

On a benchmark related assessment of the performance of mutual (investment) funds

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Abstract—We deal with the problem of how to evaluate the performance of an investment (mutual) fund, over a past period. We assume a widely assumed attitude that an investment fund is said to have performed well if its unit price have been rising faster than its benchmark during a period of growth, and has been falling slower than its benchmark during the period of decline. We develop a novel, flexible and human consistent comparison method that operates on the comparison of trends (segments) extracted from the times series representing the behavior of the fund and its benchmark. The result of comparison becomes a new data set that is linguistically summarized using the method proposed in our previous works.

Index Terms—Linguistic summaries, computing with words, fuzzy logic, mutual (investment) fund comparison.

I. INTRODUCTION

This paper is a continuation of our previous works (cf. Kacprzyk, Wilbik, Zadrozny [1], [2], [3] or Kacprzyk, Wilbik [4]) which deal with the problem of how to effectively and efficiently support a human decision maker in making decisions concerning investments in some financial instruments, notably in investment (mutual) funds. Decision makers are here basically interested in future gains/losses. We follow the decision support paradigm (as in Fig. 1), that is, assume primarily the user autonomy and a need to support, not replace, him/her. We are not concerned with the forecasting of future daily prices.

We deal with investment decisions that are made on a continuous basis over some period of time. Though the revenues resulting from those decisions are certainly very important, an omnipresent uncertainty calls for a different basis on which those decisions are to be based which is more robust under changing conditions. That is, we are less interested in exact values of quotations of the investment fund considered but more on the stability of their behavior, in relation to its benchmark. This stability is basically meant as the growth and/or decline, and variability.

Basically, in our work we advocate a view that the most promising paradigm is that of decision support, that is, that

the system should provide the decision maker with results of some additional analyses of data that may give more insight into the situation. Then, the decision maker can make decision by himself/herself.

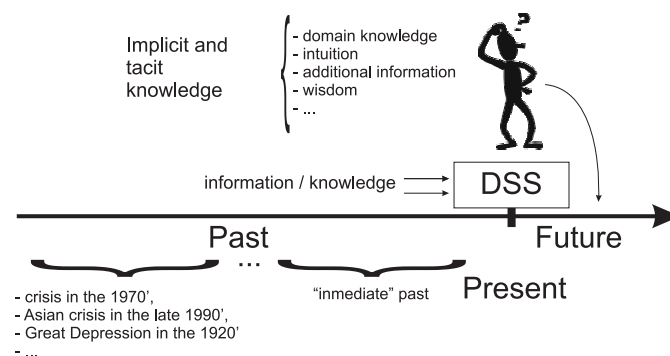


Fig. 1. Decision support paradigm

We deal with analysis of past performance of mutual funds. In our previous papers we have worked in two directions. One topic is the analysis of past performance for which we have proposed and used linguistic summaries of time series. In our works [3], [5], [4] segments (trends) derived from consecutive values have been used. We have used the following (classical) protoforms of linguistic summaries employed: "among all y 's Q are P " exemplified by "among all segments (of time series), most are slowly increasing" and "among all Ry 's Q are P " exemplified by "among all short segments (of time series), most are slowly increasing". Later we (cf. Kacprzyk and Wilbik [6]) have introduced new, so-called temporal protoforms: " E_T among all y 's Q are P " exemplified by "recently among all segments (of time series), most are slowly increasing" and " E_T among all Ry 's Q are P " exemplified by "initially, among all short segments (of time series), most are slowly increasing", that explicitly take into account importance of the time of occurrence. We have proposed several quality criteria to evaluate the linguistic summaries derived.

The second area of our analysis is the comparison of past performance of the fund and past performance of its benchmark. A mutual fund benchmark can provide information on whether the mutual fund is performing up to the expectations. Every type of mutual funds should have a benchmark that can be used for the comparison purposes.

This is obvious and mentioned in every analysis, guideline, advice, etc. For instance [7]: “Evaluate the funds performance. Every fund is benchmarked against an index Investors should compare fund performance over varying time frames vis-a-vis both the benchmark index and peers. Carefully evaluate the funds performance across market cycles particularly the downturns” or even stronger “the best way to evaluate the performance of any mutual fund is to compare it to an appropriate benchmark” [8]. Moreover, they say: “Measure your results. How do your funds stack up against the rest? One of the best ways to know is to compare the funds results with those of its benchmarks. The idea behind benchmarks is to see how your mutual fund compares with an index, one that most closely resembles the holdings of your fund. ... Another way to measure your fund results is to see how a funds returns compare with one of the major market indexes” [9], or “Often cited in news reports, market indexes can be especially helpful to mutual fund investors by offering market ‘standards’ to help them evaluate the risk and the return history of their own investments. ... While indexes are good methods of gauging how a mutual fund performs in relation to the overall market, they shouldn’t be the deciding factor in determining if a fund may meet your needs and objectives. ... The benchmark listed in your fund’s prospectus will give you a good idea of what to expect from your mutual fund. However, remember that standardized tests are just that ‘standardized’ [10]”. We could cite here much more similar articles and opinions.

However investors should be careful while choosing the benchmark, as “recent study from the University of Southern California shows that 42% of actively managed domestic stock funds list benchmarks in their prospectuses that do not match their investment style. No wonder investors have a hard time deciding on a relevant benchmark! Fund management is well aware that its results are being compared to benchmark indexes. In many cases, managers can most easily beat their benchmark by simply changing the index to which their fund is compared” [11]. However in this paper we will not deal with the problem if benchmark is proper for the analyzed fund or not.

In our previous works [12], [13] we had evaluated only the similarity between two time series. The degree of similarity of two time series was meant here as the degree to which, for instance, “most” of the long, overlapping segments are similar (for instance, such that “almost all” of their features are similar). Later [14], [15] we evaluated the similarity on linguistic summaries describing each time series. In such case the degree of similarity of two time series was meant as the degree to which, e.g., “most” of the valid summaries of the fund have the truth values similar to a majority of similar summaries of the benchmark.

Here we go one step further and assess if the fund is better then the benchmark. This is often done by calculating only the benchmark returns, i.e. a fund’s returns compared to its benchmark. However we may compare those two time series over some defined time span, and check how the fund is performing. We take a widely assumed general principle that an investment fund is said to have performed well if its unit price have been rising faster than its benchmark during a period of growth, and has been falling slower than its benchmark during the period of decline. In other words, the funds is said to outperform its benchmark when these two conditions have been met.

II. TIME SERIES SEGMENTATION

The first step in many application and also here in the proposed method is segmentation. We have assumed that segments are linear. There are many methods for a piecewise linear segmentation of time series, e.g., on-line (sliding window) methods, top-down or bottom-up strategies or employing hybrid or genetic strategies [16], [17]. Generally, the algorithm has a minor impact on the obtained segments (cf. [18]) and what really matters is the assumed granulation, expressed in the form of maximal accepted (vertical) distance from the point to the segment, ε .

For the purposes of this work we use an on-line algorithm (also called sliding window). In general the algorithms work by checking if it is possible to add the new point to the currently constructed segment, and if so, then the segment is extended. However, if not, for instance because of the exceeding of the threshold value of the accepted error, ε , then the currently constructed segment is terminated and this new point creates a new segment. In the algorithm used here the segment links two points: the first and the last covered by the segment, as proposed by Sklansky and Gonzalez [19].

Each segment is described by three features:

- dynamics of change,
- duration, and
- variability.

By *dynamics of change* we understand the speed of change of the consecutive values of time series. It may be described by the slope of a line representing the trend (segment). Thus, to quantify dynamics of change we may use the interval of possible angles $\eta \in \langle -90; 90 \rangle$.

However it might be impractical, and not human consistent, to use such a scale directly while describing dynamics of change of the trends. We may use a fuzzy granulation in order to meet the users’ needs and task specificity. The user may construct a scale of linguistic terms corresponding to various inclinations of a trend line as, e.g.:

- quickly decreasing,
- decreasing,
- slowly decreasing,
- constant,
- slowly increasing,
- increasing,
- quickly increasing

Figure 2 illustrates possible lines corresponding to the particular linguistic terms.

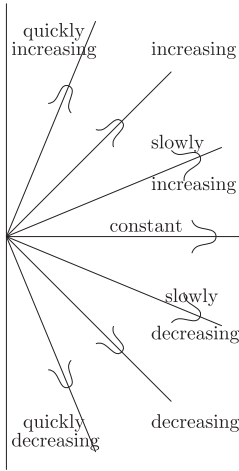


Fig. 2. A visual representation of angle granules defining the dynamics of change

In fact, each term represents a fuzzy granule of directions. In Batyrshin et al. [20], [21] there are presented many methods of constructing such a fuzzy granulation. The user may define a membership functions of particular linguistic terms depending on his or her needs.

We map a single value α characterizing dynamics of change of a trend identified by the algorithm chosen into a fuzzy set (linguistic label) best matching a given angle. We can use, for instance, some measure of a distance or similarity, cf. the book by Cross and Sudkamp [22]. Then we say that a given trend is, e.g., “decreasing to a degree 0.8”, if $\mu_{\text{decreasing}}(\alpha) = 0.8$, where $\mu_{\text{decreasing}}$ is the membership function of a fuzzy set representing “decreasing” that is the best match for angle α .

Duration describes the length of a single trend, meant as a linguistic variable whose linguistic value (label) may be exemplified by a “long trend” defined as a fuzzy set.

The definitions of linguistic terms describing the duration depend clearly on the perspective or purpose assumed by the user.

Variability refers to how “spread out” (“vertically”, in the sense of values taken on) a group of data is. We compute it as a weighted average of values taken by some measures used in statistics:

- the range (maximum – minimum),
- the interquartile range (IQR),
- the variance,
- the standard deviation,
- the mean absolute deviation (MAD).

Similarly as in the case of dynamics of change, we find for a given value of variability obtained as above a best matching fuzzy set (linguistic label) using, e.g., some measure of a distance or similarity, cf. the book by Cross and Sudkamp [22]. Again, the measure of variability is treated as a linguistic variable and expressed using linguistic terms (labels) modeled by fuzzy sets defined by the user.

III. LINGUISTIC SUMMARIES OF TRENDS

A linguistic summary of data (database) is a (usually short) sentence (or a few sentences) that captures the very essence of the data; the data is assumed here numeric, its amount is large and not comprehensible for human users. We use here Yager’s [23] basic approach, and a linguistic summary includes:

- a summarizer P ,
- a quantity in agreement Q , i.e. a linguistic quantifier,
- truth (validity) $\mathcal{T}$ of the summary and
- optionally, a qualifier R .

An extended, implementable approach was shown in Kacprzyk and Yager [24], and Kacprzyk, Yager and Zadrożny [25].

Thus, the core of a linguistic summary is a linguistically quantified proposition in the sense of Zadeh [26] which may be written, respectively, as

$$Qy's \text{ are } P \quad (1)$$

$$QRy's \text{ are } P \quad (2)$$

which may be exemplified, respectively by: “Most of employees earn *low* salary”, $\mathcal{T}=0.7$, or “Most of *young* employees earn *low* salary”, $\mathcal{T}=0.82$.

For clarity and convenience we employ Zadeh’s protoforms for dealing with linguistic summaries – cf. Kacprzyk and Zadrożny [27]. A *protoform* is a more or less abstract prototype (template) of a linguistically quantified proposition. The two basic types of protoforms of linguistic summaries of trends are:

- a simple (short) form:

$$\text{Among all segments, } Q \text{ are } P \quad (3)$$

e.g., “Among all segments, *almost all* are *slowly increasing*”.

- an extended form:

$$\text{Among all } R \text{ segments, } Q \text{ are } P \quad (4)$$

e.g., “Among all *very short* segments, *most* are *increasing*”.

The basic quality criterion is the truth value (degree) [23]. Using Zadeh’s calculus of linguistically quantified propositions [26] it is calculated as:

$$\mathcal{T}(\text{Among all } y's, Q \text{ are } P) = \mu_Q \left(\frac{1}{n} \sum_{i=1}^n \mu_P(y_i) \right) \quad (5)$$

$$\begin{aligned} \mathcal{T}(\text{Among all } Ry's, Q \text{ are } P) &= \\ &= \mu_Q \left(\frac{\sum_{i=1}^n \mu_R(y_i) \wedge \mu_P(y_i)}{\sum_{i=1}^n \mu_R(y_i)} \right) \end{aligned} \quad (6)$$

where n is the number of extracted segments, Q is a fuzzy set representing the linguistic quantifier in the sense of Zadeh [26], i.e. regular, nondecreasing and monotone and $\wedge$ is the minimum (or a t -norm, cf. Kacprzyk, Wilbik and Zadrożny [28]).

There are also some other quality criteria introduced for the evaluation of linguistic summaries, like degrees of focus,

specificity, covering or appropriateness and measure of informativeness and others. Those quality criteria were discussed in our works (cf. Kacprzyk and Wilbik [29], [30], [31], [32]).

IV. COMPARISON OF THE MUTUAL FUND AND ITS BENCHMARK

There are several possibilities how to assess if the fund is better than its benchmark. A very simple approach is to compare just the absolute return rate of the fund and benchmark after a certain period. Then we can just compare those two values to assess if the fund is better than benchmark. This is known as a *benchmark return*.

The second option is to compare the return rates of the fund and its benchmark over a certain period. First, we obtain two time series of daily return rates of an investment and its benchmark. Then we subtract the values of the time series of the benchmark from that of the time series of the fund. Next we can analyze the results, e.g., if most of the values of the new time series is above 0 or a certain value, then the fund is better than the benchmark, if most values are close to 0 then the fund is comparable with the benchmark, otherwise it is worse. Such an approach is somewhat similar to the method proposed by Castillo-Ortega, Marín and Sánchez [33].

However that approach can be inappropriate in our context of mutual funds since, usually, the difference between the fund and its benchmark may be very fluctuating. This may be caused because the daily values are compared, not the segments (or trends), and usually there is a small temporal shift between the values of the benchmark and the fund. We will show this later in the section on numerical example.

Therefore we propose to make the assessment on the basis of some trends (or linear segments), in order to avoid those rapid changes in values. We want to globally assess the interplay between the benchmark and the investment fund quotations. This is much more than the analysis of quotations and its related benchmark values but also the analysis of dynamics and behavior. Fund quotations are not directly comparable in financial terms with the values of the benchmark (the WIG index in our case), but they are related.

We assess whether the fund is better than its benchmark using the following widely accepted principle: it is said that a mutual (investment) fund is better than its benchmark when it rises faster than the benchmark and falls slower than the benchmark.

The time series is represented by its segments (trends). Each segment is described by the following four values:

- starting time, called here a partition point,
- duration of the segment (expressed in time units),
- dynamics of change (expressed as the slope of the segment),
- variability occurring in the segment, describing the variability of differences between the trend (segment line) and the particular time series values over the duration of this trend.

Let us assume that we wish to assess which of the two time series A and B is better. As a result of the segmentation procedure time series A is divided into k segments, represented

by a_j , $j = 1, \dots, k$ and time series B is divided into l segments, represented by b_j , $j = 1, \dots, l$.

A simple approach would be that the segments have the same partition points. This may be attained at the expense of using an artificial fixed point segmentation. In our case we follow a different way and assume that segmentations of the investment fund and benchmark time series are taken as they are, and there may be different partition points between the segments. This clearly complicates the analysis.

Next we create the set of partition points of the two time series considered, i.e. this set contains the time points when at least one segment has started. There are at most $k + l$ such partition points. Those times are sorted, so that the earlier ones come first.

Now between any consecutive partition points (e.g. p_i and p_{i+1}) there is only one segment (or a part of it) of each time series. We may denote those segments as a_{p_i} and b_{p_i} , i.e., a_{p_i} is the segment representing time series A that take place between p_i and p_{i+1} , as in Figure 3.

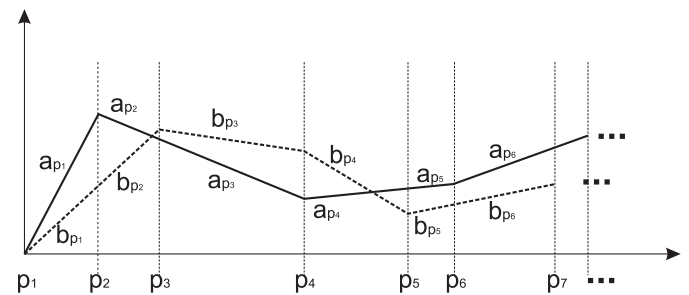


Fig. 3. Partition points for two time series

For the assessment purposes the values of the dynamic and duration are only used. First, we assess if the fund is better than its benchmark between two consecutive partition points, i.e., we compare the segments between those partition points. We subtract the value of the dynamics of change of the segment representing the benchmark's time series, from the value of the dynamics of change of the segment representing the fund's time series. This difference is treated as a linguistic variable and expressed using linguistic terms (labels) modeled by fuzzy sets defined by the user. Three linguistic labels should be sufficient, notably "better", "indifferent" and "worse". We could aggregate those partial scores, via a quantifier driven aggregation. However, the definition mentioned above distinguish 2 situations, an increase and a decrease of the values of the benchmark. Therefore it would be interesting to present this relationship in the form of a linguistic summary.

V. NUMERICAL EXAMPLE

In this numerical example we analyze the mutual fund quotations from the last two years, i.e., from January 5, 2009 until January 3, 2011, presented in Fig. 4. The investment fund considered is an equity fund that invests at least 66% assets in shares listed at the Warsaw Stock Exchange. It may also invest at the Zagreb Stock Exchange or Moscow Interbank

Currency Exchange. The fund aims at achieving a long term capital growth (the minimal saving time is recommended to be 5 years) and promises a high profit, however at a high risk, in particular in short term.

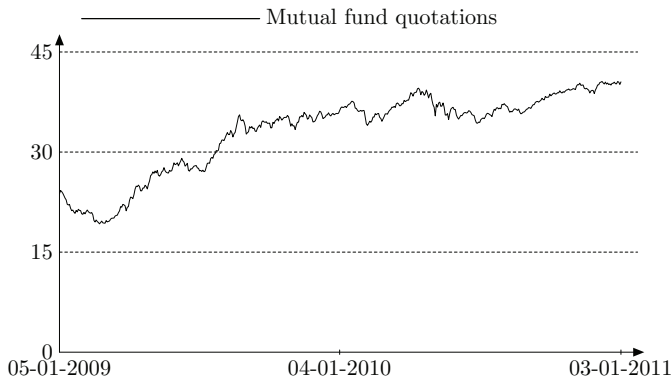


Fig. 4. Daily quotations of the investigated fund

In the analyzed time span, the benchmark of this fund was the WIG index, a total return index, which includes dividends and pre-emptive rights. More information about this index can be found at www.gpw.pl. The performance of the WIG index is presented in Fig. 5.

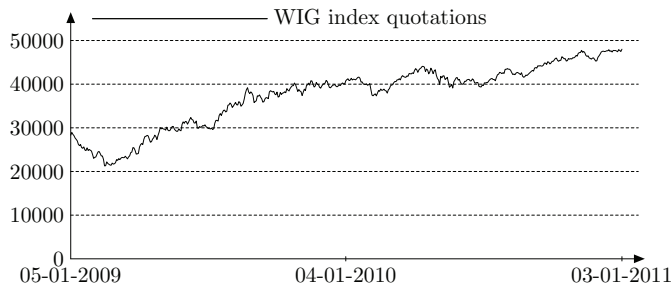


Fig. 5. Daily quotations of the WIG index

Just for information, the return rates for the fund and the WIG index over the given period are equal, respectively, to 1.60% and 1.82%. The benchmark return rate is -0.22%, so it is very small.

We could scale the both time series, i.e. interpreting the scaled values as the value of our investment of the same sum of money at a specific time point. Then we could easily subtract those values. As a result we would obtain the time series as in Fig. 6 and obtain the linguistic summaries “most values are practically the same” meaning that the difference in profit of the two instruments, meant as given above, is irrelevant in most days.

We may notice very big oscillations, and this time series looks similar to a white noise.

Let us now analyze the results of the method proposed. Using a piecewise linear segmentation method with a moderate value of the maximum accepted error we have found 28 segments for the fund quotations time series and 40 segments for the WIG quotations time series. We computed also the degree of similarity of the fund and WIG time series using

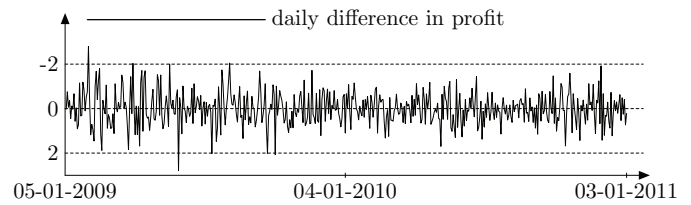


Fig. 6. Daily difference in profit of two investment instruments

a method proposed in our previous paper [12], and it is equal 0.7830, so both time series are quite similar.

Some of the linguistic summaries describing the dynamics of change of two time series and the difference of angles (slopes of the segments) are the following:

- In most cases if the dynamics of change of the investment fund was decreasing, the dynamics of change of WIG index was also decreasing, $\mathcal{T} = 1.0000$
- In most cases if the dynamics of change of the WIG index was constant, the dynamics of change of investment fund was also constant, $\mathcal{T} = 1.0000$
- In most cases if the dynamics of change of the investment fund was increasing, the dynamics of change of WIG index was also increasing, $\mathcal{T} = 0.9140$
- In most cases if the dynamics of change of the WIG index was increasing, dynamics of change of the fund was worse, $\mathcal{T} = 1.0000$
- In most cases if the dynamics of change of the WIG index was constant, dynamics of change of the fund was indifferent, $\mathcal{T} = 0.9107$
- In most cases if the dynamics of change of the WIG index was decreasing, dynamics of change of the fund was better, $\mathcal{T} = 0.7727$

The first 3 summaries describe relationships between segments describing the fund and WIG time series. The next 3 summaries compare the fund and WIG stating that, generally, the fund outperforms its benchmark, when it falls, but underperforms the benchmark when it rises. This basically means that the fund is not performing as we have wished it to do. This may be a very important clue for the funds managers. Also this may be a valuable conclusion in the context of bad situation on the economic and financial market that we have observed for a few last years.

This intuitive, human consistent assessment is a “raw estimate” and in investments practices much more sophisticated criteria are used. This is however beyond the scope of this paper.

VI. CONCLUDING REMARKS

We dealt with the problem of how to evaluate the performance of an investment (mutual) fund, over a past period. We took as a point of departure a widely assumed attitude that an investment fund was said to have performed well if its unit price had been rising faster than its benchmark during a period of growth, and had been falling slower than its benchmark during the period of decline. We developed a

novel, flexible and human consistent comparison method that operated on the comparison of trends (segments) extracted from the times series representing the behavior of the fund and its benchmark. The result of comparison became a new data set that was linguistically summarized using the method proposed in our previous works. The new method proposed was tested on an open investment fund listed on Polish and international financial market whose benchmark was the most important, general index of the Warsaw Stock Exchange. The results obtained provided a valuable insight into the very essence of the behavior of the fund's quotations in relation to its benchmark, and could be useful while making investment decisions.

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