p}}_h$. Then, the objective function of the defender for h -th core is denoted by $\bar{f}_h(\mathbf{q}, \bar{\mathbf{p}}_h)$. Hence the defender has k objective functions.

Next we represent feasible sets of the defender and the invaders. For the defender, we set m linear constraints represented as follows:

$$A\mathbf{q} \leq \mathbf{b}, \quad (3)$$

where

$$A := (\mathbf{a}_1, \dots, \mathbf{a}_m)^T = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix}, \quad (4)$$

$$\mathbf{b} := (b_1, \dots, b_m)^T, \quad (5)$$

and $a_{ij}, b_i \geq 0$ for any $i = 1, \dots, m, j = 1, \dots, n$. Hence the feasible set of the defender can be represented as follows:

$$SD := \{\mathbf{q} \in \{0, 1\}^n | A\mathbf{q} \leq \mathbf{b}\}. \quad (6)$$

On the other hand, the feasible set of the invader for scenario $s \in S$ and h -th core can be regarded as the family of sets of paths from all vertices in $\Xi(s)$ to c_h , denoted by $SI_h(s)$. Therefore, the extending DLP can be formulated as the following stochastic bilevel programming problem:

$$\left. \begin{array}{l} \underset{\mathbf{q}}{\text{minimize}} \quad (\bar{f}_1(\mathbf{q}, \bar{\mathbf{p}}_1), \dots, \bar{f}_k(\mathbf{q}, \bar{\mathbf{p}}_k))^T \\ \text{where } \bar{\mathbf{p}}_1, \dots, \bar{\mathbf{p}}_k \text{ solve} \\ \underset{\mathbf{p}_h(s)}{\text{maximize}} \quad f_h(\mathbf{q}, \mathbf{p}_h(s) | s), \\ \hspace{15em} \forall s \in S, \forall h = 1, \dots, k \\ \text{subject to} \quad \mathbf{q} \in SD, \\ \hspace{10em} \mathbf{p}_h(s) \in SI_h(s), \\ \hspace{15em} \forall s \in S, \forall h = 1, \dots, k. \end{array} \right\} \quad (7)$$

For solving (7), we give $\sigma_h \in (0, 1]$, $h = 1, \dots, k$ for each core, meaning the probability that the defender wants to guarantee for invasion to each core. Then, we transform

(7) to the following bilevel programming problem by using k auxiliary variables $\alpha_1(\mathbf{q}), \dots, \alpha_k(\mathbf{q})$:

$$\left. \begin{array}{l} \underset{\mathbf{q}}{\text{minimize}} \quad (\alpha_1(\mathbf{q}), \dots, \alpha_k(\mathbf{q}))^T \\ \text{subject to} \quad \Pr(\bar{f}_1(\mathbf{q}, \bar{\mathbf{p}}_1) \leq \alpha_1(\mathbf{q})) \geq \sigma_1, \\ \quad \quad \quad \vdots \\ \quad \quad \quad \Pr(\bar{f}_k(\mathbf{q}, \bar{\mathbf{p}}_k) \leq \alpha_k(\mathbf{q})) \geq \sigma_k, \\ \text{where } \bar{\mathbf{p}}_1, \dots, \bar{\mathbf{p}}_k \text{ solve} \\ \underset{\mathbf{p}_h(s)}{\text{maximize}} \quad f_h(\mathbf{q}, \mathbf{p}_h(s)|s), \\ \quad \quad \quad \forall s \in S, \forall h = 1, \dots, k \\ \text{subject to} \quad \mathbf{q} \in SD, \\ \quad \quad \mathbf{p}_h(s) \in SI_h(s), \\ \quad \quad \quad \forall s \in S, \forall h = 1, \dots, k. \end{array} \right\} \quad (8)$$

III. INTERACTIVE FUZZY SATISFICING METHOD

For h -th core, $h = 1, \dots, k$, we provide a satisfaction level of the defender for each objective function as the following linear membership function:

$$\mu_h(\alpha_h(\mathbf{q})) = \begin{cases} 1, & \alpha_h(\mathbf{q}) \leq \alpha_h^1, \\ \frac{\alpha_h(\mathbf{q}) - \alpha_h^1}{\alpha_h^0 - \alpha_h^1}, & \alpha_h^1 \leq \alpha_h(\mathbf{q}) \leq \alpha_h^0, \\ 0, & \alpha_h^0 \leq \alpha_h(\mathbf{q}), \end{cases} \quad (9)$$

where α_h^1 means the infimum of k -th objective function value satisfying the defender sufficiently, and α_h^0 the supremum of that no satisfying her/him.

For estimating all satisfaction levels for objective functions simultaneously, we use the concept of M-Pareto optimal solutions.

Definition 1: $\mathbf{q}^*$ is called an M-Pareto optimal solution to (8) if and only if there does not exist another $\mathbf{q}$ such that $\mu_h(\alpha_h(\mathbf{q})) \geq \mu_h(\alpha_h(\mathbf{q}^*))$ for all $h = 1, \dots, k$ and $\mu_{h'}(\alpha_{h'}(\mathbf{q})) \neq \mu_{h'}(\alpha_{h'}(\mathbf{q}^*))$ for at least one $h' \in \{1, \dots, k\}$.

Interactive fuzzy satisficing method is a method to find a satisficing solution among a set of M-Pareto optimal solutions by utilizing information of M-Pareto optimal solutions computed on the way of this method. We propose the interactive fuzzy satisficing method to the multi-objective DLPs in order to find a satisficing solution for the defender among a set of M-Pareto optimal solutions. Let $(\bar{\mu}_1, \dots, \bar{\mu}_k)$ be a k -tuple of reference membership levels of membership functions $\mu_1, \dots, \mu_k$, respectively.

Interactive fuzzy satisficing method for the stochastic DLPs

- Step 1: Set membership function (9) for each $h = 1, \dots, k$ by giving α_h^1 and α_h^0 .
Step 2: Set initial reference membership levels $(\bar{\mu}_1, \dots, \bar{\mu}_k) = (1, \dots, 1)$.

Step 3: For the given $(\bar{\mu}_1, \dots, \bar{\mu}_k)$, solve the following corresponding minimax problem:

$$\left. \begin{array}{l} \underset{\mathbf{q}}{\text{minimize}} \quad \max_{h=1, \dots, k} \{ \bar{\mu}_h - \mu_h(\alpha_h(\mathbf{q})) \\ \quad \quad \quad + \rho \sum_{l=1}^k (\bar{\mu}_l - \mu_l(\alpha_l(\mathbf{q}))) \} \\ \text{subject to} \quad \Pr(\bar{f}_1(\mathbf{q}, \bar{\mathbf{p}}_1) \leq \alpha_1(\mathbf{q})) \geq \sigma_1, \\ \quad \quad \quad \vdots \\ \quad \quad \quad \Pr(\bar{f}_k(\mathbf{q}, \bar{\mathbf{p}}_k) \leq \alpha_k(\mathbf{q})) \geq \sigma_k, \\ \text{where } \bar{\mathbf{p}}_1, \dots, \bar{\mathbf{p}}_k \text{ solve} \\ \underset{\mathbf{p}_h(s)}{\text{maximize}} \quad f_h(\mathbf{q}, \mathbf{p}_h(s)|s), \\ \quad \quad \quad \forall s \in S, \forall h = 1, \dots, k \\ \text{subject to} \quad \mathbf{q} \in SD, \\ \quad \quad \mathbf{p}_h(s) \in SI_h(s), \\ \quad \quad \quad \forall s \in S, \forall h = 1, \dots, k, \end{array} \right\} \quad (10)$$

where ρ is a sufficiently small positive number.

Step 4: If the defender is satisfied by the given solution of (10), then it is a satisficing solution of the defender. Otherwise, ask the defender to update $(\bar{\mu}_1, \dots, \bar{\mu}_k)$ by considering the current membership function values, and return to Step 3.

The following theorem introduces an important relationship between (8) and (10).

Theorem 1: (i) If $\mathbf{q}^* \in SD$ is an optimal solution of (10) with some $(\bar{\mu}_1, \dots, \bar{\mu}_k)$, then $\mathbf{q}^*$ is also an M-Pareto optimal solution of (8).

(ii) If $\mathbf{q}^* \in SD$ is an M-Pareto optimal solution of (8), then there exists $(\bar{\mu}_1, \dots, \bar{\mu}_k)$ such that $\mathbf{q}^*$ is an optimal solution of (10).

Proof: (i) if $\mathbf{q}^*$ is not an M-Pareto optimal solution of (8), then there exist a solution $\mathbf{q}$ such that $\mu_h(\alpha_h(\mathbf{q})) \geq \mu_h(\alpha_h(\mathbf{q}^*))$ for all $h = 1, \dots, k$ and $\mu_{h'}(\alpha_{h'}(\mathbf{q})) \neq \mu_{h'}(\alpha_{h'}(\mathbf{q}^*))$ for at least one $h' \in \{1, \dots, k\}$. This means that $\mathbf{q}$ is better than $\mathbf{q}^*$ for (10) with any $(\bar{\mu}_1, \dots, \bar{\mu}_k)$, and then this contradicts the optimality of $\mathbf{q}^*$.

(ii) if there does not exist $(\bar{\mu}_1, \dots, \bar{\mu}_k)$ such that $\mathbf{q}^*$ is an optimal solution of (10), then for any $(\bar{\mu}_1, \dots, \bar{\mu}_k)$, there exist a solution $\mathbf{q}$ such that $\bar{\mu}_h - \mu_h(\alpha_h(\mathbf{q})) \leq \bar{\mu}_h - \mu_h(\alpha_h(\mathbf{q}^*))$ for all $h = 1, \dots, k$ and $\bar{\mu}_{h'} - \mu_{h'}(\alpha_{h'}(\mathbf{q})) \neq \bar{\mu}_{h'} - \mu_{h'}(\alpha_{h'}(\mathbf{q}^*))$ for at least one $h' \in \{1, \dots, k\}$. This contradicts that $\mathbf{q}^* \in SD$ is an M-Pareto optimal solution of (8). ■

At Step 3 in the above method, we need to find an optimal solution of (10) for updated reference membership levels. In the next section, we propose an efficient solution method for (10).

IV. TABU SEARCH BASED ON STRATEGIC OSCILLATION

First, we consider the complexity to solve (10) strictly. For each location $\mathbf{q} \in SD$, we need to compute its objective

function values. This means that for each scenario $s \in S$, we need to find the k sets of paths $p_1(s), \dots, p_k(s)$ so as to minimize $f_1(q, p_1(s)|s), \dots, f_k(q, p_k(s)|s)$ among all sets of paths from vertices in $\Xi(s)$ to cores $c_1, \dots, c_k$. We can utilize a version of Warshall-Floyd Algorithm [1] to find such k sets of paths at a polynomial time. However, since the number of defensive locations is 2^n , solving (10) strictly needs enormous computational times for large-scale DLPs. Therefore, we propose an efficient solution method for (10) based upon tabu search algorithm with strategic oscillation.

The tabu search is one of local search methods, which replicates movement from a current solution q^{now} to a next solution q^{next} . In the proposed method, we define a move as adding/subtracting one to/from an element of q^{now} , and a neighborhood of q^{now} , denoted by $\mathcal{N}(q^{\text{now}})$, as the set of all solutions that can be transferred by only one move from it. From $\mathcal{N}(q^{\text{now}})$, q^{next} is chosen as the best solution in the sense of a given criterion, e.g. objective function value. However, without modification, a circulation of certain chosen moves may occur on the way of search, and then it only can examine a limited local area. Then, for cases that a move is chosen in the search, the tabu constraint for its opposite move is activated for given tenure, called the tabu tenure and denoted by T_1 . Then the activated moves are forbidden to choose, called tabu, even if the moves make the objective function value of (10) best in $\mathcal{N}(q^{\text{now}})$. Tabu are memorized in a tabu list through the search.

Although the tabu search method has the advantage of finding several local optimal solutions efficiently, the feasible set of (10) is generally wide and there are many local optimal solutions in the feasible set. Then, we introduce the concept of strategic oscillation for finding various types of local optimal solutions. The following function, meaning the degree of feasibility for q , plays an important role in the strategic oscillation:

$$r(q) := \max_{i=1, \dots, m} \{a_i \cdot q - b_i\} \quad (11)$$

Tabu search algorithm with strategic oscillation for the stochastic DLPs

- Step 0: Generate the initial searching matrix q^{now} , and initialize the tabu list and other parameters. If $q^{\text{now}} \in SD$, then go to Step 4.
- Step 1: Move q^{now} to q^{next} by decreasing an element of q^{now} for the aim of increasing $r(q^{\text{next}})$ as much as possible. This step is repeated until it is satisfied $r(q^{\text{next}}) \geq 0$, that is $q^{\text{next}} \in SD$.
- Step 2: Move q^{now} to q^{next} for the aim of improving the objective function value of (10). This step is repeated at a given tenure, denoted by $T_2 > 0$.
- Step 3: Move q^{now} to q^{next} by decreasing an element of q^{now} for the aim of increasing $r(q^{\text{next}})$ as much as possible. This step is repeated until $r(q^{\text{next}})$ is less than a given value, denoted by r_{low} .

- Step 4: Move q^{now} to q^{next} by increasing an element of q^{now} for the aim of improving the objective function value of (10). This step is repeated until one step short of $r(q^{\text{next}}) < 0$.
- Step 5: Do the same operations as Step 2.
- Step 6: Move q^{now} to q^{next} by increasing an element of q^{now} for the aim of improving the objective function value of (10) without the feasibility of (10). This step is repeated until $r(q^{\text{next}})$ is more than a given value, denoted by r_{upp} .
- Step 7: If the iteration count of the algorithm reaches a given number, denoted by T_3 , then terminate. Otherwise, return to Step 1.

V. NUMERICAL EXPERIMENTS

This section shows the efficiency of the method introduced in the previous sections by applying to a given numerical example of the stochastic multiobjective DLP. We consider the DLP on network $N = (V, E)$, where V and E have $n = 100$ vertices and 1000 edges, respectively, and there are $k = 3$ cores in V . For each edge $e \in E$, w_e is given a value in $\{1, \dots, 100\}$ randomly.

For the defender, the capacity of the defensive facility is $\beta = 30$, and there are the following $m = 30$ constraints: for (4), each a_{ij} , $i = 1, \dots, m$ and $j = 1, \dots, n$, is given as a value in $\{1, \dots, 100\}$ randomly, and

$$b_i = \theta_i \cdot \sum_{j=1}^n a_{ij}, \quad (12)$$

where θ_i , $i = 1, \dots, m$ is a given value in $[0.1, 0.3]$ randomly. For (8), we set $\sigma_h = 0.7$, $h = 1, 2, 3$.

For the invaders, we give $s = 3$ scenarios each of whose possibilities is given randomly. For each scenario, their sites, number of vertices, and their initial energies are given in N , $\{1, \dots, 3\}$, and $[100, 300]$, randomly and respectively.

For the tabu search algorithm in Section IV, we set $T_1 = 20$. In Steps 2 and 4 of the algorithm with strategic oscillation, we set $T_2 = 7$ and $T_3 = 100$, respectively.

For the interactive fuzzy satisficing method in Section III, at Step 0, we define three membership functions μ_1, μ_2, μ_3 whose parameters are $(\alpha_1^L, f_1^U) = (30, 90)$, $(\alpha_2^L, f_2^U) = (40, 120)$, and $(\alpha_3^L, f_3^U) = (60, 200)$, respectively. In this example of the DLP, we assume that the defender regards that c_3 is the most important, c_2 is the second, and c_1 is the last. Then, the defender thinks that she/he hopes to improve μ_3 even if μ_1 and μ_2 are worse, and similarly to improve μ_2 even if μ_1 is worse. TABLE I shows computational results for applying the method to the example of DLP, where we obtain them computed by Dell Studio XPS 8100 (CPU: Inter^R CORETM i7-860 2.80GHz $\times$ 2, RAM: 6.00GB).

In TABLE I, for the satisfaction levels of the M-Pareto optimal solution obtained at Iteration 1, the defender decreased $\bar{\mu}_1$ and $\bar{\mu}_2$ in order to improve the value of μ_3 at Iteration 2. Then, the defender can obtain the solution improving the value of $\bar{\mu}_3$. However, the solution did not satisfy the defender,

TABLE I
COMPUTATIONAL RESULTS

Iteration	1	2	3	4
$\bar{\mu}_1$	1.0	0.7	0.7	0.7
$\bar{\mu}_2$	1.0	0.9	0.8	0.8
$\bar{\mu}_3$	1.0	1.0	1.0	0.95
$\mu_1(f_1(\mathbf{q}^*, \mathbf{p}^*))$	0.564	0.356	0.475	0.501
$\mu_2(f_2(\mathbf{q}^*, \mathbf{p}^*))$	0.570	0.595	0.532	0.570
$\mu_3(f_3(\mathbf{q}^*, \mathbf{p}^*))$	0.558	0.680	0.747	0.727
CPU Time (seconds)	57.24	54.68	60.77	60.70

because the values of μ_1 for its solution is too bad. Then, at Iteration 3, the defender reduced $\bar{\mu}_2$ a little. Although the values of μ_1 for its solution is recovered, that of μ_2 is too bad. Then, the defender reduced $\bar{\mu}_3$ a little at Iteration 4. As a result, the defender obtained a satisficing solution and the method is terminated.

VI. CONCLUSION

In this paper, we have proposed a new location problem by considering a multiobjective DLP with invaders whose sites are given randomly. By representing a location region as a network, we have formulated the DLP as a stochastic multiobjective bilevel programming problem. For finding its satisficing solution of the defender, we have proposed an interactive fuzzy satisficing method with tabu search algorithm with strategic oscillation. Its efficiency has been shown by applying the proposed method for examples of the DLP.

For the proposed DLP, both the capacity of defensive facilities and the energy of invaders are given determinate values. However, it is often cases that these data include vagueness and ambiguity. Considering the DLP representing these data as fuzzy random variables [6], [13], which can be formulated as a fuzzy random programming problem, is an important future research.

REFERENCES

- [1] T. H. Cormen, C. E. Leiserson, R. L. Rivest, *Introduction to algorithms*. MIT Press, 1990.
- [2] Z. Drezner, *Competitive Location Strategies for Two Facilities*. Regional Science and Urban Economics, vol.12, vol.3, pp.485-493, 1982.
- [3] F. Glover, *Future Paths for Integer Programming and Links to Artificial Intelligence*. Computers & Operations Research, vol.5, pp.533-549, 1986.
- [4] S. L. Hakimi, *On Locating New Facilities in a Competitive Environment*. European Journal of Operational Research, vol.12, pp.29-35, 1983.
- [5] S. Hanafi, A. Freville, *An Efficient Tabu Search Approach for the 0-1 Multidimensional Knapsack Problem*. European Journal of Operational Research, vol.106, pp.659-675, 1998.
- [6] H. Katagiri, M. Sakawa, K. Kato, I. Nishizaki, *Interactive Multiobjective Fuzzy Random Linear Programming: Maximization of Possibility and Probability*. European Journal of Operational Research, vol.188, no.2, pp. 530-539, 2008.
- [7] K. Kato, M. Sakawa, H. Katagiri, C. Perkgoz, *An interactive fuzzy satisficing method based on fractile criterion optimization for multiobjective stochastic integer programming problems*. Expert Systems with Applications, vol.37, no.8, pp.6012-6017, 2010.
- [8] T. C. Miller, T. L. Friesz, R. L. Tobin, *Equilibrium Facility Location on Networks*. Springer Verlag, Berlin, 1996.
- [9] J. R. O'Hanley, R. L. Church, *Designing Robust Coverage Networks to Hedge against Worst-Case Facility Losses*. European Journal of Operational Research, vol.209, no.1, pp.23-36, 2011.

- [10] C. R. Reeves, ed., *Modern Heuristic Techniques for Combinatorial Problems*. Blackwell Scientific Press, Oxford, 1993.
- [11] S. Shioda, Z. Drezner, *A Competitive Facility Location Problem on a Tree Network with Stochastic Weights*. European Journal of Operational Research, vol. 149, no. 1, pp.47-52, 2003.
- [12] T. Uno, H. Katagiri, *Single- and Multi-Objective Defensive Location Problems on a Network*. European Journal of Operational Research, vol.188, no.1, pp.76-84, 2008.
- [13] T. Uno, H. Katagiri, K. Kato, *A Stackelberg Location Problem on a Tree Network with Fuzzy Random Demands*. Advances in Intelligent Decision Technologies (eds: G. Phillips-Wren, L.C. Jain, K. Nakamatsu, R.J. Howlett), pp.581-588, Springer, 2010.
- [14] J. Yang, M. Zhang, B. He, C. Yang, *Bi-level programming model and hybrid genetic algorithm for flow interception problem with customer choice*. Computers & Mathematics with Applications, vol.57, no.11-12, pp.1985-1994, 2009.