

Trajectory-based Control under ZMP Constraint for the 3D Biped Walking via Fuzzy Control

Hsiu-Ming Wu, *Member, IEEE*
Opto-Mechatronics Technology Center
National Taiwan University of Science and Technology
Taipei, Taiwan, R.O.C.
email: D9307205@mail.ntust.edu.tw

Chih-Lyang Hwang, *Senior Member, IEEE*
Department of Electrical Engineering
National Taiwan University of Science and Technology
Taipei, Taiwan, R.O.C.
e-mail: chih-lyang_hwang@hotmail.com

Abstract—In this study, a fuzzy control policy is presented for dynamic walking of a biped robot that is modeled as a simulated five-link biped robot. Due to complex ground contact models, it is difficult to precisely model its dynamics. Besides, a spring/damper ground model and a simplified inverted pendulum model are presented to represent the ground contact relation and the approximated model of biped robots as well. Moreover, Finite State Machine (FSM) is utilized to decompose a cycle of walking gait. Under zero moment point (ZMP) constraint, each joint follows desired trajectories in each state that are generated by quintic spline curve through the model free controller (i.e., fuzzy control). In addition, its robustness is investigated by imposing pulse perturbation on torso for some directions. The results are demonstrated in simulations as well as animation and then the proposed controller is compared with a PID controller as well.

Keywords—Biped robot; ZMP constraint; Fuzzy control; Finite state machine; PID;

I. INTRODUCTION

The biped robot possesses nonlinear dynamics and numerous degrees of freedom that are difficult to manipulate. For biped robots, it is also unavoidable to be subjected to all kinds of external disturbance (e.g. contact with ground, different ground situations). As a result, accurately modeling the dynamics of biped robot systems is quite difficult. Additionally, walking stably is also another challenge for biped robots.

A fuzzy control policy with robustness is adopted for biped walking and simultaneously a FSM approach is utilized to decompose the cycle of the walking gait. Then the biped robot system is approximated by a simplified inverted pendulum model and keeps dynamic walking based on the ZMP constraint. In summary, the contribution of this study is the development of the fuzzy control policy with robustness for the walking of the simulated 3D biped robot. The simulation and animation are done to demonstrate the validity of the proposed methodology.

A. Related Works

In recent years, the research about walking of biped robots is a popular topic. Walking stably and keeping balance are essential functions. For the dynamic walking of biped

robots, ZMP analysis is an important stability criterion for dynamic walking stably [1, 2]. Recently, [3]-[5] discuss stable biped walking through direct regulation of ZMP and generate stable walking trajectories. Besides, in order to enhance walking robustness, Morimoto and Atkeson [6] use differential dynamic programming (DDP) with a minimax criterion to make the humanoid robot generate low torque for walking and insure the policy is robust. Additionally, for keeping balance, Atkeson and Stephens [7,8] investigate ankle, hip and stepping strategies to help the humanoid robot recovery under the external push. From above studies, we can get the information that ZMP control and the walking with robustness are important for biped robots. Furthermore, due to the complex nonlinear dynamics and high dimensional state space, it's quite difficult to model the biped robot precisely. In general, it has two solvable ways. One is for the use of a simplified inverted pendulum model and some studies discuss it and have verified its effectiveness. Stramigioli et al. [9] adopt a simplification of multi-body dynamics that is achieved by instantaneously reducing it to be an equivalent single inertial entity in legged robot. In [10], Goswami introduced the Reaction Mass Pendulum (RMP) and presented an extension of the inverted pendulum that modeled the rotational inertia and angular momentum of a robot through its centroidal composite rigid body. The other one is for the use of the fuzzy controller. So far, fuzzy control has already been applied into many controlled systems including bipeds [11]-[15] as well as different kinds of commercial products successfully. The reasons are that it can handle nonlinear system well and is ease of implementation.

B. Control Policies For Walking

In general, the walking process is composed of a single support phase (SS) and a double support phase (DS) for the walking phase of a biped robot. In this work, a FSM approach is presented for the walking gaits of biped robots that decompose walking gaits into different states based on SS and DS phase. In these states, a quintic spline curve is utilized to generate smooth desired trajectories of each joint. Furthermore, a simplified inverted pendulum model is used to approximate the biped robot system and the biped robot keeps balance during walking based on the ZMP constraint. Concerning a trajectory tracking of each joint, it is achieved

by fuzzy feedback control policy. An important contribution of fuzzy systems theory is to provide a systematic procedure for transforming a set of linguistic rules into a nonlinear mapping. A fuzzy control algorithm [16]-[18] consists of a set of heuristics decision rules and is regarded as an intelligent control algorithm, which is attractive in many cases where the controlled system can't be well defined or modeled. The fuzzy theory uses "IF-THEN" rules to state the relationship between the input and output of the control plan. Also, it uses the linguistic variables instead to describe the characteristics of the system.

This paper is organized as follows: in section II, the system dynamics is constructed and trajectory planning, control policies for the biped walking robot are described in section III, and the simulation and animation results demonstrate its effectiveness under the ideal situation (i.e., without any perturbations) and external disturbance in section IV. Finally, the conclusions and further works of this paper are given in section V.

II. SYSTEM DYNAMICS AND TRAJECTORY PLANNING

The system (as in Fig. 1) is a simulated 3D biped composed of five-links: one torso, two thighs, and two calves. It is a symmetric 12 DOF system including roll and pitch for each hip, pitch for a knee and roll, pitch, and yaw for each ankle on each side. Therefore it is actuated by 12 torque-controlled joints. These joints are respectively simplified as hip abduction/adduction (HAA) that is roll angle between the pelvis and the thigh, hip flex/extend (HFE) that is the pitch angle between the pelvis and the thigh, knee flex/extend (KFE) that is pitch angle between the thigh and the calf, ankle abduction/adduction (AAA) that is the roll angle between the calf and vertical, ankle flex/extend (AFE) that is pitch angle between the calf and vertical and ankle rotation (AR) that is the rotation about the calf. Furthermore, the system states position and velocity of each DOF, so it is a 24 dimensional state space. Also, the system parameters for the simulation are similar to our real humanoid robot made by Sarcos. Its weight is 78 kg, with a 50 kg torso and 14 kg legs. The legs are 0.81 m long and the center of mass is 1 m above the ground when it stands straight. The distance between the hips is 0.213 m. In addition, the biped robot will interact with the ground during walking. Therefore, considering impact with interaction is necessary. In this paper, its ground contact relation is modeled as a spring/damper ground model as following Equations (1)-(4):

$$F_z = k_z z + b_z \dot{z} \quad (1)$$

$$F_x = k_x (x - x_0) + b_x \dot{x} \quad (2)$$

$$F_y = k_y (y - y_0) + b_y \dot{y} \quad (3)$$

$$M_z = k_a (a - a_0) + b_a \dot{a} \quad (4)$$

where x_0 and y_0 are initial contact positions and a_0 is

initial contact yaw angle of the foot of Z -axis, k_z , k_x , k_y and k_a are spring constants, b_z , b_x , b_y and b_a are damping constants, F_z , F_x , F_y and M_z are friction force and moment respectively. According to above parameters, SDFAST software is used to generate the rigid body dynamics for the biped robot. For this study, the desired trajectories of the hip, knee and ankle joints are all generated by quintic spline curves during walking. At knot points the quintic spline curves can enforce second derivative continuity of combined curve so that smooth transition, acceleration and jerk could be ensured. Each joint is driven to its desired position through fuzzy feedback controller that is described by the following section.

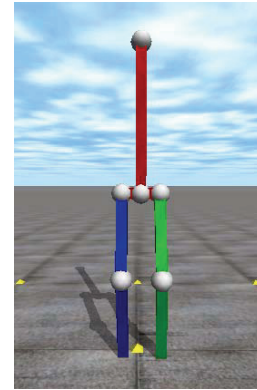


Fig. 1. Simulated 3D biped robot.

III. CONTROL POLICIES FOR BIPED ROBOT

A. Approximation Using An Inverted Pendulum Model and The ZMP Constraint

In order to simplify dynamic analysis, the biped model can be regarded as an inverted pendulum model that has been proved to be able to represent whole biped dynamics effectively. Consequently, in this study a simplified inverted pendulum model in the sagittal plane represented as Fig. 2 is described to approximate the biped robot model where forward direction is the positive X axis and above direction is positive Z axis. The height for the Center of Mass (CoM) in the walking process is assumed to be constant. From geometric relations, Equation (5) is derived where F is a net force applied on the CoM, g is the gravity acceleration and θ is the angle with vertical. Equation (6) expresses energy conservation and v is the velocity of the CoM. From Equations (5) and (6), the minimum roll distance of x direction (i.e., $x_{\min}$) is represented as Equation (7). During walking, CoM is kept within ZMP constraints via regulating the walking step and gait.

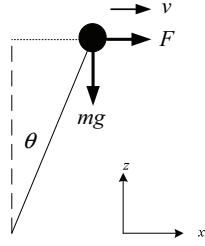


Fig.2. A simplified inverted pendulum model in the sagittal plane.

$$F/mg = \tan \theta = -x/z \quad (5)$$

$$-\int F dx = m v^2 / 2 \quad (6)$$

$$x_{\min} = \sqrt{x^2 - z v^2 / g} \quad (7)$$

Calculating $x_{\min}$ value is to predict how far the biped system will move to. Liftoff action occurs when $x_{\min} < x_{d\min}$ where $x_{d\min}$ is a desired minimum move distance. In this paper, the value of $x_{d\min}$ is assigned as 0.07. In addition, the touchdown action occur a fixed time interval after turnaround.

B. Finite State Machine

Our proposed controllers are based on a FSM, with each state having its own trajectory for internal joint angles. A walking cycle of the human walking gait is imitated and decomposed by a FSM that is employed to plan walking gaits including eleven states. (1) WAITING (2) DS1 (double support, left leg in front) (3) LIFTOFF1 (double support, lifting right leg) (4) SSL1 (first phase of single support, left leg in stance) (5) SSL2 (second phase of single support, left leg in stance) (6) DS2 (double support, right leg in front) (7) LIFTOFF2 (double support, lifting left leg) (8) SSR1 (first phase of single support, right leg in stance) (9) SSR2 (second phase of single support, right leg in stance) (10) LAUNCH1 (squat) (11) LAUNCH2 (launch state, like DS1). These states form one cycle's walking gaits of the biped robot and transfer into other state at prescribed time or conditions described in Fig. 3 where waiting time and launch1 time are prescribed time in advance. Also, foot states and CoM velocity of Y direction are transition conditions. Besides the desired smooth trajectories generated by the quintic spline curve in each state and in any given state, the joints apply torques computed by fuzzy control in order to drive each joint to its desired local angle.

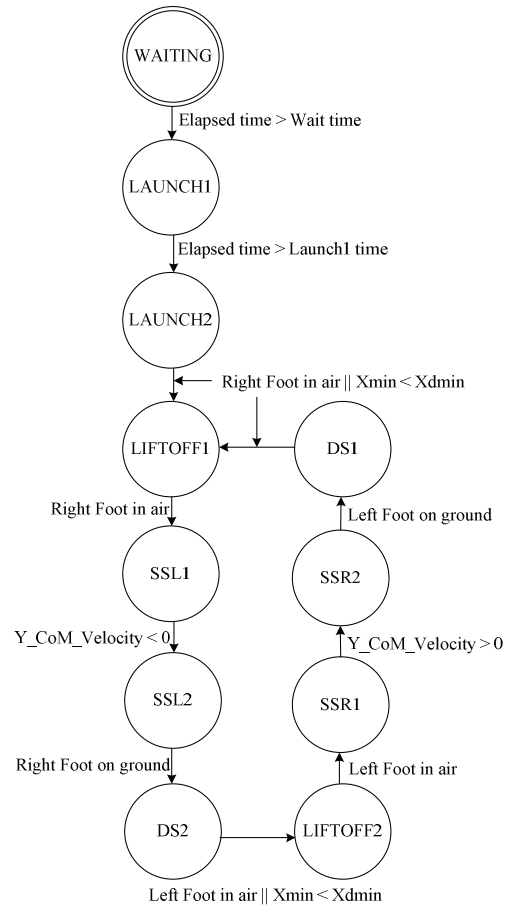


Fig. 3. FSM diagram for the biped walking.

C. Fuzzy Control Policy

The basic configuration of the fuzzy system consists of a fuzzifier, an inference engine and a defuzzifier. The fuzzifier part is to map a crisp set into a fuzzifier set. Moreover, two normalizing input scaling factors are determined. They make error and its derivative located in between -1 and 1 where error $e_i(t)$ is individual difference between desired position and actual position of each joint. Therefore, two inputs of fuzzy rule table of the i th subsystem are $\bar{e}_i(t) = g_{e_i} e_i(t)$ and $\dot{\bar{e}}_i(t) = g_{\dot{e}_i} \dot{e}_i(t)$, $i = 1 \sim 12$, where g_{e_i} and $g_{\dot{e}_i}$ are two normalizing input scaling factors respectively. $\bar{e}_i(t)$ and $\dot{\bar{e}}_i(t)$ are called fuzzy variables. They are quantized into the following eleven qualitative fuzzy variables: (i) Positive Huge (PH) (ii) Positive Big (PB) (iii) Positive Medium (PM) (iv) Positive Small (PS) (v) Positive Infinite (PI) (vi) Zero (ZE) (vii) Negative Infinite (NI) (viii) Negative Small (NS) (ix) Negative Medium (NM) (x) Negative Big (NB) (xi) Negative Huge (NH). The membership functions of the fuzzy variables are with triangular type as in Fig. 4 in this paper.

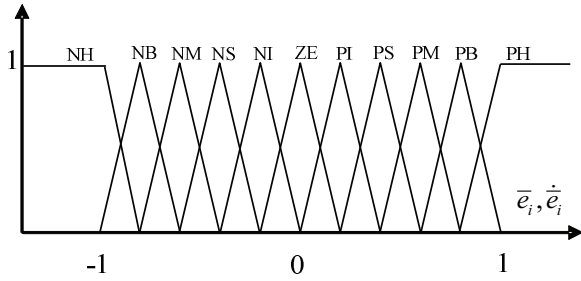


Fig. 4. Membership functions with triangular type.

In addition, the inference engine is usually composed of a series of fuzzy rules i.e., “IF-THEN” rules. The fuzzy logic subsystem i as in Fig. 5 performs a mapping from $X_i \in \mathfrak{R}^2$ to $\mathfrak{R}$.

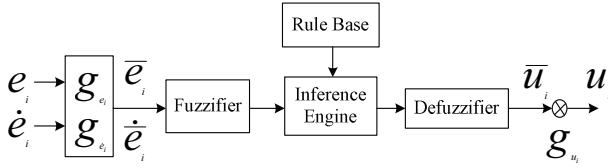


Fig. 5. Configuration of the i th fuzzy logic system.

There are l fuzzy control rules and the upper script k denotes the k th fuzzy rule:

$$\text{IF } \bar{e}_i \text{ is } F_{1i}^k \text{ and } \dot{\bar{e}}_i \text{ is } F_{2i}^k, \text{ THEN } \bar{u}_i \text{ is } G_i^k \quad (8)$$

where $[\bar{e}_i(t) \ \dot{\bar{e}}_i(t)]^T \in X_i \subset \mathfrak{R}^2$, with $\bar{e}_i(t) = g_{e_i} e_i(t)$, $\dot{\bar{e}}_i(t) = g_{\dot{e}_i} \dot{e}_i(t)$, and $\bar{u}_i(t) \in V_i \subset \mathfrak{R}$ are the inputs and output of the fuzzy logic subsystem i , respectively; F_{ji}^k ($1 \leq i, j \leq 2, 1 \leq k \leq l$) and G_i^k are labels of sets in X_i and V_i , respectively. The parameters g_{e_i} and $g_{\dot{e}_i}$ are chosen such that $\bar{e}_i(t)$ and $\dot{\bar{e}}_i(t) \in [-1, 1]$. The fuzzy inference engine performs a mapping from fuzzy sets in $X_i \subset \mathfrak{R}^2$ to fuzzy sets in $V_i \subset \mathfrak{R}$, based upon the fuzzy IF-THEN rules in the fuzzy rule base and the compositional rule of inference. The defuzzifier part is to utilize the center-average method to get the crisp control input signal $\bar{u}_i(t)$. In summary, overall fuzzy logic system can be employed to form a look-up Table 2 that directly relates the inputs $\bar{e}_i(t)$ and $\dot{\bar{e}}_i(t)$ with the output $\bar{u}_i(t)$. Also, the output $\bar{u}_i(t)$ of the look-up table ranges between from -1 and 1 that is multiplied by output scaling factor g_{u_i} to be real control output torque $u_i(t)$ as Eq. (9). Output scaling factor can be regarded as saturation limit of the actuators. Generally speaking, a larger output scaling factor is assigned, a smaller tracking error and a faster response are achieved; however, the risk of transient or oscillatory response increases after transience happen.

$$u_i(t) = g_{u_i} \bar{u}_i(t) = g_{u_i} [\bar{e}_i(t) + \Delta_i(t) \text{sgn}(\bar{e}_i)], \quad i = 1 \sim 12. \quad (9)$$

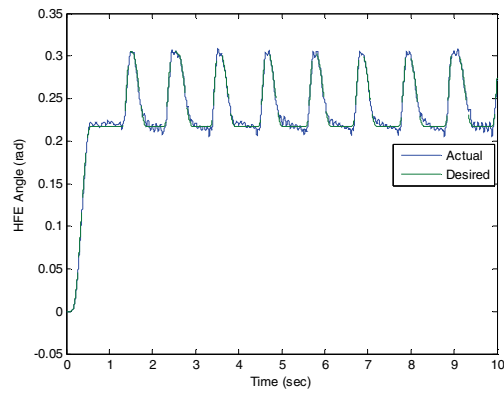
where $\bar{u}_i(t)$ is fuzzy variable of $u_i(t)$, and $\Delta_i(t) > 0 \ \forall t$

denotes the switching gain, which is related to Table 2. Furthermore, the roll torque of ankle in SS is constant. Moreover, hip roll torque needs to hold the torso up, so the desired position tracks ankle roll angles. In DS, it is occupied a very short time during the all of the walking period and then hip and ankle roll are pure torque sources (i.e., constant torques).

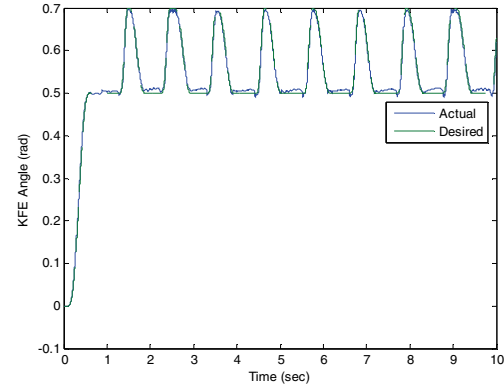
IV. SIMULATIONS AND DISCUSSIONS

Through the investigations of $e_i(t)$ and $\dot{e}_i(t)$ range from each joint, the control parameters g_{e_i} values are all selected as 0.01 and the $g_{\dot{e}_i}$ values respectively are selected as $\{0.009, 0.01, 0.01, 0.01, 0.005, 0.01\}$ at one side respectively such that their values are located between -1 and 1. They have the same values at another side. In addition, g_{u_i} values are assigned as $\{3500, 1300, 1700, 300, 45000, 200\}$ at one side. Also, these values are identical to another side. In general, a larger output scaling factor is assigned, a smaller tracking error and a faster response are achieved; however, the risk of transient response happens.

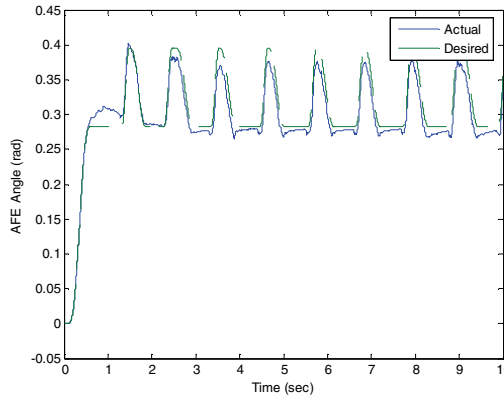
For ideal conditions (i.e., no any perturbations and uncertainties), the responses of each joint are illustrated in Fig. 6 that shows the responses of left hip, knee and ankle pitch angles. From these results shown, based on fuzzy controller and FSM method the responses of each joint have good trajectory tracking performance. In addition, PID controller is also applied to biped walking and it still obtains stable walking. For simplicity, these responses are omitted. However, to investigate further whether the fuzzy controller I propose has robustness, the external pulse force of 25 Newton is imposed on torso during 2.0~2.1 seconds from lateral push (i.e., positive Y direction) and those control parameters are the same with ones of ideal conditions. These simulation results are illustrated in Fig. 7. Moreover, Fig. 8 represents the responses under the same conditions by PID controller. It will crash at about 3 seconds. From these illustrations of the simulation results, it reveals the proposed controller is with better robustness to resist some extent disturbance and possess good performance. Except the computer simulation, it is also displayed walking of the 3D simulated biped robot using animation that reveals the proposed strategy is effective for the biped walking. In summary, they both exhibit acceptable performance under with or without uncertainty and disturbance. Further, it is also verified that the approximation of a simplified inverted pendulum model with ground contact relation for the biped robot is feasible and effective.



(a) The response of the hip pitch angle

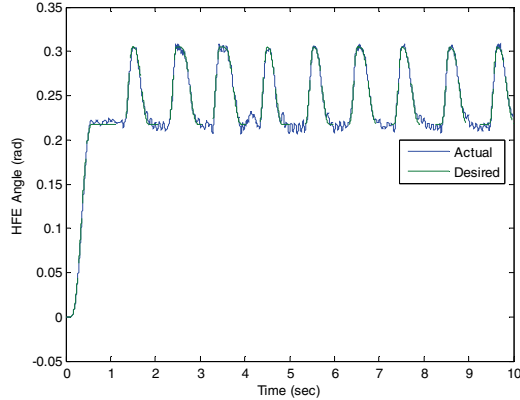


(b) The response of the knee pitch angle

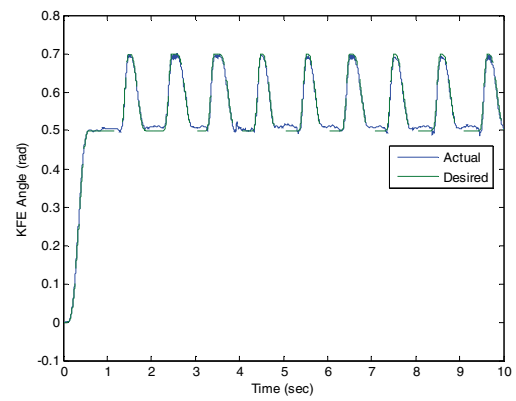


(c) The response of the ankle pitch angle

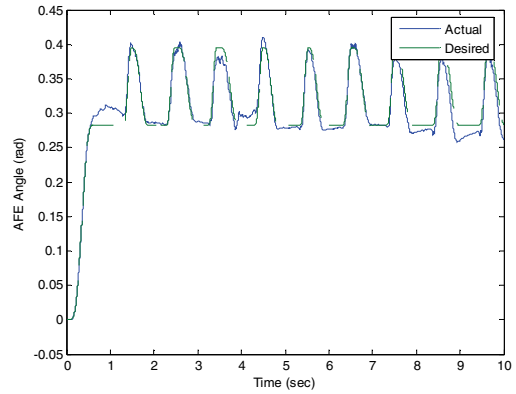
Fig. 6. The responses of internal joints in the absence of perturbation.



(a) The response of the hip pitch angle

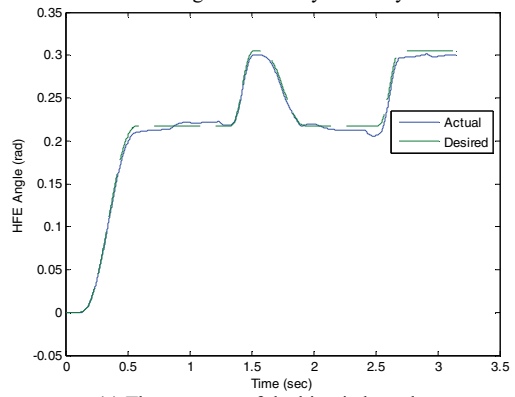


(b) The response of the knee pitch angle

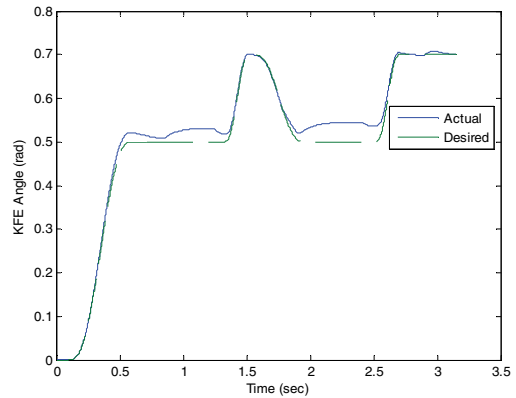


(c) The response of the ankle pitch angle

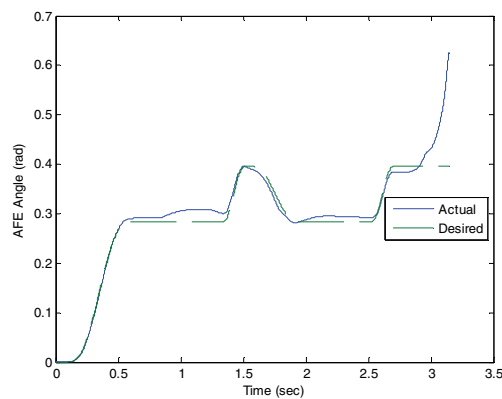
Fig. 7. The responses of joints in the presence of 25N pulse perturbation force on torso during 2~2.1 sec. by the fuzzy controller.



(a) The response of the hip pitch angle



(b) The response of the knee pitch angle



(c) The response of the ankle pitch angle
Fig. 8. The responses of Fig. 7 by the PID controller.

V. CONCLUSIONS

In this paper, the simulated 3D biped robot is considered as a simplified inverted pendulum model with ground contact relation. The fuzzy control policy and FSM approach are proposed for the periodic walking of the simulated 3D biped robot based on ZMP constraint. Under without perturbations, it has good response for the trajectory tracking on each joint. In addition, the lateral pulse perturbation force on torso is imposed to verify the robustness of the proposed controller. Finally, through the demonstration of the simulation, the responses are shown to possess good trajectory tracking as well as better robustness as compared with PID controller. Moreover, the animation also displayed that the biped robot can walk stably and periodically.

In future works, a real biped robot by the proposed control will be implemented. Additionally, it will be compared with other control strategies with respect to the robustness and performance.

ACKNOWLEDGMENT

The first author would like to appreciate the financial support from National Science Council of R.O.C. with Grant No.NSC-97-2917-I-011-101 for the Graduate Students Study Abroad Program. In addition, he also appreciates much assistance from Robotics Institute, School of Computer Science, Carnegie Mellon University, USA.

REFERENCES

- [1] M. Vukobratovic, A. A. Frank, and D. Juricic, "On the stability of biped locomotion," *IEEE Transaction on Biomedical Engineering*, vol. BME-17, no. 1, pp. 25-36, Jan. 1970.
- [2] K. Erbaturo, and O. Kurt, "Natural ZMP Trajectories for Biped Robot Reference Generation," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 3, pp. 835-845, Mar. 2009.
- [3] C. Chevallereau, D. Djuoudi, and J. W. Grizzle, "Stable Bipedal Walking With Foot Rotation Through Direct Regulation of the Zero Moment Point," *IEEE Transactions on Robotics*, vol. 24, no. 2, pp. 390-401, Apr. 2008.
- [4] D. Djuoudi, C. Chevallereau, and J. W. Grizzle, "A Path-Following Approach to Stable Bipedal Walking and Zero Moment Point

- Regulation," *IEEE International Conference on Robotics and Automation*, pp. 3597-3602, April 2007.
- [5] K. Nishiwaki and S. Kagami, "High frequency walking pattern generation based on preview control of ZMP," *IEEE International Conference on Robotics and Automation*, pp. 2667-2672, May 2006.
- [6] J. Morimoto and C. G. Atkeson, "Robust low torque biped walking using differential dynamic programming with a minimax criterion," *Proceedings of 5th International Conference on Climbing and Walking Robots*, pp. 453-459, 2002.
- [7] C. G. Atkeson and B. Stephens, "Multiple balance strategies from one optimization criterion," *IEEE-RAS International Conference on Humanoid Robots*, pp. 57-64, Nov. 2007.
- [8] B. Stephens, "Humanoid push recovery," *IEEE-RAS International Conference on Humanoid Robots*, pp. 589-595, Nov. 2007.
- [9] S. Stramigioli, V. Duindam G. van Oort and A. Goswami, "Compact analysis of 3D bipedal gait using geometric dynamics of simplified models," *IEEE International Conference on Robotics and Automation*, pp. 1978-1984, May 2009.
- [10] A. Goswami, "Kinematic and dynamic analogies between planar biped robots and the reaction mass pendulum (RMP) model," *IEEE-RAS International Conference on Humanoid Robots*, pp. 182-188, Nov. 2008.
- [11] C. W. Tao, J. S. Taur, T. W. Hsieh, and C. L. Tsai, "Design of a Fuzzy Controller with Fuzzy Swing-Up and Parallel Distributed Pole Assignment Schemes for an Inverted Pendulum and Cart System," *IEEE Transactions on Control Systems Tech.*, vol. 16, no. 6, pp. 1277-1288, Nov. 2008.
- [12] Z. Liu, Y. Zhang, and Y. Wang, "A Type-2 Fuzzy Switching Control System for Biped Robots," *IEEE Transactions on System, Man, and Cybernetics, Part C*, vol. 37, no. 6, pp. 1202-1213, Nov. 2007.
- [13] S. A. A. Moosavian, M. Alghoonch, and A. Takhmar, "Stable trajectory planning, dynamics modeling and fuzzy regulated sliding mode control of a biped robot," *IEEE-RAS International Conference on Humanoid Robots*, pp. 471-476, Nov. 2007.
- [14] R. C. Luo, C. W. Tzeng, P. Z. Cheng and K. W. Lee, "Trajectory-Tracking of Nonlinear Biped Robot System Based on Adaptive Fuzzy Sliding Mode Control," *IEEE IECON*, pp. 2789-2794, Nov. 2007.
- [15] C. L. Hwang, "A Trajectory Tracking of a Biped Robots Using Fuzzy-Model-Based Sliding-Mode Control," *IEEE Conference on Decision and Control*, pp. 203-208, Dec. 2002.
- [16] R. E. Precup and S. Preitl, "Optimization criteria in development of fuzzy controller with dynamics," *Engineering Applications of Artificial Intelligence*, vol. 17, no. 6, pp. 661-674, Sep. 2004.
- [17] K. Y. Tu, T. T. Lee and C. H. Wang, "Design of a new fuzzy suction controller using fuzzy modeling for nonlinear boundary layer," *IEEE Trans. Fuzzy Syst.*, vol. 13, no. 5, pp. 605-616, Oct. 2005.
- [18] A. Sala, T. M. Guerra and R. Babuska, "Perspectives of fuzzy systems and control," *Fuzzy Sets and Syst.*, vol. 156, no. 3, pp. 432-444, Dec. 2005.

Table 1. Rule table of the i th fuzzy inference engine.

$\frac{\dot{e}_i}{\bar{e}_i}$	PH	PB	PM	PS	PI	ZE	NI	NS	NM	NB	NH
NH	ZE	NI	NS	NM	NM	NB	NB	NB	NH	NH	NH
NB	PI	ZE	NI	NS	NM	NM	NB	NB	NB	NH	NH
NM	PS	PI	ZE	NI	NS	NM	NM	NB	NB	NB	NH
NS	PM	PS	PI	ZE	NI	NS	NM	NM	NB	NB	NB
NI	PM	PM	PS	PI	ZE	NI	NS	NM	NM	NB	NB
ZE	PB	PM	PM	PS	PI	ZE	NI	NS	NM	NM	NB
PI	PB	PB	PM	PM	PS	PI	ZE	NI	NS	NM	NM
PS	PB	PB	PB	PM	PM	PS	PI	ZE	NI	NS	NM
PM	PH	PB	PB	PB	PM	PM	PS	PI	ZE	NI	NS
PB	PH	PH	PB	PB	PB	PM	PM	PS	PI	ZE	NI
PH	PH	PH	PH	PB	PB	PB	PM	PM	PS	PI	ZE

Table 2. Look-up table of the i th fuzzy inference engine.

$\frac{\dot{e}_i}{\bar{e}_i}$	1.0	0.8	0.6	0.5	0.2	0	-0.2	-0.5	-0.6	-0.8	-1.0
-1.0	0.0	-0.05	-0.2	-0.5	-0.7	-0.8	-0.9	-0.95	-1.0	-1.0	-1.0
-0.8	0.05	0.0	-0.05	-0.2	-0.5	-0.7	-0.8	-0.9	-0.95	-1.0	-1.0
-0.6	0.2	0.05	0.0	-0.05	-0.2	-0.5	-0.7	-0.8	-0.9	-0.95	-1.0
-0.5	0.5	0.2	0.05	0.0	-0.05	-0.2	-0.5	-0.7	-0.8	-0.9	-0.95
-0.2	0.7	0.5	0.2	0.05	0.0	-0.05	-0.2	-0.5	-0.7	-0.8	-0.9
0	0.8	0.7	0.5	0.2	0.05	0.0	-0.05	-0.2	-0.5	-0.7	-0.8
0.2	0.9	0.8	0.7	0.5	0.2	0.05	0.0	-0.05	-0.2	-0.5	-0.7
0.5	0.95	0.9	0.8	0.7	0.5	0.2	0.05	0.0	-0.05	-0.2	-0.5
0.6	1.0	0.95	0.9	0.8	0.7	0.5	0.2	0.05	0.0	-0.05	-0.2
0.8	1.0	1.0	0.95	0.9	0.8	0.7	0.5	0.2	0.05	0.0	-0.05
1.0	1.0	1.0	1.0	0.95	0.9	0.8	0.7	0.5	0.2	0.05	0.0