

Fuzzy Rule Interpolation Based on Interval Type-2 Gaussian Fuzzy Sets and Genetic Algorithms

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Abstract—In this paper, we present a new method for fuzzy rule interpolation with interval type-2 Gaussian fuzzy sets for sparse fuzzy rule-based systems based on genetic algorithms. The proposed fuzzy rule interpolation method deals with the interpolation of fuzzy rules based on the multiple fuzzy rules interpolation scheme. We also present a new learning method to learn optimal interval type-2 Gaussian fuzzy sets for sparse fuzzy rule-based systems based on genetic algorithms. We apply the proposed fuzzy rule interpolation method and the proposed learning method to deal with the Mackey-Glass chaotic time series prediction problem. The experimental result shows that the proposed fuzzy rule interpolation method using the optimally learned interval type-2 Gaussian fuzzy sets obtained by the proposed learning method gets higher average accuracy rates than the existing methods to deal with the Mackey-Glass chaotic time series prediction problem.

Keywords—Fuzzy rule interpolation; sparse fuzzy rule-based systems; interval type-2 Gaussian fuzzy sets; type-1 Gaussian fuzzy sets; genetic algorithms

I. INTRODUCTION

In recent years, some methods [4], [6], [7], [17] have been presented to deal with fuzzy rule interpolation in sparse fuzzy rule-based systems based on interval type-2 fuzzy sets [18]. In [4], Chang and Chen presented a method for fuzzy rule interpolation based on the principle membership functions and the uncertainty grade functions of interval type-2 fuzzy sets. In [6], Chen and Chang presented a method for fuzzy rule interpolation based on the ratios of fuzziness of interval type-2 fuzzy sets. In [7], Chen and Chang presented a method for fuzzy rule interpolation based on the uncertainty ratios of interval type-2 fuzzy sets. In [17], Lee and Chen presented a method for fuzzy rule interpolation based on the ranking values of interval type-2 fuzzy sets.

In this paper, we present a new method for fuzzy rule interpolation with the learned interval type-2 Gaussian fuzzy sets for sparse fuzzy rule-based systems based on genetic algorithms. The proposed fuzzy rule interpolation method deals with the interpolation of fuzzy rules based on the multiple fuzzy rules interpolation scheme. We also present a new learning method to learn optimal interval type-2 Gaussian fuzzy sets for sparse fuzzy rule-based systems based on genetic algorithms. We apply the proposed learning method and the proposed fuzzy rule interpolation method to deal with the Mackey-Glass chaotic time series prediction problem. The experimental result shows that the proposed fuzzy rule interpolation method using the optimally learned interval type-

2 Gaussian fuzzy sets obtained by the proposed learning method gets higher average accuracy rates than the existing methods [2], [8], [13] to deal with the Mackey-Glass chaotic time series prediction problem.

II. PRELIMINARIES

In this section, we briefly introduce the concepts of interval type-2 Gaussian fuzzy sets [16], [18]. A type-1 Gaussian fuzzy set $\tilde{A}$ represented by the membership function $e^{-\frac{(x-m)^2}{2\sigma^2}}$ can be denoted by $\tilde{A} = \langle m, \sigma \rangle$, as shown in Fig. 1, where m and σ denote the mean and the standard deviation of the type-1 Gaussian fuzzy set $\tilde{A}$, respectively.

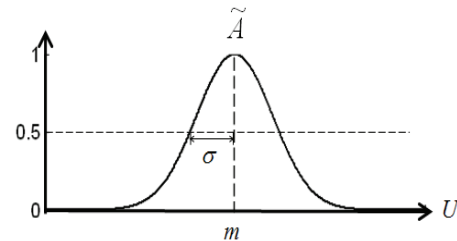


Fig. 1. A type-1 Gaussian fuzzy set $\tilde{A}$.

Definition 2.1 [18]: An interval type-2 Gaussian fuzzy set $\tilde{\tilde{A}}$ can be represented by $\langle \tilde{\tilde{A}}^U, \tilde{\tilde{A}}^L \rangle$, where $\tilde{\tilde{A}}^U$ and $\tilde{\tilde{A}}^L$ denote the upper membership function and the lower membership function of $\tilde{\tilde{A}}$, respectively, and both $\tilde{\tilde{A}}^U$ and $\tilde{\tilde{A}}^L$ are type-1 Gaussian fuzzy sets.

Let us consider the adjacent type-1 Gaussian fuzzy sets $\tilde{A}_1 = \langle m_1, \sigma_1 \rangle$ and $\tilde{A}_2 = \langle m_2, \sigma_2 \rangle$ shown in Fig. 2. Based on [16], we can get an interval type-2 Gaussian fuzzy set $\tilde{\tilde{A}} = \langle \tilde{\tilde{A}}^U, \tilde{\tilde{A}}^L \rangle$ obtained from $\tilde{A}_1$ and $\tilde{A}_2$, as shown in Fig. 3, where

$$\tilde{\tilde{A}} = \langle [m_1, m_2], \sigma_1; \sigma_2 \rangle, \quad (1)$$

$[m_1, m_2]$ denotes the interval-valued mean of the interval type-2 Gaussian fuzzy set $\tilde{\tilde{A}}$, σ_1 and σ_2 are called the left standard deviation and the right standard deviation of the interval type-2 Gaussian fuzzy set $\tilde{\tilde{A}}$, respectively, $m_1 \leq m_2$, $\sigma_1 > 0$ and $\sigma_2 > 0$.

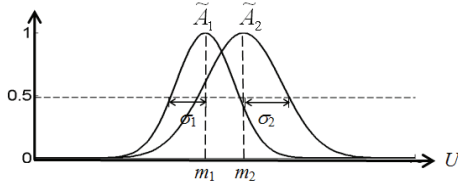


Fig. 2. Two adjacent type-1 Gaussian fuzzy sets $\tilde{A}_1 = \langle m_1, \sigma_1 \rangle$ and $\tilde{A}_2 = \langle m_2, \sigma_2 \rangle$.

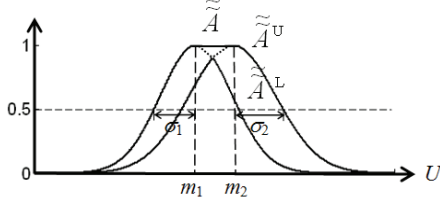


Fig. 3. An interval type-2 Gaussian fuzzy set $\tilde{\tilde{A}} = \langle [m_1, m_2], \sigma_1; \sigma_2 \rangle$.

Let us consider an interval type-2 Gaussian fuzzy set $\tilde{\tilde{A}}$ having an interval-valued mean $[m_1, m_2]$, a left standard deviation σ_1 and a right standard deviation σ_2 , as shown in Fig. 3, where $\tilde{\tilde{A}} = \langle [m_1, m_2], \sigma_1; \sigma_2 \rangle$, $m_1 < m_2$, $\sigma_1 > 0$ and $\sigma_2 > 0$. If $m_1 = m_2$ and $\sigma_1 = \sigma_2$, then the interval type-2 Gaussian fuzzy set $\tilde{\tilde{A}}$ becomes the type-1 Gaussian fuzzy set $\tilde{A}$, as shown in Fig. 1.

Definition 2.2: The representative value $Rep(\tilde{\tilde{A}})$ of the Gaussian interval type-2 fuzzy set $\tilde{\tilde{A}} = \langle [m_1, m_2], \sigma_1; \sigma_2 \rangle$ is calculated as follows:

$$Rep(\tilde{\tilde{A}}) = \frac{m_1 \times \sigma_1 + m_2 \times \sigma_2}{\sigma_1 + \sigma_2}, \quad (2)$$

where $m_1 \leq Rep(\tilde{\tilde{A}}) \leq m_2$, $\sigma_1 > 0$ and $\sigma_2 > 0$

From Eq. (2), we can see that the larger the value of the standard deviation σ_i , the closer the representative value $Rep(\tilde{\tilde{A}})$ to m_i , where $1 \leq i \leq 2$.

III. A NEW METHOD FOR FUZZY RULE INTERPOLATION BASED ON INTERVAL TYPE-2 GAUSSIAN FUZZY SETS

In this section, we present a new method for fuzzy rule interpolation based on interval type-2 Gaussian fuzzy sets. Let us consider the following multiple fuzzy rules interpolation scheme using interval type-2 Gaussian fuzzy sets:

Rule 1: If X_1 is $\tilde{\tilde{A}}_{1,1}$ and X_2 is $\tilde{\tilde{A}}_{1,2}$ and ... and X_m is $\tilde{\tilde{A}}_{1,m}$ Then Y is $\tilde{\tilde{B}}_1$

Rule 2: If X_1 is $\tilde{\tilde{A}}_{2,1}$ and X_2 is $\tilde{\tilde{A}}_{2,2}$ and ... and X_m is $\tilde{\tilde{A}}_{2,m}$ Then Y is $\tilde{\tilde{B}}_2$

...

Rule p: If X_1 is $\tilde{\tilde{A}}_{p,1}$ and X_2 is $\tilde{\tilde{A}}_{p,2}$ and ... and X_m is $\tilde{\tilde{A}}_{p,m}$ Then Y is $\tilde{\tilde{B}}_p$

Observation: X_1 is $\tilde{\tilde{A}}_1^*$ and X_2 is $\tilde{\tilde{A}}_2^*$ and ... and X_m is $\tilde{\tilde{A}}_m^*$

Conclusion: Y is $\tilde{\tilde{B}}^*$

the j th antecedent variable X_j , $\tilde{\tilde{B}}^* = \langle [m_{\tilde{\tilde{B}}^*,1}, m_{\tilde{\tilde{B}}^*,2}], \sigma_{\tilde{\tilde{B}}^*,1}; \sigma_{\tilde{\tilde{B}}^*,2} \rangle$

$\sigma_{\tilde{\tilde{B}}^*,2} >$ is the interval type-2 Gaussian fuzzy set of the fuzzy

interpolated result, m denotes the number of antecedent variables in the fuzzy rules, p denotes the number of fuzzy rules, $1 \leq j \leq m$ and $1 \leq k \leq p$. In this paper, we use the neighboring fuzzy rules [5] with respect to the observation as the closest fuzzy rules with respect to the observation for fuzzy rule interpolation, where each antecedent fuzzy set $\tilde{\tilde{A}}_{k,j}$ of the closest fuzzy rule *Rule k* is either the left nearest fuzzy set or the right nearest fuzzy set with respect to the observation fuzzy set $\tilde{\tilde{A}}_j^*$, where $1 \leq j \leq m$ and $1 \leq k \leq p$.

The proposed method for fuzzy rule interpolation using interval type-2 Gaussian fuzzy sets is now presented as follows:

Step 1: Based on Eq. (2), calculate the representative values $Rep(\tilde{\tilde{A}}_{k,j})$ of each antecedent interval type-2 fuzzy set $\tilde{\tilde{A}}_{k,j}$ appearing in fuzzy rule *Rule k* and calculate the representative values $Rep(\tilde{\tilde{A}}_j^*)$ of each observation interval type-2 fuzzy set $\tilde{\tilde{A}}_j^*$, respectively, where

$$Rep(\tilde{\tilde{A}}_{k,j}) = \frac{m_{\tilde{\tilde{A}}_{k,j},1} \times \sigma_{\tilde{\tilde{A}}_{k,j},1} + m_{\tilde{\tilde{A}}_{k,j},2} \times \sigma_{\tilde{\tilde{A}}_{k,j},2}}{\sigma_{\tilde{\tilde{A}}_{k,j},1} + \sigma_{\tilde{\tilde{A}}_{k,j},2}}, \quad (3)$$

$$Rep(\tilde{\tilde{A}}_j^*) = \frac{m_{\tilde{\tilde{A}}_j^*,1} \times \sigma_{\tilde{\tilde{A}}_j^*,1} + m_{\tilde{\tilde{A}}_j^*,2} \times \sigma_{\tilde{\tilde{A}}_j^*,2}}{\sigma_{\tilde{\tilde{A}}_j^*,1} + \sigma_{\tilde{\tilde{A}}_j^*,2}}, \quad (4)$$

$1 \leq k \leq p$ and $1 \leq j \leq m$.

Step 2: Calculate the weight W_k of the closest fuzzy rule *Rule k* with respect to the observation fuzzy sets [3], where

$$w_k = \min_{1 \leq j \leq m} \left(1 - \frac{|Rep(\tilde{\tilde{A}}_j^*) - Rep(\tilde{\tilde{A}}_{k,j})|}{|Rep(\tilde{\tilde{A}}_{l,j}) - Rep(\tilde{\tilde{A}}_{r,j})|} \right), \quad (5)$$

$$W_k = \frac{w_k}{\sum_{q=1}^p w_q}, \quad (6)$$

$Rep(\tilde{\tilde{A}}_{l,j})$ and $Rep(\tilde{\tilde{A}}_{r,j})$ are the representative values of the left nearest antecedent fuzzy sets $\tilde{\tilde{A}}_{l,j}$ and the right nearest antecedent fuzzy set $\tilde{\tilde{A}}_{r,j}$ with respect to the observation

fuzzy set $\tilde{\tilde{A}}_j^*$, respectively, $0 \leq W_k \leq 1$, $\sum_{k=1}^p W_k = 1$ and $1 \leq k \leq$

p .

Step 3: Construct an intermediate interpolated fuzzy rule, shown as follows:

If X_1 is $\tilde{\tilde{A}}_1'$ and X_2 is $\tilde{\tilde{A}}_2'$ and ... and X_m is $\tilde{\tilde{A}}_m'$ Then Y is $\tilde{\tilde{B}}'$.

where $\tilde{A}'_j = \langle [m_{\tilde{A}'_j,1}, m_{\tilde{A}'_j,2}], \sigma_{\tilde{A}'_j,1}; \sigma_{\tilde{A}'_j,2} \rangle$ is an interval type-2 Gaussian fuzzy set appearing in the j th antecedent of the constructed intermediate interpolated fuzzy rule, $\tilde{B}' = \langle [m_{\tilde{B}',1}, m_{\tilde{B}',2}], \sigma_{\tilde{B}',1}; \sigma_{\tilde{B}',2} \rangle$ is an interval type-2 Gaussian fuzzy set appearing in the consequence of the constructed intermediate interpolated fuzzy rule and $1 \leq j \leq m$. The interval-valued mean $[m_{\tilde{A}'_j,1}, m_{\tilde{A}'_j,2}]$, the left standard deviation $\sigma_{\tilde{A}'_j,1}$ and the right standard deviation $\sigma_{\tilde{A}'_j,2}$ of the interval type-2 Gaussian fuzzy set $\tilde{A}'_j = \langle [m_{\tilde{A}'_j,1}, m_{\tilde{A}'_j,2}], \sigma_{\tilde{A}'_j,1}; \sigma_{\tilde{A}'_j,2} \rangle$ are calculated as follows:

$$m_{\tilde{A}'_j,1} = \sum_{k=1}^p (W_k \times m_{\tilde{A}_{k,j},1}), \quad (7)$$

$$m_{\tilde{A}'_j,2} = \sum_{k=1}^p (W_k \times m_{\tilde{A}_{k,j},2}), \quad (8)$$

$$\sigma_{\tilde{A}'_j,1} = \sum_{k=1}^p (W_k \times \sigma_{\tilde{A}_{k,j},1}), \quad (9)$$

$$\sigma_{\tilde{A}'_j,2} = \sum_{k=1}^p (W_k \times \sigma_{\tilde{A}_{k,j},2}), \quad (10)$$

where $0 \leq W_k \leq 1$, $\sum_{k=1}^p W_k = 1$, $1 \leq k \leq p$ and $1 \leq j \leq m$. The interval-valued mean $[m_{\tilde{B}',1}, m_{\tilde{B}',2}]$, the left standard deviation $\sigma_{\tilde{B}',1}$ and the right standard deviation $\sigma_{\tilde{B}',2}$ of the interval type-2 Gaussian fuzzy set $\tilde{B}' = \langle [m_{\tilde{B}',1}, m_{\tilde{B}',2}], \sigma_{\tilde{B}',1}; \sigma_{\tilde{B}',2} \rangle$ are calculated as follows:

$$m_{\tilde{B}',1} = \sum_{k=1}^p (W_k \times m_{\tilde{B}_{k,1}}), \quad (11)$$

$$m_{\tilde{B}',2} = \sum_{k=1}^p (W_k \times m_{\tilde{B}_{k,2}}), \quad (12)$$

$$\sigma_{\tilde{B}',1} = \sum_{k=1}^p (W_k \times \sigma_{\tilde{B}_{k,1}}), \quad (13)$$

$$\sigma_{\tilde{B}',2} = \sum_{k=1}^p (W_k \times \sigma_{\tilde{B}_{k,2}}), \quad (14)$$

where $0 \leq W_k \leq 1$, $\sum_{k=1}^p W_k = 1$ and $1 \leq k \leq p$.

Step 4: Get the interval-valued mean $[m_{\tilde{B}^*,1}, m_{\tilde{B}^*,2}]$, the left standard deviation $\sigma_{\tilde{B}^*,1}$ and the right standard deviation $\sigma_{\tilde{B}^*,2}$ of the fuzzy interpolated result $\tilde{B}^* = \langle [m_{\tilde{B}^*,1}, m_{\tilde{B}^*,2}], \sigma_{\tilde{B}^*,1}; \sigma_{\tilde{B}^*,2} \rangle$, respectively, where

$$m_{\tilde{B}^*,1} = m_{\tilde{B}',1}, \quad (15)$$

$$m_{\tilde{B}^*,2} = m_{\tilde{B}',2}, \quad (16)$$

$$\sigma_{\tilde{B}^*,1} = \sigma_{\tilde{B}',1} \times \frac{\sum_{j=1}^m \sigma_{\tilde{A}'_j,1}}{\sum_{j=1}^m \sigma_{\tilde{A}'_j,1}}, \quad (17)$$

$$\sigma_{\tilde{B}^*,2} = \sigma_{\tilde{B}',2} \times \frac{\sum_{j=1}^m \sigma_{\tilde{A}'_j,2}}{\sum_{j=1}^m \sigma_{\tilde{A}'_j,2}}. \quad (18)$$

Then, we can get the fuzzy interpolated result $\tilde{B}^*$, where $\tilde{B}^* = \langle [m_{\tilde{B}^*,1}, m_{\tilde{B}^*,2}], \sigma_{\tilde{B}^*,1}; \sigma_{\tilde{B}^*,2} \rangle$.

IV. A NEW METHOD TO LEARN THE OPTIMAL INTERVAL TYPE-2 GAUSSIAN FUZZY SETS FOR SPARSE FUZZY RULE-BASED SYSTEMS BASED ON GENETIC ALGORITHMS

In this section, we present a new learning method to learn optimal interval type-2 Gaussian fuzzy sets for sparse fuzzy rule-based systems based on genetic algorithms [11], [12], [15]. Given a sparse fuzzy rule base with type-1 Gaussian fuzzy sets constructed by the existing fuzzy rule base generation method [20], we can construct a new sparse fuzzy rule base with interval type-2 Gaussian fuzzy sets, where each interval type-2 Gaussian fuzzy set corresponds to a type-1 Gaussian fuzzy set in the original sparse fuzzy rule base. Assume that the new sparse fuzzy rule base has m antecedent variables $X_1, X_2, \dots, X_m$ and has a consequence variable Y , where each antecedent variable and the consequence variable has n possible interval type-2 Gaussian fuzzy sets in the universe of discourse, the k th possible interval type-2 Gaussian fuzzy set $\tilde{A}_{k,j}$ of the j th antecedent variable X_j is denoted by $\tilde{A}_{k,j} = \langle [m_{\tilde{A}_{k,j},1}, m_{\tilde{A}_{k,j},2}], \sigma_{\tilde{A}_{k,j},1}; \sigma_{\tilde{A}_{k,j},2} \rangle$, the k th possible interval type-2 Gaussian fuzzy set $\tilde{B}_k$ of the consequence variable Y is denoted by $\tilde{B}_k = \langle [m_{\tilde{B}_k,1}, m_{\tilde{B}_k,2}], \sigma_{\tilde{B}_k,1}; \sigma_{\tilde{B}_k,2} \rangle$, $1 \leq k \leq n$ and $1 \leq j \leq m$. Given a set of P training samples, where the input observations of each training sample S_i are represented by $(x_{i1}, x_{i2}, \dots, x_{im})$, the target output of the training sample S_i is represented by y_i , $1 \leq i \leq P$ and P denotes the number of training samples. Because

the shape of an interval type-2 Gaussian fuzzy set $\tilde{\tilde{A}} = \langle [m_1, m_2], \sigma_1; \sigma_2 \rangle$ varies with its left standard deviation σ_1 and its right standard deviation σ_2 , the proposed method encodes the left standard deviations and the right standard deviations of interval type-2 Gaussian fuzzy sets appearing in the rules into the genes of a chromosome, where each chromosome in a population has $2n \times (m + 1)$ genes $g_{1,1,1}, g_{1,1,2}, g_{2,1,1}, g_{2,1,2}, \dots, g_{k,j,1}, g_{k,j,2}, \dots, g_{n,m,1}, g_{n,m,2}, g_{1,m+1,1}, g_{1,m+1,2}, g_{2,m+1,1}, g_{2,m+1,2}, \dots, g_{k,m+1,1}, g_{k,m+1,2}, \dots, g_{n,m+1,1}$ and $g_{n,m+1,2}$, where the two successive genes $g_{k,j,1}$ and $g_{k,j,2}$ represent the values $\sigma_{\tilde{\tilde{A}}_{k,j,1}}$ and

$\sigma_{\tilde{\tilde{A}}_{k,j,2}}$ of the k th possible interval type-2 Gaussian fuzzy set $\tilde{\tilde{A}}_{k,j} = \langle [m_{\tilde{\tilde{A}}_{k,j,1}}, m_{\tilde{\tilde{A}}_{k,j,2}}], \sigma_{\tilde{\tilde{A}}_{k,j,1}}; \sigma_{\tilde{\tilde{A}}_{k,j,2}} \rangle$ of the antecedent

variable X_j , respectively, $0 \leq g_{k,j,1} \leq \frac{|m_{\tilde{\tilde{A}}_{k,j,2}} - m_{\tilde{\tilde{A}}_{k,j,1}}|}{2}$, $0 \leq g_{k,j,2}$

$\leq \frac{|m_{\tilde{\tilde{A}}_{k,j,2}} - m_{\tilde{\tilde{A}}_{k,j,1}}|}{2}$, the two successive genes $g_{k,m+1,1}$ and $g_{k,m+1,2}$ represents the values of $\sigma_{\tilde{\tilde{B}}_{k,1}}$ and $\sigma_{\tilde{\tilde{B}}_{k,2}}$ of the k th possible interval type-2 Gaussian fuzzy set $\tilde{\tilde{B}}_k = \langle [m_{\tilde{\tilde{B}}_{k,1}}, m_{\tilde{\tilde{B}}_{k,2}}], \sigma_{\tilde{\tilde{B}}_{k,1}}; \sigma_{\tilde{\tilde{B}}_{k,2}} \rangle$ of the consequence variable Y , respectively, $0 \leq g_{k,m+1,1} \leq \frac{|m_{\tilde{\tilde{B}}_{k,2}} - m_{\tilde{\tilde{B}}_{k,1}}|}{2}$, $0 \leq g_{k,m+1,2} \leq \frac{|m_{\tilde{\tilde{B}}_{k,2}} - m_{\tilde{\tilde{B}}_{k,1}}|}{2}$, $1 \leq k \leq n$ and $1 \leq j \leq m$. Fig. 4 shows the genes of a chromosome encoding the left standard deviations and the right standard deviations of the interval type-2 Gaussian fuzzy sets appearing in the fuzzy rules.

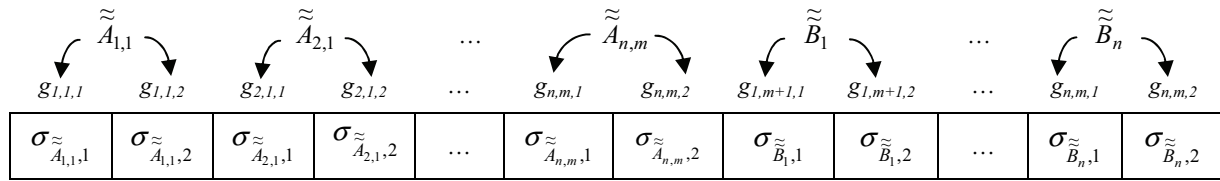


Fig. 4. A chromosome.

The proposed learning method to learn the optimal left standard deviations and the optimal right standard deviations of interval type-2 Gaussian fuzzy sets for sparse fuzzy rule-based systems is now presented as follows:

Step 1: Construct a new sparse fuzzy rule base with the interval type-2 Gaussian fuzzy sets, where each interval type-2

Gaussian fuzzy set $\tilde{\tilde{A}} = \langle [m_1, m_2], \sigma_1; \sigma_2 \rangle$ appearing in the new fuzzy rules is constructed from the corresponding type-1 Gaussian fuzzy set $\tilde{A} = \langle m, \sigma \rangle$ in the original fuzzy rule, $m_1 = m - \frac{\sigma}{t}$, $m_2 = m + \frac{\sigma}{t}$, t is a parameter determining the values m_1 and m_2 of $\tilde{\tilde{A}}$, $2 \leq t \leq \infty$, $\sigma_1 = \sigma$ and $\sigma_2 = \sigma$.

Step 2: Encode the left standard deviations and the optimal right standard deviations of the interval type-2 Gaussian fuzzy sets appearing in the fuzzy rules into the genes of the chromosomes. Randomly generate N chromosomes to form the initial population.

Step 3: Decode each chromosome of the population into the left standard deviations and the optimal right standard deviations of the interval type-2 Gaussian fuzzy sets appearing in the fuzzy rules. Apply the proposed fuzzy rule interpolation method to generate the interpolated consequence fuzzy set with respect to the input observations ($x_{i1}, x_{i2}, \dots, x_{im}$) of each training sample S_i and then calculate the representative value of the fuzzy interpolated result as the inferred output with respect to the input observations of the training sample S_i , where $1 \leq i \leq P$ and P denotes the number of training samples. Based on the target output and the inferred output of each training

sample, calculate the fitness value *Fitness* of each chromosome using the following fitness function:

$$RMSE = \sqrt{\frac{\sum_{i=1}^P (y_i - y_i^*)^2}{P}}, \quad (19)$$

$$Fitness = \frac{1}{RMSE}, \quad (20)$$

where *RMSE* denotes the root mean square error between the inferred outputs $y_1^*, y_2^*, \dots, y_P^*$ and the target outputs $y_1, y_2, \dots, y_P$ of the training samples $S_1, S_2, \dots$ and S_P , and P denotes the number of training samples. The larger the fitness value *Fitness* of a chromosome, the better the chromosome.

Step 4: Select chromosomes from the current population using the binary tournament selection method [12] to form a mating pool. The system repeatedly randomly selects two chromosomes from the current population and puts the chromosome having the larger fitness value into the mating pool and then puts the two selected chromosomes back into the current population. Repeatedly perform the above process until the number of chromosomes in the mating pool is equal to N , where N is the number of chromosomes in a population.

Step 5: Apply the multi-points crossover method [15] to the chromosomes in the mating pool. For each chromosome C_i in the mating pool, the system randomly generates a real value between 0 and 1. If the randomly generated real value is smaller than a predefined crossover rate γ , where $\gamma \in [0, 1]$, then the system randomly chooses another chromosome C_j

from the mating pool. Then, the system randomly chooses $\left\lfloor \frac{m \times n}{4} \right\rfloor$ crossover points to replace the values of the genes of the crossover points between the selected chromosomes C_i and C_j .

Step 6: Decode each chromosome of the mating pool into the left standard deviations and the optimal right standard deviations of the interval type-2 Gaussian fuzzy sets appearing in the fuzzy rules. Apply the proposed fuzzy rule interpolation method to generate the interpolated consequence fuzzy set with respect to the input observations $(x_{i1}, x_{i2}, \dots, x_{im})$ of each training sample S_i and then calculate the representative value of the fuzzy interpolated result as the inferred output with respect to the input observations of the training sample S_i , where $1 \leq i \leq P$ and P denotes the number of training samples. Based on the target outputs and the inferred outputs of training samples, calculate the fitness value *Fitness* of each chromosome based on the fitness function shown in Eq. (20).

Step 7: Perform the elitist replacement method [11] to form the population of the next generation. The system selects the chromosomes having the top N largest fitness values in the current population and the mating pool to form the population of the next generation. If the number of selected chromosomes from the mating pool is equal to zero, go to **Step 8**. Otherwise, go to **Step 9**.

Step 8: Reinitialize the population using the cataclysmic mutation method [11]. The system puts the best chromosome having the largest fitness value in the current population into the population of the next generation. Mutates the remaining $N - 1$ chromosomes of the current population to generate the $N - 1$ chromosomes in the population of the next generation.

Step 9: If the number of generations is larger than a predefined number of generations, then go to **Step 10**, else go to **Step 3**.

Step 10: Decode the best chromosome having the largest fitness value in the current population into the left standard deviations and the right standard deviations of the interval type-2 Gaussian fuzzy sets appearing in the fuzzy rules. Output the fuzzy rules represented by the optimally learned interval type-2 Gaussian fuzzy sets for the sparse fuzzy rule-based system.

V. EXPERIMENTAL RESULTS

In this section, we apply the proposed fuzzy rule interpolation method and the proposed learning method to deal with the Mackey-Glass chaotic time series prediction problem [10]. We also compare the experimental results of the proposed method with the results obtained by the existing methods [2], [8], [13]. The goal of the Mackey-Glass chaotic time series prediction problem [10] is to achieve a high prediction accuracy for forecasting the time series based on the following equation:

$$\frac{dx(t)}{dt} = \frac{0.2x(t-\tau)}{1+x^{10}(t-\tau)} - 0.1x(t). \quad (21)$$

In [10], Cowder extracted 1000 input-output data pairs using four past values for predicting the Mackey-Glass chaotic time series, shown as follows:

$$[x(t-18), x(t-12), x(t-6), x(t); x(t+6)], \quad (22)$$

where $x(t-18)$, $x(t-12)$, $x(t-6)$ and $x(t)$ are the four input attributes of the data pair at time $t-18$, $t-12$, $t-6$ and t , respectively, $x(t+6)$ is the output attribute of the data pair at time $t+6$, $\tau=17$, $x(0)=1.2$ and $0.42 \leq x(t) \leq 1.31$. In this paper, we also use the 1000 input-output data pairs (i.e., from $x(1)$ to $x(1000)$) as the data set of the Mackey-Glass chaotic time series prediction problem, where the first 500 input-output data pairs (i.e., from $x(1)$ to $x(500)$) are used as the training data set and the remaining 500 input-output data pairs (i.e., from $x(501)$ to $x(1000)$) are used as the testing data set. We apply the fuzzy rules generation method presented in [20] to construct a sparse fuzzy rule base consisting of 116 fuzzy rules using type-1 Gaussian fuzzy sets for the Mackey-Glass chaotic time series prediction problem based on the 500 training input-output data pairs, where each linguistic variable has seven possible type-1 Gaussian fuzzy sets $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4, \tilde{A}_5, \tilde{A}_6$ and $\tilde{A}_7$, as shown in Fig. 5, which are evenly distributed over the universe of discourse, where $\tilde{A}_1 = \langle 0.42, 0.07 \rangle$, $\tilde{A}_2 = \langle 0.57, 0.07 \rangle$, $\tilde{A}_3 = \langle 0.72, 0.07 \rangle$, $\tilde{A}_4 = \langle 0.87, 0.07 \rangle$, $\tilde{A}_5 = \langle 1.02, 0.07 \rangle$, $\tilde{A}_6 = \langle 1.17, 0.07 \rangle$ and $\tilde{A}_7 = \langle 1.31, 0.07 \rangle$. Then, we apply the rule-cutting threshold 0.3 to reduce the constructed sparse fuzzy rule base of 116 fuzzy rules into a more compact sparse fuzzy rule base of 51 fuzzy rules by removing the fuzzy rules whose degrees of brief are smaller than 0.3.

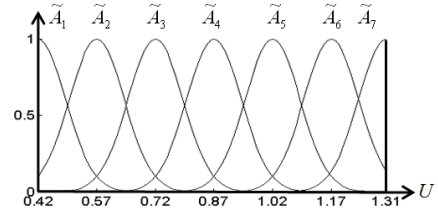


Fig. 5. Seven type-1 Gaussian fuzzy sets used in each variable for the Mackey-Glass chaotic time series prediction problem.

Based on Step 1 of the proposed learning method shown in Section IV, we can construct a new sparse fuzzy rule base using interval Gaussian type-2 fuzzy sets, where each interval type-2 Gaussian fuzzy set $\tilde{\tilde{A}}_i = \langle [m_{i,1}, m_{i,2}], \sigma_{i,1}; \sigma_{i,2} \rangle$ appearing in the fuzzy rules corresponds to the original type-1 Gaussian fuzzy set $\tilde{A}_i = \langle m_i, \sigma_i \rangle$ appearing in the fuzzy rules

of the original sparse fuzzy rule base, $m_{i,1} = m_i - \frac{\sigma_i}{4}$, $m_{i,2} = m_i + \frac{\sigma_i}{4}$, $\sigma_{i,1} = \sigma_i$, $\sigma_{i,2} = \sigma_i$ and $1 \leq i \leq 7$. Fig. 6 shows the seven possible interval type-2 Gaussian fuzzy sets used for each variable for the Mackey-Glass chaotic time series prediction problem.

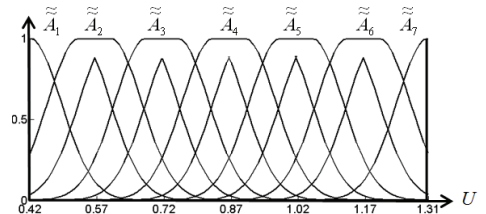


Fig. 6. Seven interval type-2 Gaussian fuzzy sets used in each variable for the Mackey-Glass chaotic time series prediction problem.

After applying Step 1 of the proposed learning method, the above 51 fuzzy rules using type-1 Gaussian fuzzy sets are transformed into the 51 fuzzy rules using interval type-2 Gaussian fuzzy sets, where $\tilde{A}_1 = \langle [0.42, 0.42], [0.07, 0.07] \rangle$, $\tilde{A}_2 = \langle [0.53, 0.6], [0.07, 0.07] \rangle$, $\tilde{A}_3 = \langle [0.68, 0.75], [0.07, 0.07] \rangle$, $\tilde{A}_4 = \langle [0.83, 0.9], [0.07, 0.07] \rangle$, $\tilde{A}_5 = \langle [0.98, 1.05], [0.07, 0.07] \rangle$, $\tilde{A}_6 = \langle [1.13, 1.2], [0.07, 0.07] \rangle$ and $\tilde{A}_7 = \langle [1.31, 1.31], [0.07, 0.07] \rangle$, as shown in Fig. 6. Because there are four antecedent variables and one consequence variable appearing in the fuzzy rules and because each antecedent variable and the consequence variable have seven possible interval type-2 Gaussian fuzzy sets shown in Fig. 5, there are a total of 35 (i.e., $(4 + 1) \times 7 = 35$) individual interval type-2 Gaussian fuzzy sets appearing in the 51 fuzzy rules. Therefore, we encode the left standard deviations and the right standard deviations of the 35 individual interval type-2 Gaussian fuzzy sets into chromosomes, and each chromosome has 70 genes representing the left standard deviations and the right standard deviations of the 35 interval type-2 Gaussian fuzzy sets. Each two successive genes represents the left standard deviation $\sigma_{i,1}$ and the right standard deviation $\sigma_{i,2}$ of the corresponding Gaussian interval type-2 fuzzy set $\tilde{A}_i = \langle [m_{i,1}, m_{i,2}], \sigma_{i,1}; \sigma_{i,2} \rangle$, where $1 \leq i \leq 7$. Based on the new sparse fuzzy rule base with the interval type-2 Gaussian fuzzy sets and the training data set of the 500 training data pairs of the Mackey-Glass chaotic time series prediction problem, we apply the proposed learning method to learn the optimal left standard deviations and the optimal right standard deviations of the interval type-2 Gaussian fuzzy sets appearing in the fuzzy rules, where the population size is 60, the number of evolutions is 300 and the crossover rate is 0.9. Fig. 7 shows an example of the learned optimal interval type-2 Gaussian fuzzy sets obtained by the proposed learning method.

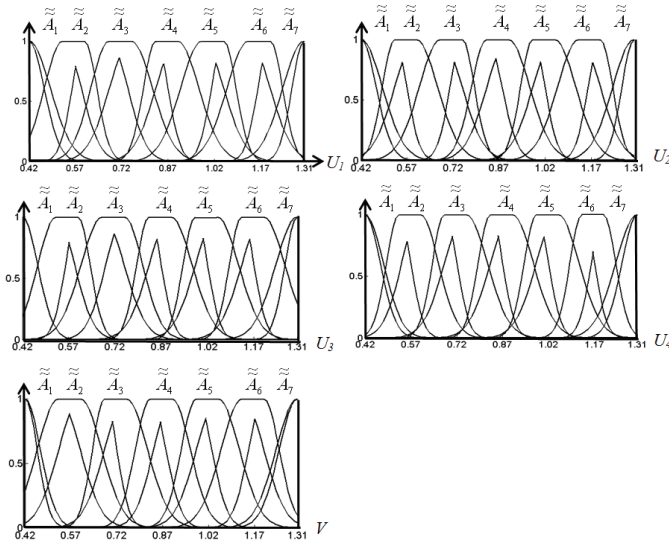


Fig. 7. Optically learned interval type-2 Gaussian fuzzy sets for the Mackey-Glass chaotic time series problem based on the proposed learning method.

Based on the 51 fuzzy rules with the optimally learned interval type-2 Gaussian fuzzy sets obtained by the proposed

learning method, we apply the proposed fuzzy rule interpolation method to produce the interpolated outputs with respect to the testing samples in the 500 testing data pairs. For example, based on the 51 fuzzy rules with the optimally learned interval type-2 Gaussian fuzzy sets shown in Fig. 7, the proposed fuzzy rule interpolation method has the root mean square error 0.0486 with respect to the testing data set.

We have implemented a program based on the proposed learning method and the proposed fuzzy rule interpolation method using Eclipse Version 3.4.2 in a Core 2 duo x86 PC. The system executes the program ten times and the mean of the RMSEs of these ten runs is used as the average RMSE of the proposed method. We then compare the experimental results of the proposed method with the HS method [13], the CCL method [2] and the CK method [8]. Table I shows a comparison of the average root mean square errors for different methods for dealing with the Mackey-Glass chaotic time series prediction problem [10]. From Table I, we can see that the proposed fuzzy rule interpolation method using the optimally learned interval type-2 Gaussian fuzzy sets gets a smaller average root mean square error than the HS method [13], the CCL method [2] and the CK method [8].

TABLE I
A COMPARISON OF THE AVERAGE ROOT MEAN SQUARE ERRORS FOR DIFFERENT METHODS FOR THE MACKEY-GLASS CHAOTIC TIME SERIES PREDICTION PROBLEM

Methods	HS Method [13] (Using Type-1 Gaussian Fuzzy Sets)	CCL Method [2] (Using Type-1 Gaussian Fuzzy Sets)	CK Method [8] (Using Type-1 Gaussian Fuzzy Sets)	The Proposed Method (Using Type-1 Gaussian Fuzzy Sets)	The Proposed Method (Using the Optimally Learned Interval Type-2 Gaussian Fuzzy Sets)
Average RMSE	0.0546	0.1547	0.0630	0.0515	0.0485

Note: The system executes the proposed fuzzy rule interpolation method and the proposed learning method to deal with the Mackey-Glass chaotic time series prediction problem ten times, where the population size of the genetic algorithm is 60, the number of evolutions is 300 and the crossover rate is 0.9.

VI. CONCLUSION

In this paper, we have presented a new method for fuzzy rule interpolation with the learned interval type-2 Gaussian fuzzy sets for sparse fuzzy rule-based systems based on genetic algorithms. The proposed fuzzy rule interpolation method deals with the interpolation of fuzzy rules based on the multiple fuzzy rules interpolation scheme. We also have presented a new learning method to learn optimal Gaussian interval type-2 fuzzy sets for sparse fuzzy rule-based systems based on genetic algorithms. We also have applied the proposed learning method and the proposed fuzzy rule interpolation method to deal with the Mackey-Glass chaotic time series prediction problem. From Table I, we can see that the proposed fuzzy rule interpolation method using the optimally learned interval type-2 Gaussian fuzzy sets obtained by the proposed learning method gets a higher average accuracy rate than the existing methods [2], [8], [13].

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