

Stability Analysis and Synthesis of Markovian Jump Nonlinear Systems with Incomplete Transition Descriptions via Fuzzy Control

Min Kook Song

Department of Electrical and
Electronic Engineering, Yonsei University,
Shinchon-Dong, Seodaemum-Gu,
Seoul, Korea
Email: MinKookSong@gmail.com

Jin Bae Park

Department of Electrical and
Electronic Engineering, Yonsei University,
Shinchon-Dong, Seodaemum-Gu,
Seoul, Korea
Email: jbpark@yonsei.ac.kr

Young Hoon Joo

School of Electronic and
Information Engineering, Kunsan National
University, Kunsan, Chonbuk,
573-701, Korea
Email: yhjoo@kunsan.ac.kr

Abstract—This paper is concerned with exploring an extended approach for the stability analysis and synthesis for Markovian jump nonlinear systems (MJNLSs) with incomplete transition descriptions via fuzzy control. In this paper, not all the elements of the rate transition matrices (RTMs) are assumed to be known. By fully considering the properties of the RTMs and the convexity of the uncertain domains, sufficient criteria of stability and stabilization are obtained. The proposed stability conditions are much less conservative than most of the existing results and stabilization conditions with a mode-dependent fuzzy controller are derived for Markovian jump fuzzy systems (MJFSs) in terms of linear matrix inequalities (LMIs). Finally, illustrative numerical examples are provided to demonstrate the effectiveness of the proposed approach.

Index Terms—Markovian fuzzy systems, partially known transition matrix, linear matrix inequality

I. INTRODUCTION

Markovian jump nonlinear systems (MJNLSs) are a class of multi-modal systems in which the transitions among different modes are governed by a Markov chain. The studies of these systems are motivated by the powerful modeling capability of Markov chains in practical applications, and many useful results have been obtained, see [2]-[6] for instance. However, in most of the studies, complete knowledge of the mode transitions is required as a prerequisite for analysis and synthesis of MJNLSs. This means that the transition probabilities (TPs) of the underlying Markov chain are assumed to be completely known.

To relax the assumption that all the TPs are known, a new concept for Markovian jump linear systems (MJLSs) with partially unknown TPs was proposed in [17] and a series of studies have been carried out [18]-[20]. Meanwhile, as two extreme cases, the so-called switched systems under arbitrary switching [18]-[20] and the conventional Markov jump systems was covered in the framework. However, although the works laid a conceptual foundation for analysis and synthesis of MJLSs, the proposed approach still have room for improvement in terms of conservatism. In fact, the properties of the transition rate matrix (RTM) has not been fully used. And most of existing results are dedicated to the MJLSs, while few results are available

for the control design of Markovian jump nonlinear systems (MJNLSs).

In this paper, a new approach will be explored for the analysis and synthesis of MJNLSs with incomplete description of their transitions via fuzzy control. The proposed systems are more general and because it includes the completely known or unknown transition probabilities. Using the properties that the sum of each row is zero in a RTM, together with the convexity of the uncertain domains, necessary and sufficient conditions for the stability analysis and stabilization synthesis problems are first derived. The stabilization condition is expressed in terms of linear matrix inequalities (LMIs), which can be solved efficiently via existing LMI-optimization techniques.

II. PROBLEM FORMULATION AND PRELIMINARIES

Given the probability space $(\Omega, \mathcal{F}, \mathbb{P})$: and consider continuous-time and discrete-time MJLNS, which can be described by the following fuzzy models:

$$\begin{aligned} R_i: & \text{ IF } z_1(t) \text{ is } \Gamma_{i1} \text{ and } \cdots \text{ and } z_p(t) \text{ is } \Gamma_{ip} \\ \text{ THEN } & \dot{x}(t) = A_{0i}(\eta(t))x(t) + B_i(\eta(t))u(t) \quad (1) \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ constitutes the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $R_i, i \in \mathcal{I}_R = \{1, 2, \dots, r\}$, denotes the i th fuzzy rule, $z_h(t), h \in \mathcal{I}_P = \{1, 2, \dots, p\}$, is the h th premise variable, $\Gamma_{ih}, (i, h) \in \mathcal{I}_R \times \mathcal{I}_P$, is the fuzzy set of $z_h(t)$ in R_i , the system matrices of the i th rule are denoted by (A_{0i}, B_i) , which are assumed known and some constant matrices of compatible dimensions. The stochastic process $\{\eta(t), t \geq 0\}$ (respectively, the Markovian chain $\eta(k), k \geq 0$), taking values in a finite space state denoted by $\mathbb{T} = \{1, 2, \dots, \mathcal{N}\}$. The random form process $\eta(t)$ is a continuous-time has the following mode transition probabilities:

$$\Pr(\eta(t + \Delta t) = s | \eta(t) = l) = \begin{cases} \lambda_{ls}\Delta + o(\Delta), & s \neq l \\ 1 + \lambda_{ls}\Delta + o(\Delta), & s = l \end{cases}$$

where $\Delta > 0$, $\lim_{\Delta \rightarrow 0} (o(\Delta t)/\Delta t) = 0$ ($l, s \in \mathbb{T}, l \neq s$) denotes the switching rate from mode l at time t to mode s at

time $t + \Delta t$, and $\lambda_{ll} = -\sum_{s=1, s \neq l} \lambda_{ls}$ for all $l \in \mathbb{T}$. Hence, the rate transition matrix (RTM) in the Markov process is given by

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2N} \\ & & \ddots & \\ \lambda_{N1} & \lambda_{N2} & \cdots & \lambda_{NN} \end{bmatrix} \quad (2)$$

The set $\mathbb{T}$ contains N modes of system (1) for $\eta(t) = l \in \mathbb{T}$, the system matrices of the l th modes are denoted by $(A_{0i,l}, B_{i,l})$, which are real known with appropriate dimensions.

In addition, the transition rates or probabilities of the Markov chain in this paper are considered to be partially known, namely, some elements in matrix Λ or $\mathbf{1}$ are time-invariant but unknown. We denote

$$\mathbb{T}_{\mathbb{K}}^l = \{s : \text{if } \lambda_{ls} \text{ is known}\} \quad (3)$$

$$\mathbb{T}_{\mathbb{UK}}^l = \{s : \text{if } \lambda_{ls} \text{ is unknown}\} \quad (4)$$

Moreover, if $\mathbb{T}_{\mathbb{K}}^l \neq \emptyset$, it is described as

$$\mathbb{T}_{\mathbb{K}}^l = \{\mathbb{E}_1^l, \dots, \mathbb{E}_m^l\}, \quad 1 \leq m \leq N \quad (5)$$

where $\mathbb{E}_m^l$ represents the m th element with the index $\mathbb{E}_m^l$ in the l th row of matrix Λ . In the continuous-time case, when λ_{ll} is unknown, it is necessary to provide a lower bound $\lambda_d^{(l)}$ for it and we have $\lambda_d^{(l)} < -\lambda_{\mathbb{E}}^{(l)}$. Let the mode at t be l . Using the center-average defuzzifier, product inference, and singleton fuzzifier, (1) is inferred as

$$\dot{x}(t) = \sum_{i=1}^r \theta_i(z(t)) (A_{0i}(\eta(t))x(t) + B_i(\eta(t))u(t)) \quad (6)$$

where $\theta_i(z(t)) = w_i(z(t)) / \sum_{i=1}^r w_i(z(t))$, $w_i(z(t)) = \prod_{h=1}^p \mu_{\Gamma_{ih}}(z_h(t))$, and $\mu_{\Gamma_{ih}}(z_h(t)) : U_{z_h} \subset \mathbb{R} \rightarrow \mathbb{R}_{[0,1]}$ is the membership function of $z_h(t)$ on the compact set U_{z_h} . Some basic properties are $\theta_i(z(t)) \geq 0$ and $\sum_{i=1}^r \theta_i(z(t)) = 1$.

Throughout this paper, it is assumed that state $x(t)$ or $x(k)$ are available and the mode $\eta(t)$ or $\eta(k)$ are not available for feedback control. For each possible value of $\eta(t) = l$ or $\eta(k) = l, l \in \mathbb{T}$ in the succeeding discussion, for the convenience of notations, we will denote

$$A_{0i,l}(t) = \sum_{i=1}^r \theta_i(z(t)) A_{0i}(\eta(t))$$

$$B_{i,l}(t) = \sum_{i=1}^r \theta_i(z(t)) B_i(\eta(t))$$

In this paper, we will derive the stochastic stability criteria for system (1) when the transition rates or probabilities are partially known, and to determine the feedback gains $K_j (j \in \mathcal{I}_r)$ such that the resulting closed-loop system is stochastically stable via the mode-independent fuzzy controller

$$R_i: \text{IF } z_1(t) \text{ is } \Gamma_{i1} \text{ and } \cdots \text{ and } z_p(t) \text{ is } \Gamma_{ip} \\ \text{THEN } u(t) = K_j(\eta(t))x(t), \quad j \in \mathcal{I}_R$$

whose defuzzified output is given by

$$u(t) = \sum_{j=1}^r \theta_j(z(t)) K_j(\eta(t)) x(t) \\ = K_{j,l}(t) x(t) \quad (7)$$

With the mode-independent fuzzy control law (7), the overall closed-loop MJFS when $\eta(t) = l, l \in \mathbb{T}$ can be written as

$$\dot{x}(t) = (A_{0i,l}(t) + B_{i,l}(t)K_{j,l}(t))x(t) \quad (8)$$

Let $x(t, x_0, \eta_0)$ denote the trajectory of the state $x(t)$ from the initial state $x(0) = x_0$ and the initial mode $\eta(0) = \eta_0$.

III. STABILITY ANALYSIS AND STABILIZATION

In this section, a stochastic stability condition for system (1) will be developed when the transition rates or probabilities are partially unknown, and a state feedback fuzzy controller will be designed for the MJFSs (8) such that the closed-loop systems are stochastically stable. First, we provide the following previous stability results for MJFSs when the transition rates or probabilities are partially unknown:

Lemma 1: System (1) is stochastically stable if and only if there exist matrices $Q_l = Q_l^T \succ 0$, $Q_s = Q_s^T \succ 0$, $l, s \in \mathbb{T}$ such that

$$A_{0i,l}^T(t)Q_l + Q_l A_{0i,l}(t) + \sum_{s=1}^N \lambda_{ls} Q_s < 0 \quad (9)$$

Let us first give the stability result for the unforced system ($u(t) \equiv 0$). The following theorem presents a necessary and sufficient condition on the stochastic stability of the considered system with partially known transition rates.

Theorem 1: Consider the unforced MJFS (1) with $u(k) = 0$ and known transition probabilities. The corresponding system is stochastically stable if there exist matrices $P_{i,l} \succ 0$, $i \in \mathcal{I}_r$, $l, s \in \mathbb{T}$,

$$A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_{\mathbb{K}}^l - \sum_{s \in \mathbb{T}_{\mathbb{K}}^l} \lambda_{ls} P_{i,s} < 0 \quad (10)$$

$$\forall s \in \mathbb{T}_{\mathbb{UK}}^l, \text{ if } l \in \mathbb{T}_{\mathbb{K}}^l$$

$$A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_{\mathbb{K}}^l + \lambda_d^l P_{i,l} \\ - \lambda_d^s P_{i,s} - \sum_{s \in \mathbb{T}_{\mathbb{K}}^l} \lambda_{ls} P_{i,s} < 0 \quad (11)$$

$$\forall s \in \mathbb{T}_{\mathbb{UK}}^l, \text{ if } l \in \mathbb{T}_{\mathbb{UK}}^l$$

where λ_d^l is a given lower bound for the unknown diagonal element.

Proof: We consider the proof into two cases, that is $l \in \mathbb{T}_{\mathbb{K}}^l$ and $l \in \mathbb{T}_{\mathbb{UK}}^l$.

1) Case I: $l \in \mathbb{T}_{\mathbb{K}}^l$ It should be noted that one has $\sum_{s \in \mathbb{T}_{\mathbb{K}}^l} \lambda_{ls} \leq 0$. Since $\sum_{s \in \mathbb{T}_{\mathbb{K}}^l} \lambda_{ls} = 0$ means the elements in the i^{th} row of the transition rates are known, we only need to consider $\sum_{s \in \mathbb{T}_{\mathbb{UK}}^l} \lambda_{ls} < 0$ here. Now we rewrite the left-hand side of (9) as

$$\begin{aligned}
\Theta_l &= A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l \\
&= A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l + \sum_{s \in \mathbb{T}_{\text{UK}}^l} \hat{\lambda}_{ls} P_{i,s} \\
&= A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l - \lambda_K^l \sum_{s \in \mathbb{T}_{\text{UK}}^l} \frac{\hat{\lambda}_{ls}}{-\lambda_K^l} P_{i,s}
\end{aligned}$$

where $\mathcal{P}_K^l = \sum_{s=1}^N \lambda_{ls} P_s$ and the elements $\hat{\lambda}_{ls}$, $s \in \mathbb{T}_{\text{UK}}^l$ are unknown. Since we have $0 \leq \frac{\hat{\lambda}_{ls}}{-\lambda_K^l} \leq 1$ and $\sum_{s \in \mathbb{T}_{\text{UK}}^l} \frac{\hat{\lambda}_{ls}}{-\lambda_K^l} = 1$, we know that

$$\Theta_l = \sum_{s \in \mathbb{T}_{\text{UK}}^l} \frac{\hat{\lambda}_{ls}}{-\lambda_K^l} [A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l - \lambda_K^l P_{i,s}]$$

Therefore, for $0 \leq \hat{\lambda}_{ls} \leq -\lambda_K^l$, $\Theta_l < 0$ is equivalent to $A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l - \lambda_K^l P_{i,s} < 0$, $\forall s \in \mathbb{T}_{\text{UK}}^l$, which implies that, in the presence of unknown elements $\hat{\lambda}_{ls}$, the system stability is ensured if and only if (10) holds.

2) CaseII : $\hat{\lambda}_{ii}$ is unknown, $\hat{\lambda}_{ii}^l \geq 0$ and $\hat{\lambda}_{ll} \leq -\lambda_K^l$. Also, we consider $\hat{\lambda}_{ll} < -\lambda_K^l$ since if $\hat{\lambda}_{ll} = -\lambda_K^l$, then the i_{th} row of the transition rates matrix is completely known. Now, the left-hand side of the stability condition in (9) can be rewritten as

$$\begin{aligned}
\Theta_l &= A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l \\
&= A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l + \hat{\lambda}_{ii} P_{i,l} + \sum_{s \in \mathbb{T}_{\text{UK}}^l, s \neq l} \hat{\lambda}_{ls} P_{i,s} \\
&= A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l + \hat{\lambda}_{ii} P_{i,l} \\
&\quad + \left(-\hat{\lambda}_{ll} - \lambda_K^l \right) \sum_{s \in \mathbb{T}_{\text{UK}}^l, s \neq l} \frac{\hat{\lambda}_{ls}}{-\hat{\lambda}_{ll} - \lambda_K^l} P_{i,s}
\end{aligned}$$

Likewise, since we have $0 \leq \hat{\lambda}_{ls}/(-\hat{\lambda}_{ll} - \lambda_K^l) \leq 1$ and $\sum_{s \in \mathbb{T}_{\text{UK}}^l, s \neq l} \frac{\hat{\lambda}_{ls}}{(-\hat{\lambda}_{ll} - \lambda_K^l)} = 1$, we know that

$$\begin{aligned}
\Theta_l &= \sum_{s \in \mathbb{T}_{\text{UK}}^l, s \neq l} \frac{\hat{\lambda}_{ls}}{-\hat{\lambda}_{ll} - \lambda_K^l} [A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l \\
&\quad + \hat{\lambda}_{ii} P_{i,l} - \hat{\lambda}_{ii} P_{i,s} - \lambda_K^l P_{i,s}]
\end{aligned}$$

which means that $\Theta_l \leq 0$ is equivalent to $\forall s \in \mathbb{T}_{\text{UK}}^l$, $s \neq l$

$$A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l + \hat{\lambda}_{ii} P_{i,l} - \hat{\lambda}_{ii} P_{i,s} - \lambda_K^l P_{i,s} \quad (12)$$

As $\hat{\lambda}_{ll}$ is lower bounded by λ_d^l , we have

$$\lambda_d^l \leq \hat{\lambda}_{ll} \leq -\lambda_K^l$$

which implies that $\hat{\lambda}_{ll}$ may take any value between $[\lambda_d^l, -\lambda_K^l + \epsilon]$ for some $\epsilon < 0$ arbitrarily small. Then $\hat{\lambda}_{ll}$ can be further written as a convex combination

$$\hat{\lambda}_{ll} = -\alpha \lambda_K^l + \alpha \epsilon + (1 - \alpha) \lambda_d^l$$

where α takes values arbitrarily in $[0, 1]$. Thus (12) holds if and only if $\forall s \in \mathbb{T}_{\text{UK}}^l$, $s \neq l$

$$A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l - \lambda_K^l P_{i,l} + \lambda_K^l P_{i,s} - \lambda_K^l P_{i,s}$$

$$+ \epsilon (P_{i,l} - P_{i,s}) \leq 0 \quad (13)$$

and

$$A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l + \lambda_d^l P_{i,l} - \lambda_d^l P_{i,s} - \lambda_K^l P_{i,s} < 0 \quad (14)$$

simultaneously hold. Since ϵ is arbitrarily small, (13) holds if and only if

$$A_{0i,l}^T P_{i,l} + P_{i,l} A_{0i,l} + \mathcal{P}_K^l - \lambda_K^l P_{i,l} < 0 \quad (15)$$

which is the case in (14) when $s = l$, $s \in \mathbb{T}_{\text{UK}}^l$. Hence (12) is equivalent to (11). Therefore, in the presence of unknown elements in the transition rates matrix, one can readily conclude that the system is stable if and only if (10) and (11) hold for $l \in \mathbb{T}_{\text{UK}}^l$ and $l \in \mathbb{T}_{\text{UK}}^l$, respectively. ■

Now let us consider the stabilizing controller design in the presence of unknown elements in the transition rates matrix. The following theorem presents a sufficient criterion for the existence of a mode-dependent stabilizing controller of the form in (7).

Theorem 2: Consider the system (1) with partially known transition rates. If there exist matrices $X_{i,l} > 0$ and $Y_{i,l}$, $i, j \in \mathcal{I}$, $l \in \mathbb{T}$, Z and a positive scalar μ such that

$$\Upsilon_{ii,ls} < 0, \quad i \in \mathcal{I}, \forall s \in \mathbb{T}_{\text{UK}}^l, \text{ if } l \in \mathbb{T}_{\text{UK}}^l \quad (16)$$

$$\frac{1}{r-1} \Upsilon_{ij,ls} + \frac{1}{2} (\Upsilon_{ij,ls} + \Upsilon_{ji,ls}) < 0 \quad (17)$$

$$i \neq j \in \mathcal{I}, \forall s \in \mathbb{T}_{\text{UK}}^l, \text{ if } l \in \mathbb{T}_{\text{UK}}^l$$

$$\Xi_{ii,ls} < 0 \quad i \in \mathcal{I}, \forall s \in \mathbb{T}_{\text{UK}}^l, \text{ if } l \in \mathbb{T}_{\text{UK}}^l \quad (18)$$

$$\frac{1}{r-1} \Xi_{ij,ls} + \frac{1}{2} (\Xi_{ij,ls} + \Xi_{ji,ls}) < 0 \quad (19)$$

$$i \neq j \in \mathcal{I}, \forall s \in \mathbb{T}_{\text{UK}}^l, \text{ if } l \in \mathbb{T}_{\text{UK}}^l$$

where, $\Upsilon_{ij,ls}, \Xi_{ij,ls}$ are shown at the top of the next page,

$$\mathcal{X}_{\text{UK}}^l = \text{diag} [X_{\text{UK}_1}, \dots, X_{\text{UK}_{m_l}}] \quad (20)$$

$$\mathcal{T}_{\text{UK}}^l = [\sqrt{-\lambda_{\text{UK}_1}} X_{i,l}, \dots, \sqrt{-\lambda_{\text{UK}_{m_l}}} X_{i,l}] \quad (21)$$

and $\forall e \in \{1, 2, \dots, m_l\}$, K_e is described in (7), $K_e \neq l$, then there exists a mode-dependent stabilizing controller of the form in (7) such that the closed-loop system is stochastically stable. Moreover, if the LMIs in (16)- (19) have solutions, a fuzzy controller gain is give by

$$K_j = Y_{j,l} Z^{-1} \quad (22)$$

Proof: Consider the system (1) with the control input (7) and replace $A_{0i,l}$ by $A_{0i,l} + B_{i,l} K_{j,l}$ in (10) and (11), respectively. Then if $l \in \mathbb{T}_{\text{UK}}^l$, performing a congruence transformation to (10) by $P_{i,l}^{-1}$, we can obtain

$$\begin{aligned}
&(A_{0i,l} + B_{i,l} K_{j,l}) P_{i,l}^{-1} + P_{i,l}^{-1} (A_{0i,l} + B_{i,l} K_{j,l})^T \\
&\quad + P_{i,l}^{-T} \mathcal{P}_{\text{UK}}^l P_{i,l}^{-1} - P_{i,l}^{-T} \left(\sum_{s \in \mathbb{T}_{\text{UK}}^l} \lambda_{ls} P_{i,s} \right) P_{i,l}^{-1} \quad (25)
\end{aligned}$$

Setting $X_{i,l} = P_{i,l}^{-1}$, $Y_{i,l} = K_{j,l} Z$ and considering (21). Applying the lemma 3, one can obtain that (25), then the l th

$$\Upsilon_{ij,ls} = \begin{bmatrix} -Z - Z^T & Z^T A_{0i,l}^T + Y_{j,l}^T B_{i,l}^T + X_{i,l} & Z^T & 0 & 0 \\ * & -\mu^{-1} X_{i,l} + \pi_{ll} X_{i,l} & 0 & \mathcal{T}_{\mathbb{K}}^l & \sqrt{-\lambda_{\mathbb{K}}^l} X_{i,l} \\ * & * & -\mu X_{i,l} & 0 & 0 \\ * & * & * & -\mathcal{X}_{\mathbb{K}}^l & 0 \\ * & * & * & * & -X_{i,s} \end{bmatrix} \quad (23)$$

$$\Xi_{ij,ls} = \begin{bmatrix} -Z - Z^T & Z^T A_{0i,l}^T + Y_{j,l}^T B_{i,l}^T + X_{i,l} & Z^T & 0 & 0 \\ * & -\mu^{-1} X_{i,l} + \pi_d X_{i,l} & 0 & \mathcal{T}_{\mathbb{K}}^l & \sqrt{-\lambda_d^l - \lambda_{\mathbb{K}}^l} X_{i,l} \\ * & * & -\mu X_{i,l} & 0 & 0 \\ * & * & * & -\mathcal{X}_{\mathbb{K}}^l & 0 \\ * & * & * & * & -X_{i,s} \end{bmatrix} \quad (24)$$

inequality of (10) holds for some matrices $X_{i,l} = X_{i,l}^T > 0$ if and only if there exist a scalar $\mu > 0$ and a matrix Z such that (26), at the top of page.

By Schur complement, we can obtain that (26) is equivalent to (27). In a similar way, if $s \in \mathbb{T}_{\mathbb{UK}}^l$, (18) and (19) can be worked out from (11). (10)-(11) will be satisfied in Theorem 1 such that the closed-loop system (8) is stochastically stable. Meanwhile, due to $Y_{i,l} = K_{j,l}Z$, the desire controller gain is given by (22). ■

IV. SIMULATIONS

In this section, we present an numerical example to demonstrate the effectiveness of the proposed results. Consider a single-link robot arm in, in which the dynamic equation is given by

$$\ddot{\theta}(t) = -\frac{MgL}{J} \sin(\theta(t)) - \frac{D(t)}{J} \dot{\theta}(t) + \frac{1}{J} u(t) \quad (28)$$

where $\theta(t)$ is the angle position of the arm, and $u(t)$ is the control input. M is the mass of the payload. J is the moment of inertia, g is the acceleration of gravity, L is the length of the arm, and $d(t)$ is the coefficient of viscous friction. The values of parameters g and L are given by $g = 9.81$ and $L = 0.5$. It is assumed that the parameter $D(t) = D = 2$ is time invariant, and the parameters M and J have four different modes as shown in Table 1. Let $x_1(t) = \theta(t)$ and $x_2(t) = \dot{\theta}(t)$. Under the condition $-179.4270 < \theta(t) < 179.4270$, the nonlinear term $\sin(\theta(t))$ can be exactly represented as

$$\sin(\theta(t)) = h_1(x_1(t)) \cdot x_1(t) + h_2(x_1(t)) \cdot \beta \cdot x_1(t) \quad (29)$$

with $\beta = 10^{-2}/\pi$, where $h_1(x_1(t))$, $h_2(x_1(t)) \in [0, 1]$, and $h_1(x_1(t)) + h_2(x_1(t)) = 1$. By solving the equations, the membership functions $h_1(x_1(t))$ and $h_2(x_1(t))$ are obtained as follows:

$$h_1(x_1(t)) = \begin{cases} \frac{\sin(x_1(t)) - \beta \cdot x_1(t)}{x_1(t) \cdot (1 - \beta)}, & x_1(t) \neq 0 \\ 1, & x_1(t) = 0. \end{cases} \quad (30)$$

$$h_2(x_1(t)) = \begin{cases} \frac{x_1(t) - \sin(x_1(t))}{x_1(t) \cdot (1 - \beta)}, & x_1(t) \neq 0 \\ 0, & x_1(t) = 0. \end{cases} \quad (31)$$

It can be seen from the above equations that $h_1(x_1(t)) = 1$ and $h_2(x_1(t)) = 0$, when $x_1(t)$ is about 0 rad, and $h_1(x_1(t)) = 0$

and $h_2(x_1(t)) = 1$, when $x_1(t)$ is about π rad or $-\pi$ rad. Then the nonlinear Markovian system (28) can be represented by the following T-S fuzzy model:

Plant Rule 1 :

IF $x_1(t)$ is "about 0 rad," THEN

$$\dot{x}(t) = A_{1,l}x(t) + B_{1,l}u(t)$$

Plant Rule 2 :

IF $x_1(t)$ is "about π rad or $-\pi$ rad," THEN

$$\dot{x}(t) = A_{2,l}x(t) + B_{2,l}u(t)$$

where

$$\begin{aligned} x(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, A_{1,1} = \begin{bmatrix} 0 & 1 \\ -gl & -D \end{bmatrix}, \\ A_{1,2} &= \begin{bmatrix} 0 & 1 \\ -gL & -0.8D \end{bmatrix}, A_{1,3} = \begin{bmatrix} 0 & 1 \\ -gL & -0.5D \end{bmatrix}, \\ A_{1,4} &= \begin{bmatrix} 0 & 1 \\ -gL & -0.4D \end{bmatrix}, A_{2,1} = \begin{bmatrix} 0 & 1 \\ -\beta gL & -D \end{bmatrix}, \\ A_{2,2} &= \begin{bmatrix} 0 & 1 \\ -\beta gL & -0.8D \end{bmatrix}, A_{2,3} = \begin{bmatrix} 0 & 1 \\ -\beta gL & -0.5D \end{bmatrix}, \\ A_{2,4} &= \begin{bmatrix} 0 & 1 \\ -\beta gL & -0.4D \end{bmatrix}, \\ B_{1,1} &= B_{2,1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_{1,2} = B_{2,2} = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}, \\ B_{1,3} &= B_{2,3} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, B_{1,4} = B_{2,4} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix} \end{aligned}$$

TABLE I
MODES OF THE PARAMETERS M , AND J

Mode l	M	J
1	1	1
2	1.25	1.25
3	2	2
4	2.5	2.5

The two cases for the transition rates matrix are considered in Table 2. The proposed theorem 2 is to design a mode-independent fuzzy controller such that the closed-loop system is stochastically stable with partially unknown transition

$$\begin{bmatrix} -Z - Z^T & Z^T (A_{0i,l} + B_{i,l} K_{j,l})^T + X_{i,l} & Z^T \\ * & -\mu^{-1} X_{i,l} + X_{i,l}^T P_{\mathbb{K}}^l X_{i,l} - X_{i,l}^T \left(\sum_{s \in \mathbb{T}_{\mathbb{K}}^l} \lambda_{ls} P_{i,s} \right) X_{i,l} & 0 \\ * & * & -\mu X_{i,l} \end{bmatrix} < 0 \quad (26)$$

$$\Upsilon_{ij,ls} = \begin{bmatrix} -Z - Z^T & Z^T A_{0i,l}^T + Y_{j,l}^T B_{i,l}^T + X_{i,l} & Z^T & 0 & 0 \\ * & -\mu^{-1} X_{i,l} + \pi_{ll} X_{i,l} & 0 & \mathcal{T}_{\mathbb{K}}^l & \sqrt{-\lambda_{\mathbb{K}}^l} X_{i,l} \\ * & * & -\mu X_{i,l} & 0 & 0 \\ * & * & * & -\mathcal{X}_{\mathbb{K}}^l & 0 \\ * & * & * & * & -X_{i,s} \end{bmatrix} < 0 \quad (27)$$

probabilities. With the choice of $\mu = 0.2$, we can obtain the following controller gain matrices by solving (16)-(19) in Theorem 2:

Case I

$$\begin{aligned} K_{1,1} &= \begin{bmatrix} -4.7256 \\ -6.2890 \end{bmatrix}^T, K_{1,2} = \begin{bmatrix} -4.8434 \\ -6.1849 \end{bmatrix}^T \\ K_{2,1} &= \begin{bmatrix} -7.1346 \\ -93.1655 \end{bmatrix}^T, K_{2,2} = \begin{bmatrix} -7.1646 \\ -8.8974 \end{bmatrix}^T \\ K_{3,1} &= \begin{bmatrix} -5.4878 \\ -10.0789 \end{bmatrix}^T, K_{3,2} = \begin{bmatrix} -5.4967 \\ -10.1987 \end{bmatrix}^T \\ K_{4,1} &= \begin{bmatrix} -7.1648 \\ -13.1846 \end{bmatrix}^T, K_{4,2} = \begin{bmatrix} -7.1649 \\ -14.7868 \end{bmatrix}^T \end{aligned}$$

Case II

$$\begin{aligned} K_{1,1} &= \begin{bmatrix} -3.1648 \\ -5.1464 \end{bmatrix}^T, K_{1,2} = \begin{bmatrix} -3.1684 \\ -6.1879 \end{bmatrix}^T \\ K_{2,1} &= \begin{bmatrix} -0.7978 \\ -2.4798 \end{bmatrix}^T, K_{2,2} = \begin{bmatrix} -0.4798 \\ -2.4987 \end{bmatrix}^T \\ K_{3,1} &= \begin{bmatrix} -14.1654 \\ -19.7934 \end{bmatrix}^T, K_{3,2} = \begin{bmatrix} -14.4684 \\ -21.4987 \end{bmatrix}^T \\ K_{4,1} &= \begin{bmatrix} -6.7913 \\ -13.2890 \end{bmatrix}^T, K_{4,2} = \begin{bmatrix} -5.8434 \\ -13.1201 \end{bmatrix}^T \end{aligned}$$

TABLE II
DIFFERENT TRANSITION PROBABILITIES MATRICES

Case I	1	2	3	4
1	-0.5	?	0.3	?
2	?	-0.6	?	0.3
3	0.2	0.3	?	?
4	?	?	0.3	?
Case II	1	2	3	4
1	-0.7	?	0.1	0.1
2	?	-0.4	0.1	?
3	0.5	0.3	?	?
4	?	?	0.4	?

Figures 1-2 show the result of the changing between modes during the simulation with the initial mode at mode 1. Applying the fuzzy controller (7) with the above gain matrices, the state response and control inputs of the closed-loop system are shown in figures under given initial condition

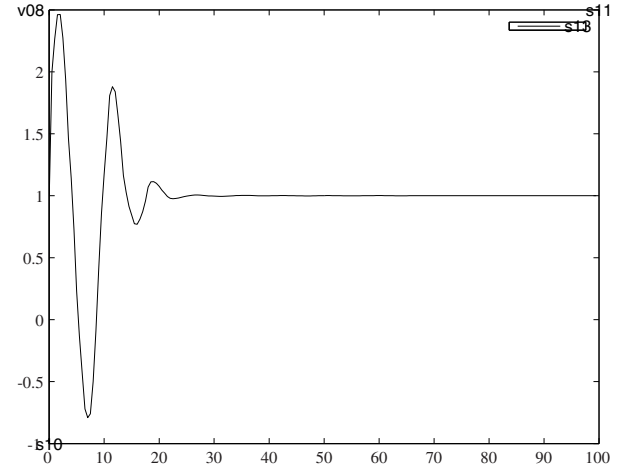


Fig. 1. The trajectories of states x_1

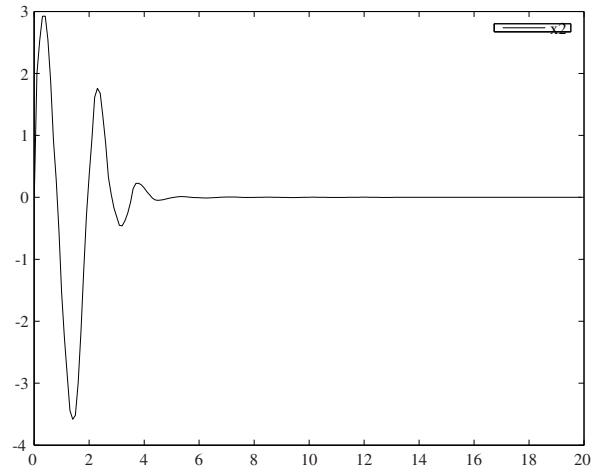


Fig. 2. The trajectories of states x_2

$x_0 = [0.5\pi, -2]^T$. It is seen from the curves in Figures 1-2 that, despite the partially unknown transition rates, the designed controllers stabilize the system (28) effectively.

V. CONCLUSION

In this paper, we have revisited the analysis and synthesis problems of Markov jump nonlinear system with incomplete transition descriptions via fuzzy control. Necessary and sufficient criteria are obtained for MJFSs by fully exploiting the properties of the transition rates matrix and the transition probabilities matrix. The conservatism of the approach developed previously, which only leads to sufficient conditions for the system, is reduced by the newly developed approach. The proposed stabilization conditions with a mode-dependent fuzzy controller are derived for Markovian jump fuzzy systems in terms of LMIs, which can be solved readily by using existing LMI optimization techniques. Finally, numerical examples have verified the theoretical results given in the paper.

ACKNOWLEDGMENT

REFERENCES

- [1] H. Kopka and P. W. Daly, *A Guide to L^AT_EX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.
- [2] Y. Ji and H. J. Chizeck, "Controllability, stabilizability, and continuous-time Markovian jump linear quadratic control," *IEEE Trans. Automat. Control*, vol. 35, no. 7, pp. 777-788, 1990.
- [3] J. Xiong, J. Lam, H. Gao, and D. W. Ho, "On robust stabilization of Markovian jump systems with uncertain switching probabilities," *Automatica*, vol. 41, no. 5, pp. 897-903, 2005.
- [4] O. L. V. Costa, M. D. Fragoso, and R. P. Marques, "Discrete-time Markovian jump linear systems," *London: Springer-Verlag*, 2005.
- [5] J. L. Xiong, J. Lam, H. J. Gao, and W. C. Daniel, "On robust stabilization of Markovian jump systems with uncertain switching probabilities," *Automatica*, vol. 41, no. 5, pp. 897-903, 2005.
- [6] L. Zhang, Y. Shi, T. Chen, and B. Huang, "A new method for stabilization of networked control systems with random delays," *IEEE Trans. Autom. Control*, vol. 50, no. 8, pp. 1177-1181, 2005.
- [7] K. Tanaka and M. Sano, "A robust stabilization problem of fuzzy control systems and its application to backing up control of a truck-trailer," *IEEE Trans. Fuzzy Syst.*, vol. 2, no. 2, pp. 119-134, 1994.
- [8] Y. Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 2, pp. 200-211, 2000.
- [9] Y. Y. Cao and P. M. Frank, "Stability analysis and synthesis of nonlinear time-delay systems via linear Takagi-Sugeno fuzzy models," *Fuzzy Sets Syst.*, vol. 124, no. 2, pp. 213-229, 2001.
- [10] T. M. Guerra and L. Vermeiren, "LMI-based relaxed nonquadratic stabilization conditions for nonlinear systems in the Takagi-Sugeno form," *Automatica*, vol. 40, no. 5, pp. 823-829, 2004.
- [11] N. S. D. Arrifano and V. A. Oliveira, "State feedback fuzzy-model-based control for Markovian jump nonlinear systems," *Controle Automacao: Revista da Sociedade Brasileira de Automacao*, vol. 15, no. 3, pp. 279-290, 2004.
- [12] S. Natache, N. S. D. Arrifano and V. A. Oliveira, "Robust H_∞ fuzzy control approach for a class of Markovian jump nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 13, no. 6, pp. 738-754, 2006.
- [13] N. S. D. Arrifano and V. A. Oliveira, "State feedback fuzzy-model-based control for Markovian jump nonlinear systems," *Controle Automacao: Revista da Sociedade Brasileira de Automacao*, vol. 15, no. 6, pp. 279, 2004.
- [14] S. Nguang, W. Assawinchaichote, and P. Shi, "Robust H_∞ control design for fuzzy singularly perturbed systems with Markovian jumps: an LMI approach," *IET Control Theory and Applications*, vol. 1, no. 4, pp. 893-908, 2007.
- [15] H. N. Wu, and K. Y. Cai, "Robust fuzzy control for uncertain discrete-time nonlinear Markovian jump systems without mode observations," *Information sciences.*, vol. 177, no. 6, pp. 1509-1522, 2007.
- [16] H. J. Kushner, "Stochastic Stability and Control", *New York: Academic*, 1967.
- [17] L. X. Zhang and E. K. Boukas, "Stability and stabilization of Markovian jump linear systems with partly unknown transition probability," *Automatica*, vol. 45, no. 2, pp. 463-468, 2009.
- [18] L. X. Zhang and E. K. Boukas, " H_∞ control for discrete-time Markovian jump linear systems with partly unknown transition probabilities," *Int. J. Robust Nonlin. Control*, vol. 19, no. 5, pp. 868-883, 2009.
- [19] L. X. Zhang and E. K. Boukas, " H_∞ control of a class of extended Markov jump linear systems," *IET Control Theory Appl.*, vol. 3, no. 7, pp. 834-842, 2009.
- [20] L. X. Zhang, E. K. Boukas, and J. Lam, "Analysis and synthesis of Markov jump linear systems with time-varying delays and partially known transition probabilities," *IEEE Trans. Autom. Control*, vol. 53, no. 10, pp. 2458-2464, 2008.
- [21] L. Sheng and M. Gao, "Stabilization for Markovian jump nonlinear systems with partly unknown transition probabilities via fuzzy control," *Fuzzy Sets and Systems*.