

Nonlinear Cause-Effect Relationships In Fuzzy Cognitive Maps

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Abstract—Fuzzy Cognitive Maps (FCMs) have been widely used for a plethora of applications, exploiting its ability to represent the knowledge and the dynamics of a system. The diversity of inference mechanisms, which have been proposed until nowadays, discloses the effort for an effective concept value calculation methodology. In contrast with the most research efforts which consider a linear relation of the influence that a concept exercise to another concept, in this paper a nonlinear representation of that influence is introduced. The importance which is associated with the proposed methodology is that a nonlinear cause-effect relationship strengthens the behavior of an FCM through the simulation process. The analysis of this proposal through a progressive reasoning is followed by appropriately selected problems.

Index Terms—Cognitive inference, Concept influence, Generalized logistic function, Cause-effect relationships.

I. INTRODUCTION

Knowledge representation, in a complex system, is a demanding and composite procedure. Experts, from different fields of interest, are invited to offer their knowledge and experience for the modeling of the system. Conforming to market requirements, researchers systematically use qualitative approaches for the representation of the systems. The first attempt to represent knowledge through graphs was introduced by Alexrod [1]. Specifically, Alexrod in [1] proposed the technique of Cognitive Maps (CMs) as a tool of decision making in social and political systems. Kosko in [2] enriched the CMs with fuzzy logic and proposed Fuzzy Cognitive Maps (FCM). CM, a soft computing tool, constricts its contribution to system analysis because of the binary representation of cause-effect relationships. Kosko in [2] transformed the signed directed graphs to fuzzy signed directed graphs and overcame the issues related to systems' rules about knowledge-mining. Moreover, the reliability of causal relationship matrix is increased according to [3], which resulted from the synthesis of multiple experts' knowledge. The introduction of FCMs provoked the interest of the scientific community and challenged many researchers to notice the particular field. Taber and Siegel in [4] and Taber in [5] proposed a methodology of estimating experts' credibility weight, while Kosko and Burgess in [6] developed the inference mechanism of FCMs and applied the Differential Hebbian Law for unsupervised learning. The conception of

combining different FCMs was inserted by Dickerson and Kosko [7] enhancing the architectural configuration of FCMs.

To continue with, Pelaez and Bowles [8] supported the idea that various factors, such as integrity constraints and time lag, should be taken into consideration during the inference process. The introduction of these constraints was the beginning of a race, in order for dynamic (non-linear) systems to be represented using FCM. Tsadiras and Margaritis in [9] introduced Certainty Neuron Fuzzy Cognitive Maps (CNFCM), which reveal the existence of concepts' memory related to its previous states and used for the computation of its current states. Andreou et al. [10] extended the previous model by combining CNFCM and genetic algorithms in order to surpass the weaknesses lie in the recalculation of a concept's weights every time a new strategy is applied.

A dynamic version of FCM proposed by Miao et al. [11] in order to explore the dynamic nature of concepts and to determine the strength of the cause and the cause-effect relationship as much as the degree of influence. Evolutionary strategies, combined with fuzzy cognitive maps, revealed efficiency and credibility increment when applied in ambiguous and multi-stimulus situations ([12], [13], [14], [15]). An integrated research, elaborated by Koulouriotis et al. [16], combines decay mechanisms, inertial forces, diminishing effects and biases, such as synergies and conditional effects ([17]), in order to describe and comprehend nonlinear and asymmetric cause-effect relationships, achieving simultaneously controllability. Fuzzy cognitive maps have several applications, either by implementing a learning procedure ([33], [18]) or not ([19], [20], [21], [23], [22]). Finally, the participation of agents in FCM process ([24], [25], [26]) intends to a non-centralized detection of FCM stable state and to a modification process using asynchronous calculations.

In the current article, an integrated inference mechanism is presented which is based on influence modification in order to meet the requirements of dynamic systems. In section II, a concise presentation of the theoretical background of FCM is quoted, offering an introductory view and the key points of inference mechanism development. The subsequent sections deal with the presentation of the proposed methodology and

the experimental analysis, while the concluding remarks are given in the last section.

II. FUZZY COGNITIVE MAPS: A THEORETICAL BACKGROUND

Fuzzy Cognitive Maps (FCMs), proposed by Kosko [2], are fuzzy signed directed graphs with feedback, emphasizing both to the representation of the cause-effect relationships between concepts (i.e., positive, negative, zero) and the influence degree of this relationship expressed in linguistic values. Concretely, a FCM is a fuzzy graph which models a system and its behavior using the concepts and their relationships. A concept i indicates a state, a procedure, an event or a variable of the system and is represented with C_i ($i = 1, 2, \dots, n$). The directed edge between concepts C_i and C_j denotes the causality between concepts i and j and is represented with the weight w_{ij} , that can be a fuzzy value within $[-1, 1]$ or a trivalent logic within $\{-1, 0, 1\}$. There are three possible types that a weight (w_{ij}) can be distinguished in: a) positive ($w_{ij} > 0$), which means that an increase (decrease) in the value of concept C_i leads to an increment (decrement) of value of C_j , b) negative ($w_{ij} < 0$), which means that an increase (decrease) in the value of concept C_i leads to a decrement (increment) of value of C_j , and c) neutral ($w_{ij} = 0$), which means that there is no relationship between C_i and C_j . The graphic illustration of the modeled system and the corresponding adjacency matrix is shown in Fig.1 and Fig.2.

A general rule for computing the value of a concept in each simulation step is the calculation of the influence caused by other concepts that are connected to the particular concept. The value A_i^{t+1} denotes the effect that concept C_i caused at time $t+1$ by other concepts [27] and is calculated by the following equation:

$$A_i^{t+1} = f \left(A_i^t + \sum_{j=1, j \neq i}^n A_j^t \cdot w_{ji} \right) \quad (1)$$

where, A_i^t is the value of concept C_i at time t , A_j^t is the value of concept C_j at time t , w_{ji} is the weight of the interconnection between concepts C_i and C_j , and f is a threshold function which limits the result of the multiplication in the interval $[-1, 1]$.

Tsadiras in [28] analyzed and compared different FCMs according to the type of threshold function which was implemented. After the initialization of the system by the experts and defuzzifying the inserted data, the concepts are ready to interact according to the determined relation network. The moment that a concept is activated, the other concepts, which are connected with, are influenced. The interaction between concepts continues until a fixed-point equilibrium is reached; a limit cycle is reached; or chaotic behavior is exhibited [9]. Tsadiras and Margaritis [9], developed a procedure by which the appropriate threshold function can be chosen according to concept's value and the effect influence field of definition. The interpretation of (2-4) is that a concept which is influenced by

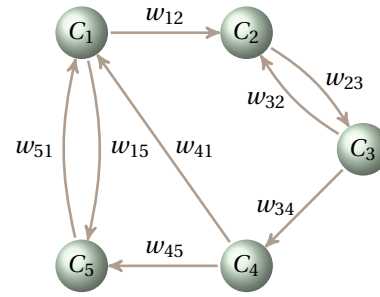


Fig. 1. A simple Fuzzy Cognitive Map

$$W = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0 & w_{12} & 0 & 0 & w_{15} \\ 0 & 0 & w_{23} & 0 & 0 \\ 0 & w_{32} & 0 & w_{34} & 0 \\ w_{41} & 0 & 0 & 0 & w_{45} \\ w_{51} & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Fig. 2. The Adjacency matrix of FCM

external effect can be activated to the degree that is not already activated.

$$f(A_i^t, S_i^t) = \begin{cases} A_i^t + S_i^t(1 - A_i^t), & A_i^t, S_i^t \geq 0 \\ A_i^t + S_i^t(1 + A_i^t), & A_i^t, S_i^t < 0, |A_i^t|, |S_i^t| \leq 1 \\ \frac{A_i^t + S_i^t}{1 - \min(|A_i^t|, |S_i^t|)}, & \text{otherwise} \end{cases} \quad (2)$$

$$A_i^{t+1} = f(S_i^t, A_i^t) \quad (3)$$

$$S_i^t = \sum_{j=1, j \neq i}^n A_j^t w_{ji} \quad (4)$$

where A_i^t is the value of concept C_i at the moment t , and S_i^t is the total influence which has applied from other concepts at moment t . The problem which arises from (4) is that S_i^t should belong within space $[-1, 1]$. The influence of the other concepts, S_i^t , has the maximum value when $|A_i^t| = 1$ and all A_j^t , $j = 1, \dots, n$, have the same sign with the corresponding weight w_{ji} . So,

$$S_{i,\max} = \sum_{j=1}^n |A_j^t| |w_{ji}| = \sum_{j=1}^n |w_{ji}|, \quad i = 1, \dots, n \quad (5)$$

The maximum value of $S_{i,\max}$ can be calculated as follows:

$$S_{\max} = \max(S_{1,\max}, S_{2,\max}, \dots, S_{n,\max}) \quad (6)$$

A new matrix w' is created which its elements w_{ji} can be calculated using the following equation:

$$w'_{ji} = \frac{w_{ji}}{S_{\max}} \quad (7)$$

Tsadiras and Margaritis [9], using (7), ensured that the influence value S_i^t will never be out of space $[-1, 1]$. Considering that the third part of (2) could possibly lead to division with

zero, Andreou et al. [29] proposed a transformation by which the absolute values of variables are replaced by their normal values. The third part of (2) transformed and acquired the form $\frac{A_i^t + S_i^t}{1 - \min(A_i^t, S_i^t)}$.

Moreover, Yaman and Polat in [30] modified the third part of (2) and the one that Andreou et al. [29] proposed, inserting the mathematical expression $A_i^t + S_i^t$. In the case that the particular condition is satisfied, the sum of $A_i^t + S_i^t$ is impossible to be out of the continue space $[-1, 1]$. As it is obvious, (3) and (4) are also part of this proposition. Generally, the influence degrees are expected to be computed at the end of the determined influence durations. In the proposed methodology, links' influence duration is reduced by one unit and unit influence degree is recalculated for every step.

Finally, Koulouriotis et al. [16] proposed an enhancement on Dynamic Cognitive Networks (DCNs), combining adaptive characteristics which portraiture a dynamic system. These extensions are nonlinear and asymmetric cause-effect relationships, decaying mechanisms, inertial forces, diminishing effects and biases. Referring to nonlinear and asymmetric cause-effect relationships, Koulouriotis et al. [16] introduced a formula which is based on sigmoid function and is used for the computation of the effect that a concept A cause to a concept B (8).

$$EFFECT = ef_{AB}(C_A) = \frac{2}{1 + e^{-\alpha C_A}} - 1 \quad (8)$$

$A \rightarrow B$

where α is a control parameter whose role is to adjust the rate of the impact that concept A exerts on B. The methodology by which authors [16] calculated the value of concept is given by (9):

$$A_i^{t+1} = \min \left(1, \max \left(-1, A_i^t + \sum_{j=1, j \neq i, A_j^t > 0}^n w_{ji}^+ A_j^t + \sum_{j=1, j \neq i, A_j^t < 0}^n w_{ji}^- A_j^t \right) \right) \quad (9)$$

Similarly, Koulouriotis et al. [16] imported decaying mechanisms, inertial forces, diminishing effects and biases in inference procedure, specifying the necessity of meticulously defined systems.

III. PROPOSED METHODOLOGY

Considering the above presentation of Fuzzy Cognitive Maps, we presume that A_j is the value of concept C_j and the weight w_{ji} indicates the relationship between concepts C_j and C_i , where $A_j, w_{ji} \in [-1, 1]$. The influence F_{ji} that a concept C_j causes to C_i is denoted with

$$F_{ji} = w_{ji} \cdot A_j, \quad F_{ji} \in [-1, 1] \quad (10)$$

The graphic representation of (10) is illustrated in Fig.3 and reflects the linear function $y = a \cdot x$. Analytically, the vertical axis represents the influence F_{ji} that a concept C_j cause to concept C_i and the horizontal axis represents the value A_j of concept C_j . The line ϵ (red line in Fig.3) represents the linear

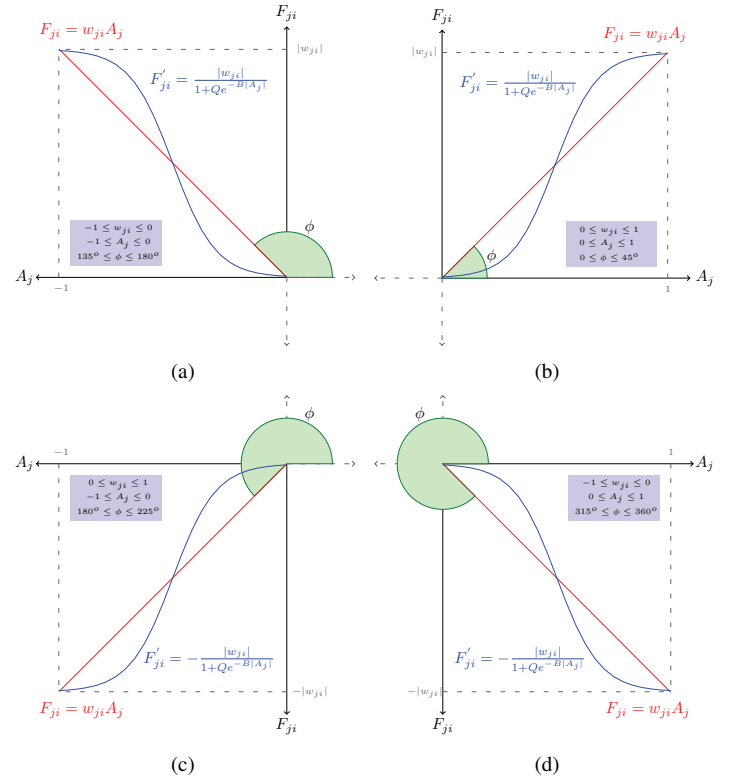


Fig. 3. The influence degree of concept C_j to C_i expressed with the generalized logistic function for a) $135^\circ \leq \phi \leq 180^\circ$, b) $0^\circ \leq \phi \leq 45^\circ$, c) $180^\circ \leq \phi \leq 225^\circ$, d) $315^\circ \leq \phi \leq 360^\circ$.

relation of F_{ji} and A_j . From the above statements, it can easily be deduced that w_{ji} is the slope of line ϵ .

$$w_{ji} = \tan(\phi) \quad (11)$$

In order for F_{ji} to belong within $[-1, 1]$, given that $A_j, w_{ji} \in [-1, 1]$, the field of definition of ϕ is $\phi \in \{[0^\circ, 45^\circ] \cup [135^\circ, 225^\circ] \cup [315^\circ, 360^\circ]\}$. Considering the current perception, the relation between influence F_{ji} and the value A_j is linear. In the present article, it is proposed a nonlinear representation of this relation, using the generalized logistic function.

$$f(x) = L + \frac{U - L}{(1 + Qe^{-B(x-M)})^{1/\nu}} \quad (12)$$

where, L is the lower asymptote, U is the upper asymptote, B is the growth rate, ν affects near which asymptote maximum growth occurs, Q depends on the value $f(0)$ and M is the time of maximum growth if $Q = \nu$.

According to the above considerations(11, 12), the influence F_{ji} that a concept C_j causes to a concept C_i , where $A_j, w_{ji} \in [-1, 1]$, can be formulated using the generalized

logistic function (12).

$$F'_{ji} = \begin{cases} \frac{|w_{ji}|}{1+Qe^{-B|A_j|}}, & \text{for } 0 \leq w_{ji}, A_j \leq 1 \\ & \text{or} \\ & -1 \leq w_{ji}, A_j \leq 0 \\ -\frac{|w_{ji}|}{1+Qe^{-B|A_j|}}, & \text{for } 0 \leq A_j \leq 1, -1 \leq w_{ji} \leq 0 \\ & \text{or} \\ & -1 \leq A_j \leq 0, 0 \leq w_{ji} \leq 1 \end{cases} \quad (13)$$

where, $L = 0$, $U = |w_{ji}|$, $M = 0$, $\nu = 1$ and Q, B are two free parameters in order to adjust the shape of the sigmoid curve. The graphic illustration of influence F'_{ji} , in relation to A_j , is shown in Fig.3. In the first quadrant Fig.3(a), the linear relation of F_{ji} and A_j is represented with the red line. The angle ϕ ($0 \leq \phi \leq 45^\circ$) shows the space that line $F_{ji} = w_{ji}A_j$ can move in order to satisfy the limitation $0 \leq w_{ji} \leq 1$, considering (11). Moreover, the nonlinear relation of F_{ji} and A_j is represented with the generalized logistic function (sigmoid function), with upper limit $F_{ji_{max}} = |w_{ji}|$. Similarly, the other three quadrants are defined in Figs.3(b), 3(c) and 3(d), verifying the expression (13). Taking into consideration the above definitions in (13) and Fig.3, the value A_i^{t+1} of concept C_i at time $t+1$ can be computed using the following equation:

$$A_i^{t+1} = f \left(A_i^t + \sum_{j=1, j \neq i}^n F'_{ji}(A_j^t, w_{ji}) \right) \quad (14)$$

However, this methodology can be applied to every concept value computational methodology due to the fact that it not changes the methodology but the cause-effect relationship between two concepts.

The particular methodology proposes a nonlinear way to represent influence between concepts considering its appliance to dynamic systems. Essentially, the proposed methodology which is based on generalized sigmoid function, is a generalization of the concepts causal-effect relationship, which is reported in [16]. The nonlinear relationship between concepts transforms their values in such a way that results in a smoother convergence. However, this behavior might cause slower convergence to an equilibrium state. Furthermore, appropriately selected parameters (L, U, Q, B, M, ν) could lead the map to an equilibrium state even if a linear causal-effect relationship is not able to reach a stable state.

IV. EXPERIMENTAL ANALYSIS

In order to apply, analyze and compare the proposed methodology, two different scenarios have been chosen which implement real-world and synthetic data.

A. Scenario 1

The first scenario [33] is referred to a simple control process in chemical industry. It comprises one tank and three valves that influence the amount of liquid in the tank. The particular procedure can be described as follows: Valves 1 and 2 pour two different liquids into the tank, producing a new liquid while a

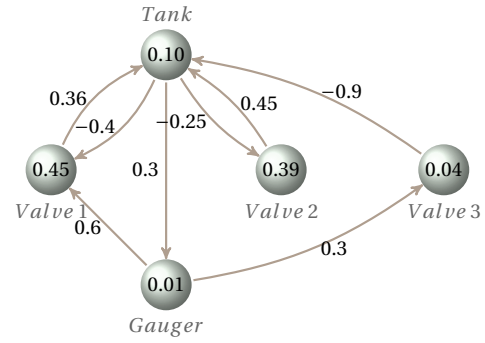


Fig. 4. The Fuzzy Cognitive Map derived from scenario 1

chemical reaction is triggered. When the new liquid produced reaches a specific level, measured by a sensor, valve 3 empties the tank. The FCM model is derived from the above control problem, according to a group of experts, and is illustrated in Fig.4. As we can assume from Fig.4, there are five concepts that constitute the FCM. The first concept (C_1) indicates that the amount of the liquid that Tank contains is dependent on the operational state of Valves 1, 2 and 3. Moreover, the state of Valves 1, 2 and 3 (C_1, C_2 and C_3 correspondingly) according to the predefined problem, could be closed, open or partially opened. Finally, the specific gravity of the liquid into the tank typifies the last concept (C_5). The initial values of concepts are given in matrix $A_{initial} = [0.4 \ 0.7 \ 0.6 \ 0.7 \ 0.3]$, the initial weight matrix ($W_{initial}$) is derived from the FCM of Fig.4 and the values of concepts C_1 and C_5 must be inside the ranges $0.68 \leq C_1 \leq 0.74$ and $0.74 \leq C_5 \leq 0.80$. In the current experimental analysis (Scenario 1, 2), the values of the free parameters Q and B (13) which are used in the procedure are $Q = 200$, $B = 10.5966$.

In [33], a linear cause-effect relationship between concepts (1) is used during the training process (Nonlinear Hebbian Learning - NHL) of the above FCM. In [33] the desired equilibrium state $A_{NHL}^1 = [0.683 \ 0.7632 \ 0.6538 \ 0.7534 \ 0.744]$ is reached and the NHL algorithm returns the updated weight matrix (W_{NHL}^1). In order for the proposed methodology to be compared with the common procedure, the above FCM is trained again with the difference that the NHL uses the proposed methodology (14) to compute the concepts values. In this case the equilibrium state $A_{prop} = [0.7108 \ 0.7789 \ 0.6480 \ 0.7764 \ 0.7660]$ is reached and the NHL algorithm returns the weight matrix W_{prop} (Fig.5). The values of concepts C_1 and C_5 which have been resulted from the aforementioned training processes are illustrated in Table I.

In order to evaluate the weight matrix W_{prop} , 1000 random initial vectors (A_{random}) are produced. These vectors are used as initial states of the FCM models which have as weighted matrices the W_{prop} and W_{NHL}^1 correspondingly. The minimum, maximum and mean values of concepts C_1 and C_5 as well as the standard deviations, which have been derived from the random examples, are presented in Table II.

By examining the mean values of Table II can be observed

TABLE I
EQUILIBRIUM STATES OF TRAINING PROCESSES

	Training Process Equilibrium State	
	Tank (C_1)	Gauger (C_5)
A_{NHL}^1	0.6830	0.7440
A_{Prop}	0.7108	0.7660

TABLE II
RANDOM EXAMPLES OF C_1, C_5

			1000 random examples			
			A_{NHL}^1		A_{Prop}	
Tank (C_1)	min	max	0.6833	0.6843	0.7106	0.7113
	Mean	St.Dev.	0.6836	1.29e-004	0.7109	1.29e-004
Gauger (C_5)	min	max	0.7447	0.7454	0.7657	0.7663
	Mean	St.Dev.	0.7450	1.53e-004	0.7660	1.24e-004

$$W_{prop} = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0 & -0.2112 & -0.0417 & 0 & 0.4649 \\ 0.5362 & 0 & 0 & 0 & 0 \\ 0.5617 & 0 & 0 & 0 & 0 \\ -0.8274 & 0 & 0 & 0 & 0 \\ 0 & 0.7110 & 0 & 0.4966 & 0 \end{pmatrix} \end{matrix}$$

Fig. 5. The weight matrix of FCM using the proposed methodology

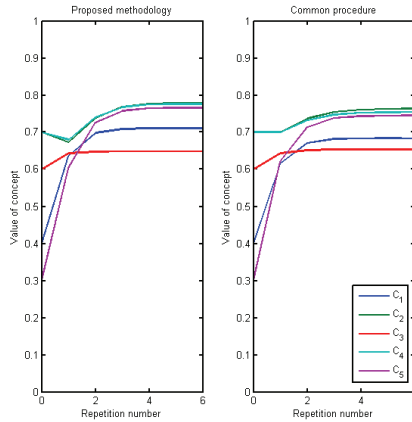


Fig. 6. Simulation process of Scenario 1

that the usage of the proposed methodology (nonlinear cause-effect relationships between concepts) in the NHL learning process leads to an equilibrium state for C_1 and C_5 very close to the midpoint of the desired bounds. It can further be concluded, from the minimum and maximum values as well as the standard deviations of 1000 random examples, that the consideration of a nonlinear cause-effect relationship between concepts leads to a FCM model as stable as the traditional methodology.

B. Scenario 2

An engine for producing scenarios with different properties (number of concepts, initial concept values, weight matrices) has been developed for the paper purposes. In order to investigate the simulation results, two different cases of the created scenarios have been selected. In the first case, the function $f(n) = \frac{1}{1+e^{-n}}$ is used as a threshold function (concept values inside the range $[0, 1]$), whereas in the second case the threshold function is $f(n) = \frac{2}{1+e^{-2n}} - 1$ (concept values inside the range $[-1, 1]$). The proposed methodology is compared with a common procedure in which (1) is used for calculating concepts values. Figures 8, 10 and 12 consist of two parts. The left part shows the simulation results of the proposed methodology while the right indicates the simulation results of (1).

In the first case, the proposed methodology presents very similar results with the common procedure (Fig.8). The initial weight matrix and the initial values of concepts which have been chosen for the simulation as much as the equilibrium state values (values of concepts) of the compared methodologies are illustrated in Fig.7 and Table III, respectively.

However, in the second case, the proposed methodology illustrates not only similar results with the common procedure, but also results which enhance its strength inference nature against the common procedure (Fig.12). The first example of the second case (when threshold function is $f(n) = \frac{2}{1+e^{-2n}} - 1$) is consisted of five concepts. The weight matrix, the initial concepts values as well as the equilibrium state of the two methodologies are presented in Fig.9 - Fig.10 and Table IV. It can be observed that although the two methodologies reach a similar equilibrium state, the proposed methodology needs more steps to finish the simulation process. Furthermore, the curves in Fig.10 follow a slightly different path to reach the final state.

Information about the initial and the equilibrium state of the second example of case two are given in Fig.11 - Fig.12 and Table V. By examining the aforementioned figures and

$$W_{\text{initial}} = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1764 & 0 & 0.3342 & 0 \\ 0 & 0 & 0 & -0.1011 & 0.1729 & -0.8323 \\ -0.3024 & 0 & 0 & 0 & 0 & 0.9496 \\ -0.0973 & -0.7245 & 0.0076 & 0 & 0 & 0.3027 \\ 0 & 0.6734 & -0.0208 & 0.9459 & 0.2406 & 0 \end{pmatrix} \end{matrix}$$

Fig. 7. Initial weight matrix for Scenario 2 - Case 1

TABLE III
INITIAL AND EQUILIBRIUM STATE VALUES OF CONCEPTS FOR SCENARIO 2
- CASE 1

Concepts	Initial state	Equilibrium state	
	A_{Initial}	A_{Proposed}	A_{Common}
C_1	0.3569	0.5372	0.5588
C_2	0.6627	0.6416	0.6439
C_3	0.2815	0.6959	0.6882
C_4	0.2304	0.8386	0.8133
C_5	0.7111	0.8128	0.7869
C_6	0.6246	0.7781	0.7699

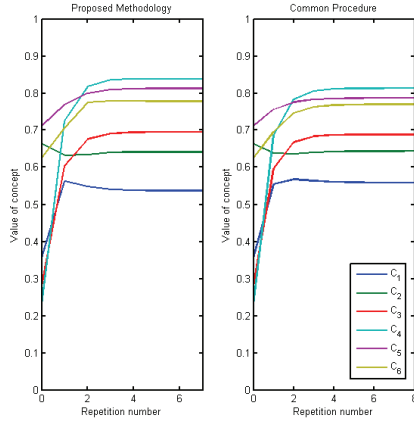


Fig. 8. Simulation process of Scenario 2 - Case 1

the corresponding table can be concluded that the selection of the threshold function leads to different results between the two methodologies. In Fig.12 can be observed that the proposed methodology reached an equilibrium fixed-point in contrast to the common procedure which reaches a limit cycle. This is feasible with an appropriate adjustment of the proposed methodology parameters (12-13). This fact, in combination with the first example of case two, highlights the enhanced, by the proposed methodology, dynamic behavior of the FCM. To continue with, the adjustable character (parameters Q , B) of the proposed influence mechanism introduce additional adaptability to similar situations.

V. CONCLUSIONS AND FUTURE WORK

Fuzzy cognitive maps are a remarkable simulation methodology primarily based on a linear cause-effect relationship between concepts. The subject of the particular research is the nonlinear representation of influence between concepts in order to achieve more effective simulation of real-world situations. The characteristics of the proposed methodology,

$$W_{\text{prop}} = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0 & -0.0999 & -0.1133 & 0 & 0.9380 \\ -0.1444 & 0 & 0.1095 & 0 & 0.6938 \\ 0 & 0 & 0 & 0 & 0.0636 \\ 0 & 0 & 0.7425 & 0 & 0 \\ 0 & 0 & 0 & 0.3017 & 0 \end{pmatrix} \end{matrix}$$

Fig. 9. Initial weight matrix for Scenario 2 - Case 2 - Example 1

TABLE IV
INITIAL AND EQUILIBRIUM STATE VALUES OF CONCEPTS FOR SCENARIO 2
- CASE 2 - EXAMPLE 1

Concepts	Initial state	Equilibrium state	
	A_{Initial}	A_{Proposed}	A_{Common}
C_1	0.6455	0.6048	0.5646
C_2	0.2268	-0.5647	-0.5198
C_3	-0.1384	0.8955	0.8616
C_4	0.8747	0.7952	0.7527
C_5	-0.9476	0.8046	0.7506

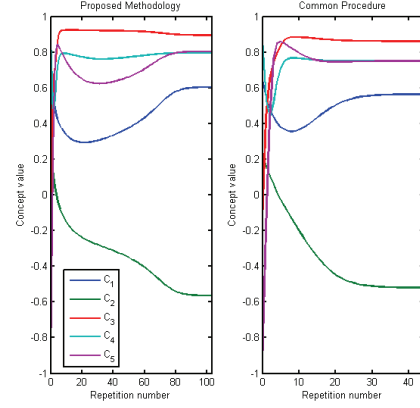


Fig. 10. Simulation process of Scenario 2 - Case 2 - Example 1

$$W_{\text{prop}} = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0 & -0.4750 & -0.3156 & 0 & 0.4645 \\ 0 & 0 & -0.3722 & 0 & 0.6527 \\ -0.4407 & 0 & 0 & -0.7656 & 0 \\ 0 & 0 & 0.0968 & 0 & 0 \\ -0.2620 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Fig. 11. Initial weight matrix for Scenario 2 - Case 2 - Example 2

TABLE V
INITIAL AND EQUILIBRIUM STATE VALUES OF CONCEPTS FOR SCENARIO 2
- CASE 2 - EXAMPLE 2

Concepts	Initial state	Equilibrium state	
	A_{Initial}	A_{Proposed}	A_{Common}
C_1	-0.0978	0.7304	-
C_2	-0.5402	-0.8613	-
C_3	-0.0484	0.2923	-
C_4	-0.4572	-0.5676	-
C_5	-0.2925	-0.7408	-

which exploit experts' knowledge and experience, as well as its adjustability, highlight the enhanced inference mechanism.

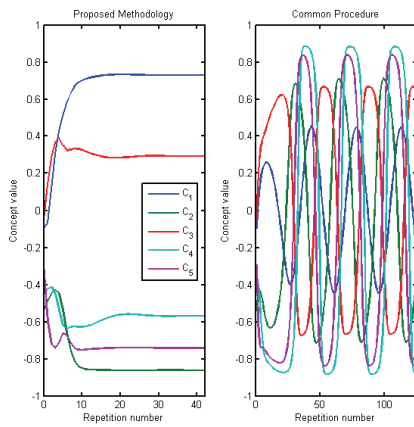


Fig. 12. Simulation process of Scenario 2 - Case 2 - Example 2

Improved learning results and randomly generated indicative examples, illustrate the ability of the proposed mechanism to converge into fixed states and efficiently manage realistic systems, even when the conventional approach falling into repeated cycle state. The proposed methodology stimulates the future research to a direction which is associated with the targeted and extensive analysis of real-world and other problems, especially when limit cycle or chaotic behavior is detected. Furthermore, the proposed methodology depends on a set of free parameters (L, U, Q, B, M, ν), thus a sensitivity analysis is going to be explored in a future work. Moreover, a thorough study of learning algorithms using the nonlinear influence mechanism in conjunction with its property of ad-justability, could improve the whole process.

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