

Formalizing Object Membership in Fuzzy Ontology with Property Importance and Property Priority

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Abstract—In this paper, we formalize the object membership in fuzzy ontology with property importance and property priority, while previous models lack building blocks to handle the importance and priority of properties. A formal mechanism used to measure object memberships in concepts is proposed. Such a mechanism can measure object memberships in concepts defined by properties with importance or priority well. We show that our model is more reasonable in measuring object memberships in concepts than previous models by examples and experiments.

Index Terms—Fuzzy ontology, Property importance, Property priority

I. INTRODUCTION

Traditional ontology is represented as a crisp set of objects [1] and objects are considered either to belong to or not to belong to a concept. However, there are many fuzzy concepts (e.g., ‘hot water’ and ‘yellow car’) in reality, and traditional ontology cannot handle these fuzzy concepts. Fuzzy ontologies are proposed to extend the representation ability of ontologies to handle fuzzy concepts such as [2] [3] [4]. Object membership is an important measurement of objects in fuzzy concepts and it refers to the degree of an object belonging to a concept. However, object memberships are given by users manually or obtained by fuzzy functions defined by users in these fuzzy ontologies. While concepts, objects and properties are building blocks of ontologies, these fuzzy ontologies lack of a formal mechanism to determine memberships of objects in concepts automatically based on the defining properties of concepts and properties which objects possess.

Au Yeung and Leung [5] formalize object memberships in concepts based on theories in cognitive psychology and present a mechanism to measuring object memberships based on properties of concepts and objects automatically. Their work is extended by Cai and Leung [6]. Besides, all these works consider that properties used to define a concept are equivalent. In other words, all properties have the same importance or the same priority, and we name them as **equivalent properties**. However, some properties are more important than other properties in many situations. We name the properties with different importance in concepts as **weighted properties**. For example, when some one wants to rent a house, he may consider price of the house is more important than the location of the house. In addition, some properties are prior to other properties in measuring object memberships. We name the

properties with different priorities in concepts as **prioritized properties**. For example, when measuring how great an athlete is in the Olympic Games, the number of goal medals the athlete obtained is prior to the number of silver medals the athlete obtained. The effect of an athlete obtaining one goal medal cannot be substituted by that of no matter how many silver medals the athlete obtained. Although different properties having different importance or priorities are prevalent, previous works cannot handle object memberships in concepts defined by weighted properties or prioritized properties well. For the ripeness of property importance and priority, we consider that it is necessary to model them in ontology.

In this paper, we formalize the object membership in fuzzy ontology with property importance and property priority. The formal mechanism we proposed for measuring object memberships in concepts can handle not only concepts defined by equivalent properties, but also concepts defined by weighted properties or prioritized properties well, which cannot be handled well in previous models. Besides, we formally model property importance and property priority in fuzzy ontology, and point out and discuss the differences between property importance and property priority. Experiments are conducted to validate the proposed model. To the best of our knowledge, it is the first work of modeling object memberships in fuzzy ontology with weighted properties and prioritized properties.

II. BACKGROUND AND RELATED WORK

A. Classical View of Concept Representation

In cognitive psychology, it is generally accepted that concepts are characterized by properties [7]. One important model of concept representation based on properties is classical view. The classical view [8] [9] of concepts posits that each concept is defined by a set of properties which are individually necessary and collectively sufficient. Properties are atomic units which are the basic building blocks of concepts. In classical view, there are clear-cut boundaries between members and non-members of the category. As a result, the classical view cannot handle the vague concepts.

B. Fuzzy Ontology and Object Membership

Currently, most ontologies are based on DLs (description logics) [10] and concepts are represented as crisp sets of ob-

jects (e.g., ontologies written in OWL DL) [11]. These ontologies cannot represent the fuzzy concepts. Several fuzzy DLs are proposed to handle the fuzzy concepts by combining fuzzy set theory [12] and description logics. For example, Straccia proposes a fuzzy $\mathcal{ALC}$ in [2] and a fuzzy $\mathcal{SHOIN}(\mathcal{D})$ in [3]. Stoilos et al. present a fuzzy $\mathcal{SHIN}$ in [4]. These fuzzy DLs vary in possessing different expressive power, complexity and reasoning capabilities. Some fuzzy ontologies are constructed based on fuzzy DLs or fuzzy logic [13]. Besides, some works apply fuzzy ontologies for some applications. For instance, Cross and Voss [14] explore the potential that fuzzy mathematics and ontologies have for improving performance in multilingual document exploitation. These works lack a formal mechanism to obtain the membership degrees of objects in concepts automatically, and they do not take how people representing concepts in their mind into consideration.

Recently, Au Yeung and Leung [5] propose a formal model for fuzzy ontology based on the idea of classical view. They have formalized the membership degrees of objects in concepts by constructing several vectors in ontologies. To overcome previous models' limitations, Cai and Leung [6] propose a novel formal model of fuzzy ontology with property hierarchy and object membership. However, all these works cannot handle the concepts defined by properties which have different importance degrees or priorities. They lack formal mechanisms for measuring object memberships in concepts defined by weighted properties and prioritized properties. Besides, Bobillo and Straccia explore an expressive fuzzy description logic reasoner [15], which include the weighted sum constructor.

III. LIMITATIONS OF PREVIOUS MODELS

We use some motivating examples to illustrate the limitations of previous models and make our motivation more clear in this section.

Example 1. Suppose that Bob wants to rent a room for 10 months from a real estate agent when he visits the Chinese University of Hong Kong. The rent of such a room would be the best if it is less than or equal to 2800 HKD each month, and he cannot offer too much more for the rent. He would like the room with a short walk distance to the university and there are some living facilities such as a bed, internet, TV, refrigerator and so on. He takes these requirements into consideration synthetically and defines the concept 'The suitable room to rent' (denoted as S_r) by some properties which are 'Price', 'Rental period', 'Time cost of walk' and 'Facilities'. In his consideration, some properties are more important than other properties. He concentrates more on the 'Price' and less on 'Rental period', 'Time cost of walk' and 'Facilities' for the reason that he has limited budget for rent. The importance weights of these properties are given in table I. The concept S_r is defined as following:

$$S_r : [A]_{0.95}, [B]_{0.9}, [C]_{0.85}, [D]_{0.8}$$

where the subscript of each property is the minimal requirement of the property. Objects (e.g., all candidate rooms) satisfying all minimal requirements of defining properties of

TABLE I
IMPORTANCE WEIGHTS OF PROPERTIES OF THE CONCEPT S_r .

	A	B	C	D
Price				
Rental period				
Time cost of walk				
Facilities				
weight	0.4	0.3	0.2	0.1

TABLE II
PROPERTIES OF THE OBJECTS O_1 AND O_2 .

	Price	Rental period	Time cost of walk	Facilities
O_1	2600 HKD	10 months	11 minutes	basic
O_2	3500 HKD	12 months	8 minutes	convenience

S_r (e.g., 'The suitable room to rent') belong to S_r to a degree 1 [5] [6]. Suppose there are two candidate rooms denoted by O_1 and O_2 as following (properties of O_1 and O_2 are shown in table II).

$$O_1 : [A]_1, [B]_1, [C]_{0.9}, [D]_{0.4}; O_2 : [A]_{0.6}, [B]_{0.8}, [C]_1, [D]_1$$

The subscript of each property is the degree of an object possessing the property. We want to calculate object memberships for two rooms O_1 and O_2 in concept S_r .

We use equations in AuYeung-Leung model [5] to measure the memberships of O_1 and O_2 and obtain the following results:

$$\mu_{S_r}(O_1) = 0.52, \mu_{S_r}(O_2) = 0.604$$

Besides, we also calculate the memberships of O_1 and O_2 in S_r according to the axioms and equations in Cai-Leung model [6] and get the results as following:

$$\mu_{S_r}(O_1) = 0.5, \mu_{S_r}(O_2) = 0.6136$$

We can find that the results calculated by previous models are not intuitive. The room O_1 satisfies properties A and B to high degrees and these properties are considered as top two important properties for Bob. Although O_2 possesses properties C and D to high degrees, it possesses the more important (more considerable) properties A and B to lower degrees. What is more, the rent of O_2 also are too much more than the budget of rent that Bob can offer, and it requires Bob to rent the room for at least 12 months which is longer than the visiting period of Bob. Thus, it is clearly that O_1 should be a better candidate than O_2 for Bob to rent and should be given a higher object membership in S_r . Previous models take all properties into account equivalently and cannot reflect the importance of properties, thus the object memberships in concepts defined by properties with different importance are not intuitive and reasonable.

Example 2. Suppose in an IT company, a secretary need to find a suitable time slot to arrange a meeting for discussing a project. The CTO of the company, the project team leader, project coordinator and the team members must attend the meeting. To which a time slot is a suitable one for the meeting depends on preference degrees (PDs) of the people who attend the meeting. Besides, from the point of view of the secretary, the priorities of preferences of these people are different. The preference of the CTO is prior to that of others. The

TABLE III
DEFINING PROPERTIES AND THEIR PRIORITIES IN CONCEPT S_t

A	PDs of CTO	first priority
B	PDs of team leader	second priority
C	PDs of project coordinator	second priority
D	PDs of team member	third priority

preference of the team leader is as the same as that of the project coordinator, and is priori to the team members. Thus, for the concept ‘suitable time slot for the meeting’ denoted by S_t , it is defined by four properties with priorities shown in table III.

We consider that the concept S_t is defined as following:

$$S_t : [A]_1, [B]_{0.95}, [C]_{0.9}, [D]_{0.8}$$

where the subscript of each property is the minimal requirement of the property. For three time slots denoted by O_{t1} , O_{t2} , O_{t3} as following, we want to calculate object memberships for them in concept S_t .

$$O_{t1} : [A]_1, [B]_{0.9}, [C]_{0.65}, [D]_{0.5}$$

$$O_{t2} : [A]_{0.9}, [B]_{0.72}, [C]_{0.65}, [D]_1$$

$$O_{t3} : [A]_{0.9}, [B]_{0.85}, [C]_{0.75}, [D]_{0.6}$$

The subscript of each property is the degree of an object possessing the property. We use equations in AuYeung-Leung model [5] to measure the memberships of O_{t1} , O_{t2} and O_{t3} and obtain the following results:

$$\mu_{S_t}(O_{t1}) = 0.6, \mu_{S_t}(O_{t2}) = 0.685, \mu_{S_t}(O_{t3}) = 0.68$$

Similarly, we calculate the memberships of O_{t1} , O_{t2} and O_{t3} in S_t according to the axioms and equations in the [6] and get the results as following:

$$\mu_{S_t}(O_{t1}) = 0.6667, \mu_{S_t}(O_{t2}) = 0.8667, \mu_{S_t}(O_{t3}) = 0.8$$

We can find that the results calculated by previous models are not intuitive. The CTO and the team leader prefer time slot O_{t1} to a higher degree than other time slots, while the project coordinator prefer O_{t3} and other team members prefer O_{t2} . O_{t1} should be a more suitable time slot for the meeting than O_{t2} and O_{t3} for the reason that the CTO has the highest priority. Similarly, O_{t3} should have higher membership in S_t than O_{t2} for the similar reason.

According to the above examples, we can find that previous models cannot handle the cases of which defining properties of concepts have different importance degrees or priority degrees.

IV. A FORMAL MODEL OF ONTOLOGY WITH PROPERTY IMPORTANCE AND PROPERTY PRIORITY

In this section, we propose a novel formal model of fuzzy ontology with property importance and property priority.

A. A Conceptual Model of Fuzzy Ontology

Following the works in [6], we consider a fuzzy ontology O in a particular domain Δ consisting of a set of fuzzy concepts, a set of fuzzy properties and a set of objects.¹

An object in an ontology is represented by a set of properties named **object property vector**. An object property vector $\vec{P}_a$ of an object a is a vector of *property*^{value} pairs as following:

$$\vec{P}_a = (p_{a,1}^{v_{a,1}}, p_{a,2}^{v_{a,2}}, \dots, p_{a,n}^{v_{a,n}}), 1 \leq i \leq n$$

where $p_{a,i}$ is a property a possessing, $v_{a,i}$ is the degree to which a possesses property $p_{a,i}$. The relation among properties in the object property vector is conjunction.

We combine the classical view and fuzzy set theory so that our model can handle vague concepts. We consider that all members of a concept should possess all defining properties of the concept to some degrees. A concept can be defined by a set of properties. Besides, there is a minimal requirement for each defining property of concepts. If an object possesses all defining properties of a concept to higher degrees, then it means that the object satisfies the minimal requirements of defining properties to higher degrees. Thus the object is given a higher membership degree in the concept.

Based on classical view and fuzzy set theory, the generation of representation of a concept C is as following [6]:

$$\vec{C} = (\vec{S}_1, \vec{S}_2, \dots, \vec{S}_m), 1 \leq i \leq m$$

and

$$\vec{S}_i = (p_{i,1}^{w_{i,1}}, p_{i,2}^{w_{i,2}}, \dots, p_{i,n_i}^{w_{i,n_i}}), 1 \leq j \leq n_i$$

where n_i is the number of properties in $\vec{S}_i$. A $\vec{S}_i$ is named a **characteristic vector** of C which consists of a set of defining properties. The relation among characteristic vectors is union, and the relation among defining properties in a $\vec{S}_i$ is conjunction. $p_{i,j}$ is a defining property in a $\vec{S}_i$. $w_{i,j}$ is considered as a minimal requirement of property $p_{i,j}$ and $w_{i,j} \in (0, 1]$. Thus, a concept can be defined by some characteristic vectors, and such a representation of a concept can handle conjunction concepts, disjunction concepts and combination concepts [6].

B. Modeling Property Importance

The membership of an object in a concept is affected by the satisfaction degrees of the object satisfying minimal requirements of all defining properties of the concept. Among all defining properties, some properties have more affection on the object membership measurement than others and we consider that such properties are more important than other properties for object membership calculation. For different concepts, their defining properties are different and the importance of those defining properties are different also. The effect caused by decreasing of the degree of an object possessing a property with a higher importance may be compensated by (be substituted by) increasing more degrees of the object possessing other properties with relative lower priority in a

¹In the rest of this paper, all concepts, and properties are referred to fuzzy concepts and fuzzy properties respectively unless otherwise specified.

concept. For example, although the location of a house is worse than that of another one, if its rent is much more cheaper than that of the other one then it still is a better candidate to rent.

According to some main dictionaries, importance means a quality or aspect having great worth or significance, and it implies a value judgment of the superior worth or influence of something or someone. What is more, weight implies a judgment of the immediate relative importance of something. Thus, we can consider that there is a weight distribution among all defining properties, and such weights are corresponding to the importance of the corresponding defining properties, the sum of all weights is 1.

The importance of properties in a concept are represented by a **property importance vector** in which values are used to indicate effect of corresponding defining properties on object membership measurement in the concept.

Definition 1. A **property importance vector** of a concept c , denoted by $\vec{t}^c$, is a vector of *property^{value}* pairs as following:

$$\vec{t}^c = (p_1^{r_1}, p_2^{r_2}, \dots, p_n^{r_n}), 1 \leq i \leq n, r_i \in (0, 1]$$

where n is the number of properties in the domain, r_i is the importance value of property p_i .

If a property p_i is more important than another property p_j in a concept definition, i.e., p_i has more affection on object membership than p_j , then importance value r_i of p_i is greater than that (i.e., r_j) of p_j in $\vec{t}^c$. If $r_i = 1$ and $\sum_{i=1}^n r_i = 1$ means that there is only one defining property for the concept, and this single defining property is absolutely important. $\sum_{i=1}^n r_i = 1$ means that measuring memberships of objects in a concept is based on all defining properties of the concept. In different application domains, users have different perspective for concepts. Thus, such a property importance vector is given by users.

C. Modeling Property Priority

According to main English dictionaries, priority is used to measure the relative precedence among things in a group. The higher the priority of one thing, the earlier the thing should be handled or the more preferred is the thing.

According to people's intuition and the explanation of main English dictionaries, the essential meaning of priority is that, the effect of a property with higher priority on object membership is in a higher level than that of property with lower priority. In other words, the effect caused by any decreasing of the degree of an object possessing a property with a higher priority cannot be compensated by increasing no matter to what degrees the object possessing other properties with lower priority in a concept. If one wants to raise the object membership of an object in a concept, a defining property with a relatively higher priority must be sufficiently satisfied prior to a defining property with a relatively lower priority.

We consider that priorities of properties in a concept are represented by a property priority vector of the concept, in which are used to indicate preference order of satisfying properties on measuring object memberships.

Definition 2. A **property priority vector** of a concept c is a vector of *property^{value}* pairs as following:

$$\vec{p}^c = (p_1^{u_1}, p_2^{u_2}, \dots, p_n^{u_n}), 1 \leq i \leq n, u_i \in \Omega$$

where n is the number of properties in the domain, u_i is the priority value of property p_i and Ω is a total order set.

Such a property priority vector is given by users for the reason that different users have different consideration on property priorities. In the property priority vector, u_i is the priority value of corresponding property p_i , and it is used to indicate the priority (preference order) of p_i . Furthermore, we present two propositions for priority ordering.

Proposition 1. If a property p_i is **prior to** (more preference than) property p_j in concept c , then u_i is succeeded by u_j in Ω , denoted by $u_i \succ_c u_j$.

Proposition 2. If a property p_i is **prior to or have the same priority with** property p_j in a concept c , then u_i is succeeded by u_j or u_i is equal to u_j , denoted by $u_i \succeq_c u_j$.

V. FORMALIZING OBJECT MEMBERSHIPS

As mentioned above, the membership of an object in a concept depends on aggregating satisfaction degrees of the object satisfying the minimal requirements of all defining properties of the concept.

A. Local Satisfaction Degrees

Definition 3. The **local satisfaction degree** (LSD) of an object a for a properties p_i in a characteristic vector $\vec{S}_x$ is the degree of object a satisfying the minimal requirement of p_i in $\vec{S}_x$.

The local satisfaction degree (LSD) of object a for property p_i in a characteristic vector $\vec{S}_x$ is a degree in the range of $[0,1]$ and it is given by a local satisfaction function ℓ .

$$\ell : Q \times O \rightarrow [0, 1]$$

where Q is the set of defining properties, O is the set of object property vector. If $\ell(p_i, \vec{P}_a) = 1$, it means that object a definitely satisfies the minimal requirement of property p_i . If $\ell(p_i, \vec{P}_a) = 0$, it means that object a does not satisfy the minimal requirement of property p_i at all. We present some axioms for the local satisfaction function ℓ to observe.

Axiom 1. For an object a , a defining property $p_{S,i}$ in characteristic vector $\vec{S}$, if a possesses property $p_{S,i}$ to a degree greater than the minimal requirement of $p_{S,i}$, i.e., $v_{S,i}^a \geq w_{S,i}$, then $\ell(p_{S,i}, \vec{P}_a) = 1$.

Axiom 2. For an object a , a defining property $p_{S,i}$ in characteristic vector $\vec{S}$, if a possesses property $p_{S,i}$ to a degree 0, i.e., $p_{S,i}^a = 0$, then $\ell(p_{S,i}, \vec{P}_a) = 0$.

Axiom 3. For an object a , two defining properties $p_{S,i}$ and $p_{S,j}$ in characteristic vector $\vec{S}$, $v_{S,i}^a < w_{S,i}$ and $v_{S,j}^a < w_{S,j}$, and $v_{S,i}^a = v_{S,j}^a$, $w_{S,i} \geq w_{S,j}$, then $\ell(p_{S,i}, \vec{P}_a) \leq \ell(p_{S,j}, \vec{P}_a)$

Axiom 4. For an object a , two defining properties $p_{S,i}$ and $p_{S,j}$ in characteristic vector $\vec{S}$, $v_{S,i}^a < w_{S,i}$, $v_{S,j}^a < w_{S,j}$ and $v_{S,i}^a \geq v_{S,j}^a$, if $w_{S,i} = w_{S,j}$, then $\ell(p_{S,i}, \vec{P}_a) \geq \ell(p_{S,j}, \vec{P}_a)$

Axiom 5. For two objects a and b , a defining property $p_{S,i}$ in characteristic vector $\vec{S}$, $v_{S,i}^b \leq v_{S,i}^a < w_{S,i}$, then $\ell(p_{S,i}, \vec{P}_a) \geq \ell(p_{S,i}, \vec{P}_b)$

Theorem 1. For an object a , two defining properties $p_{S,i}$ and $p_{S,j}$ in characteristic vector $\vec{S}$, $v_{S,i}^a < w_{S,i}$, $v_{S,j}^a < w_{S,j}$ and $v_{S,i}^a \geq v_{S,j}^a$, if $w_{S,i} \leq w_{S,j}$, then $\ell(p_{S,i}, \vec{P}_a) \geq \ell(p_{S,j}, \vec{P}_a)$

Axioms 1 and 2 specify the boundary cases of local satisfaction degrees of objects for properties. Axioms 3 specifies the influence of minimal requirements of properties, and it means that the higher minimal requirement of a property is the stricter requirement of the property, thus the less LSDs for an object for the property. Axioms 4 and 5 specify the influence of degrees of objects possessing properties, and they show the monotonic increase of the LSDs of objects for properties as the increase of degrees of objects possessing properties.²

We propose a possible function which satisfies the Axioms 1 to 5.

$$\ell(p_{S,i}, \vec{P}_a) = \begin{cases} v_{S,i} + (1 - w_{S,i}) & v_{S,i}^a < w_{S,i} \\ 1 & v_{S,i}^a \geq w_{S,i} \end{cases} \quad (1)$$

B. Global Satisfaction Degrees with Weighted Properties

After obtaining all local satisfaction degrees of an object for all defining properties in a characteristic vector, we need to aggregate all of them and the aggregation degree is named as **Global Satisfaction Degree** (GSD). There are two kinds of Global Satisfaction Degree. One is corresponding to characteristic vectors defined with weighted properties and another is corresponding to characteristic vector defined with prioritized properties.

Definition 4. The global satisfaction degree (GSD) of an object a for a characteristic vector $\vec{S}$ defined with weighted properties is an aggregation degree of aggregating all local satisfaction degrees of a for properties in $\vec{S}$, which is in the range of $[0,1]$.

While a concept is defined by a set of weighted properties, the global satisfaction degree of an object for a characteristic vector depends on aggregating not only all local satisfaction degrees, but also the importance of the corresponding properties. The higher degree of an object satisfying an property and the more importance of the property are, the higher object membership.

We consider that the global satisfaction degree of an object for a characteristic vector consisting of weighted properties is given by a function:

$$g^t : (T \times L)^n \rightarrow [0, 1]$$

where T is the set of property importance vectors, L is the set of LSDs of objects for properties and n is the set of numbers of defining properties in characteristic vectors.

We also can represent such a function as the following:

$$g_{S,a}^t = \oplus_{i=1}^n \{(r_{S,i}, \ell(p_{S,i}, \vec{P}_a)) | p_{S,i} \in \vec{S}\}$$

²We do not show the verification of all axioms and the proof of all theorems in the paper for the limitation of space.

where $p_{S,i}$ is a defining property of a characteristic vector $\vec{S}$, $r_{S,i}$ is the importance value of property p_i in a concept C where $\vec{S} \in \vec{C}$ and $\oplus$ is an aggregation function.

Properties with higher importance have more effects on the global satisfaction degrees than other properties with lower importance while aggregating all local satisfaction degrees. It means that an object can possess a higher GSD if it satisfies properties which are more important to higher degrees. This is in line with the intuition of people and the essence of the importance.

The function $g_{S,a}^t$ should observe the following axioms:

Axiom 6. For an object a and a characteristic vector $\vec{S}$, if $\exists i$ where $p_{S,i} \in \vec{S}$, $\ell(p_{S,i}, \vec{P}_a) = 0$, then $g_{S,a}^t = 0$

Axiom 7. For an object a and a characteristic vector $\vec{S}$, if $\forall i$ where $p_{S,i} \in \vec{S}$, $\ell(p_{S,i}, \vec{P}_a) = 1$, then $g_{S,a}^t = 1$

Axiom 8. For two objects a and b , a characteristic vector $\vec{S}$, if for some properties $p_{S,i} \in \vec{S}$, $\ell(p_{S,i}, \vec{P}_a) \geq \ell(p_{S,i}, \vec{P}_b)$, and $\forall x$ where $x \neq i$, $p_{S,x} \in \vec{S}$ and $\ell(p_{S,x}, \vec{P}_a) = \ell(p_{S,x}, \vec{P}_b)$, then $g_{S,a}^t \geq g_{S,b}^t$

Axiom 9. For an object a , two characteristic vectors $\vec{S}$ and $\vec{K}$, if for property $p_{S,i} \in \vec{S}$ and property $p_{K,j} \in \vec{K}$, $i \neq j$, $r_{S,i} = r_{K,j}$, $\ell(p_{S,i}, \vec{P}_a) \geq \ell(p_{K,j}, \vec{P}_a)$, and for all other properties x in $\vec{S}$ are also shared by $\vec{K}$, $x \neq i \neq j$, $\ell(p_{S,x}, \vec{P}_a) = \ell(p_{K,x}, \vec{P}_a)$, then $g_{S,a}^t \geq g_{K,a}^t$

Axiom 10. For two objects a and b , two defining properties $p_{S,i}$ and $p_{S,j}$ in a characteristic vector $\vec{S}$, $r_{S,i} > r_{S,j}$, if $r_{S,i} - r_{S,j} = \gamma$, $\gamma > 0$, $\ell(p_{S,i}, \vec{P}_a) - \ell(p_{S,i}, \vec{P}_b) = \delta$, $\delta \geq 0$ and $\ell(p_{S,j}, \vec{P}_b) - \ell(p_{S,j}, \vec{P}_a) = \delta'$, $\delta' \geq 0$, and $\forall x$ where $x \neq i \neq j$, $\ell(p_{S,x}, \vec{P}_a) = \ell(p_{S,x}, \vec{P}_b)$, and $\exists \epsilon \geq 0$, $\delta' - \delta = \epsilon$, then $g_{S,b}^t \leq g_{S,a}^t$.

Axioms 6 and 7 specify the boundary cases of global satisfaction degrees of objects for characteristic vectors. Axioms 8 and 9 specify the influence of the LSDs of objects for properties on GSDs of objects for characteristics vectors consisting of weighted properties. They show the monotonicity of the aggregation, i.e., the global satisfaction degree of an object for a characteristic vector should increase when the local satisfaction degrees of the objects for defining properties in the characteristic vector increase.

Axiom 10 shows the essential of importance of properties. The effect of a more important property on GSDs of objects for a characteristic vector is greater than that of less importance properties. Besides, the effects reduced by decreasing the importance of properties can be compensated by increasing LSDs of objects for other properties.

We present a possible function which satisfies Axiom 6 to 10 as following:

$$g_{S,a}^t = \begin{cases} \sum_{i=1}^n r_{S,i} \times \ell(p_{S,i}, \vec{P}_a) & \text{others} \\ 0 & \exists i, \ell(p_{S,i}, \vec{P}_a) = 0 \end{cases} \quad (2)$$

If all properties in a characteristic vector have the same importance (i.e., the same weight), it is a special case of characteristic vector with weighted properties. In such a special case, all properties in a characteristic vector are equivalent properties and the function $g_{S,a}^t$ does not need to satisfy Axiom

10 (only need to observe Axioms 6 to 9) for the reason that all property importance are the same. In this special case, the GSD of an object for a characteristic vector defined by equivalent properties is given by a function denoted by $g_{S,a}^l$:

$$g_{S,a}^l = \Phi_{i=1}^n \{ \ell(p_{S,i}, \vec{P}_a) | p_{S,i} \in \vec{S} \}$$

where Φ is an aggregation function only satisfying Axioms 6-9. We present a possible function for $g_{S,a}^l$ as following:

$$g_{S,a}^l = \begin{cases} \frac{\sum_{i=1}^n \ell(p_{S,i}, \vec{P}_a)}{n} & \text{others} \\ 0 & \exists i, \ell(p_{S,i}, \vec{P}_a) = 0 \end{cases} \quad (3)$$

Equation 3 observes Axioms 6 to 9 and does not observe Axiom 10, and it is also a special case of Equation 2. In such a special case, for Equation 2, the weight of each property in characteristic vector $\vec{S}$ is $\frac{1}{n}$, i.e., $\forall i, r_{S,i} = \frac{1}{n}$.

C. Global Satisfaction Degrees with Prioritized Properties

Property priority is different from property importance. The affections of properties with different priority on object membership are in different level. Each priority value in Ω is corresponding to a priority level. The effect of increase of LSDs of an object for lower priority properties cannot be cumulated so as to compensate that of decrease of LSDs of the object for higher priority properties. In other words, if the local satisfaction degree of an object O_a for the highest priority property in a concept C is greater than that of another object O_b , then the object membership of O_a in C is greater than that of O_b no matter to what degrees the LSDs of these two objects for other properties with lower priorities are. For example, the vetoes of permanent member of the UN Security Council (i.e., USA, UK, France, Russia and China) have higher priority than that of other members, and if a proposal is not approved by any one of the permanent members, then the proposal cannot be passed even if all non-permanent members agree to it. Thus, we cannot combine the LSDs of objects for higher priority properties with that of the LSDs of objects for lower priority properties by simply aggregation as introduced in Section 5.2.

To reflect the characteristics of priority of properties, we consider that there is a corresponding global satisfaction degree for each priority level.

Definition 5. The i th-level global satisfaction degree of object a for a characteristic vector $\vec{S}$ denoted by $g_{S,a}^{\rho,i}$ is an aggregation degree of aggregating all local satisfaction degrees of the object for properties with a priority i ($i \in \Omega$) in $\vec{S}$, which is in the range of $[0,1]$.

There may be some properties which have the same priority. In other words, there are some properties in i th priority level, and these properties are considered as equivalent properties in i th priority level. We consider that i th-level global satisfaction degree of an object a for a characteristic vector $\vec{S}$ is determined as following:

$$g_{S,a}^{\rho,i} = \Phi_{x=1}^n \{ \ell(p_{S,x}, \vec{P}_a) | p_{S,x} \in \vec{S}, u_{S,x} = i \} \quad (4)$$

where $\ell(p_{S,x}, \vec{P}_a)$ is the local satisfaction degree of object a for property $p_{S,x}$, $u_{S,x}$ is the priority of $p_{S,x}$ and Φ is an aggregation function as introduced in Section 5.2.

Definition 6. The global satisfaction degree of object a for a characteristic vector $\vec{S}$ with prioritized properties is a tuple consists of all i th-level GSDs, i.e.,

$$g_{S,a}^{\rho} = (g_{S,a}^{\rho,1}, g_{S,a}^{\rho,2}, \dots, g_{S,a}^{\rho,n})$$

where n is the number of priority levels of properties in $\vec{S}$.

For the reason that global satisfaction degrees of objects for a characteristic vectors with prioritized properties are tuples, we present some axioms for comparing them.

Axiom 11. For two objects a and b , a characteristic vector $\vec{S}$ with prioritized properties, if for a priority level i , $g_{S,a}^{\rho,i} > g_{S,b}^{\rho,i}$, and for those priority levels x where $u_{S,x} \succeq u_{S,i}$ we have $g_{S,a}^{\rho,x} = g_{S,b}^{\rho,x}$, then $g_{S,a}^{\rho} > g_{S,b}^{\rho}$.

Axiom 12. For two objects a and b , a characteristic vector $\vec{S}$ with prioritized properties, if for all priority levels i where $g_{S,a}^{\rho,i} = g_{S,b}^{\rho,i}$, then $g_{S,a}^{\rho} = g_{S,b}^{\rho}$.

Theorem 2. For two objects a and b , a characteristic vector $\vec{S}$ with prioritized properties, if for all priority levels i , $g_{S,a}^{\rho,i} \geq g_{S,b}^{\rho,i}$, then $g_{S,a}^{\rho} \geq g_{S,b}^{\rho}$.

Axiom 11 shows the essential of priorities of properties. If the LSD of an object a for a property with a priority (i.e., priority level i) is greater than that of another object b , and the LSDs of a and b for properties with higher priorities are the same (i.e., priority level x), then the GSD of a will greater than that of b no matter to which degree of object a and b satisfy other properties with lower priority than i . Axiom 12 specifies the case that when two objects' GSDs are the same. If all LSDs of the two objects for all defining properties in a characteristic vector are the same, then the GSDs of them for the characteristic vector are the same.

If all properties in a characteristic vector have the same priority, it is a special case of characteristic vector with prioritized properties. In such a special case, there is only one priority level (i.e., 1st-level), and all properties in a characteristic vector are equivalent properties in the same priority level. Thus, the GSD of an object a for a characteristic vector $\vec{S}$ is equal to the 1st-level GSD (a tuple consisting only the 1st-level global satisfaction degree), i.e.,

$$g_{S,a}^{\rho} = g_{S,a}^{\rho,1} = \Phi_{i=1}^n \{ \ell(p_{S,i}, \vec{P}_a) | p_{S,i} \in \vec{S}, u_{S,i} = 1 \}$$

D. Measuring Object Membership

We consider the fuzzy membership of an object a in fuzzy concept C depends on the following equation:

$$\mu_C(a) = \max(g'_{S_1,a}, g'_{S_2,a}, \dots, g'_{S_m,a}) \quad (5)$$

where m is the number of characteristic vectors in concept C and $g'_{S_i,a}$ is the global satisfaction degree of object a for characteristic $\vec{S}_i$ in concept C . If the properties in $\vec{S}_i$ are weighted properties, then $g'_{S_i,a} = g_{S_i,a}^l$, which is a numerical value defined in Section 5.2; if the properties in $\vec{S}_i$ are prioritized properties, then $g'_{S_i,a} = g_{S_i,a}^{\rho}$, which is a tuple defined in Section 5.3.

An object may satisfy all the property minimal requirements of more than one characteristic vectors. We use the maximal value of $g'_{S_i,a}$ as the membership of a in C because that the relation among $\vec{S}_i$ is disjunction.

TABLE IV
LSDs of O_1 and O_2 in the concept S_r

	$PS_{r,A}$	$PS_{r,B}$	$PS_{r,C}$	$PS_{r,D}$
O_1	1	1	1	0.65
O_2	0.65	0.9	1	1

TABLE V
LSDs of O_1 , O_2 and O_3 in S_t

	$PS_{t,A}$	$PS_{t,B}$	$PS_{t,C}$	$PS_{t,D}$
O_1	1	0.95	0.75	0.7
O_2	0.9	0.77	0.75	1
O_3	0.9	0.9	0.85	0.8

VI. ILLUSTRATING EXAMPLES

Let us revisit the Example 1 in Section 3. For our model, the property importance vector of S_r is given as following:

$$\vec{t}^{S_r} = (A^{0.4}, B^{0.3}, C^{0.2}, D^{0.1})$$

Based on Equation 1, we obtain the local satisfaction degrees of O_1 and O_2 for properties in S_r shown in table IV.

We use Equations 2 and 5 to calculate the object memberships for O_1 and O_2 and get the following results:

$$\mu_{S_r}(O_1) = 0.96, \mu_{S_r}(O_2) = 0.83$$

Such a result is more intuitive than previous models. Based on our model, the room O_1 satisfies the top two importance properties A and B to higher degrees than O_2 even though it possesses properties C and D to lower degrees than O_2 , thus the global satisfaction degree of O_1 for concept S_r is greater than that of O_2 . Previous models do not take property importance into consideration and cannot handle objects memberships in concepts with weighted properties.

For the Example 2 in Section 3, in our model, the property importance vector of S_t is given as following:

$$\vec{\rho}^{S_t} = (A^{first}, B^{second}, C^{third}, D^{fourth})$$

Based on Equation 1, we obtain the local satisfaction degrees of O_{t1} , O_{t2} and O_{t3} for properties in the concept S_t shown in table V.

We use Equation 3, 4 and 5 to obtain the global satisfaction degree of object O_{t1} , O_{t2} and O_{t3} as following:

$$\mu_{S_t}(O_{t1}) = (1, 0.85, 0.7), \mu_{S_t}(O_{t2}) = (0.9, 0.76, 1)$$

$$\mu_{S_t}(O_{t3}) = (0.9, 0.875, 0.8)$$

Based on Axioms 11 and 12, the object membership comparison result of O_{t1} , O_{t2} and O_{t3} is as following:

$$\mu_{S_t}(O_{t1}) > \mu_{S_t}(O_{t3}) > \mu_{S_t}(O_{t2})$$

Such a result is more reasonable than previous models.

In the above two examples, both concepts S_r and S_t have only one characteristic vector (i.e., in Equation 5, $m = 1$), and the object memberships of objects in these two concepts depend on the object property vectors, the unique characteristic vector for each concept and property importance vector (or property priority vector).

VII. EXPERIMENT

A. Evaluation on Concepts with Property Importance

We collect the information of 200 rental houses in Hong Kong. We want to measure the membership degrees of these 200 houses for the concept of ‘suitable houses to rent’ for users. There are four properties for each house, which are ‘Price’, ‘Rental period’, ‘Time cost of walk’ and ‘Facilities’. We let each user input his personal importance degrees of properties for measurement. Different users may give different importance weights for properties.

In our experiment, the standard result is a ranking of object memberships given by users through questionnaires. It is not suitable to ask people to rank too many objects based on memberships because people may be confused with too many objects in a questionnaire, so we randomly select 10 houses from those 200 houses for questionnaires of users. For each user, the randomly selected 10 houses may be different. We evaluate the calculation results of our model on the 10 houses by comparing them with the standard results given by users.

We propose a measure function *Precision* for evaluating the calculation results based on the questionnaire.

$$Precision = 1 - \frac{\sum_{i=1}^n |CRank_i - QRank_i|}{m}$$

where n is the number of objects, m is the possible maximal difference between the calculation ranking of objects and questionnaire ranking of objects, $CRank_i$ is the rank number of the i th object obtained by calculation, $QRank_i$ is the rank number of the i th object given by the user. The *Precision* function is a decreasing function of average of distances between corresponding objects’ memberships rankings in calculation and questionnaire. There are 10 objects and thus m (the denominator in the equation) is 50. The precision comparison of our model and the model in [6] on concepts with property importance are shown in Fig. 1.

According to Fig. 1, our method is more accurate than previous method without property importance. The reason is that our method takes the property importance into consideration and this is in line with users. Previous method just treats all properties equivalently, thus the results of membership ranking are the same for all users, i.e., it does not distinguish different users’ requirements on property importance.

B. Evaluation on Concepts with Property Priority

To evaluate our method on handling the cases with property priority, we collect the medal information of 87 countries that have medals in 2008 Olympic Games. We want to measure the membership degrees of these countries for the concept of ‘Outstanding Country in 2008 Olympic Games’. There are three properties for each country, which are ‘Number of Gold Medals’, ‘Number of Silver Medals’, and ‘Number of Bronze Medals’. In our method, we give the first priority to ‘Number of Gold Medals’, the second priority to ‘Number of Silver Medals’ and third priority to ‘Number of Bronze Medals’. We also use the precision metric and the standard answer is obtained from the ‘medal table’ in 2008 Olympic Games



Fig. 1. Comparison of precision with concepts with property importance

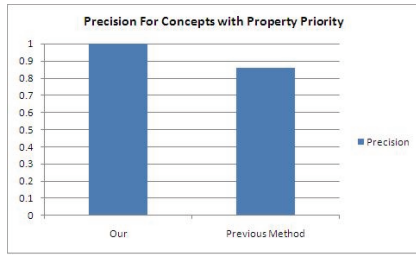


Fig. 2. Comparison of precision with concepts with property priority

website.³ The precision comparison of our method and the method in [6] on concepts with property importance are shown in Fig. 2. According to Fig. 2, similar to Fig. 2, our method is more accurate (the same ranking as the standard answer) than previous method without property priority.

VIII. DISCUSSIONS

Importance and priority are two different subjective judgments of people on properties.

Property importance is used to indicate the influence effects of corresponding properties on a cognitive task (e.g., measuring object membership in this paper). One interesting characteristic of weighted property is that the effect of a weighted property on object membership measurement can be substituted (compensated) by effects of other weighted properties. For a concept defined by weighted properties, the membership of an object in the concept is the combination degree of the object satisfying all defining properties. Properties having different importance in a characteristic vector can form a weight distribution among properties, the more important the more weight of a property. All the weighted properties can affect the object membership measurement simultaneously instead of a sequence comparison of defining properties. For instance, in Example 1, the membership of a candidate room in concept S_r depends on aggregating the local satisfaction degrees of the house for all defining properties of S_r simultaneously.

Property priority is used to indicate the preference order of satisfying defining properties for increasing object memberships. For a concept defined by prioritized properties, object membership of an object in the concept is a tuple of degrees instead of a number in the range of [0,1], and the object membership comparison depends on a sequence comparison

from properties with higher priority to properties with lower priority. For a prioritized property with higher priority, the effect of it on object membership measurement cannot be substituted by effects of other prioritized properties with lower priority values. Thus, it is more reasonable to model the membership of an object in a concept defined by prioritized properties as a tuple consisting of different level GSDs instead of a number and compare the memberships of two objects level-by-level (from higher to lower priority) as a sequence comparison. For instance, in Example 2, the choose of a suitable time slot for the meeting depends on comparing the different level GSDs from high to low priority level.

IX. CONCLUSION

In this paper, we formalize object membership in fuzzy ontology with property importance and property priority. We also show that our model is more reasonable and intuitive in measuring object memberships in concepts with property importance or property priority than previous models by examples. Our formal mechanism used to measure object memberships in concepts can handle not only concepts defined by equivalent properties, but also concepts defined by weighted properties or prioritized properties well, which cannot be handled well in previous models.

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³http://match.2008.sina.com.cn/bj2008/all_medal.php