

Some Refinements for Non quadratic Stabilization of continuous TS Models

T.M. Guerra¹, A. Jaadari^{1,2},

J. Pan^{1,3}, A. Sala²

¹ LAMIH FRE CNRS 3304, University of Valenciennes
Hainaut-Cambrésis, France
{guerra Abdelhafidh.Jaadari}@univ-valenciennes.fr,

² Dept. of Systems Engineering & Control, polytechnic
University of Valencia, Spain
asala@isa.upv.es

³ School of Automation, Southeast University, Nanjing, China
jtpan@yahoo.cn,

Abstract – Quadratic stability / stabilization / estimation for Takagi-Sugeno models have now reached maturity. Lots of results concerning performances (H_2 , H_∞ , D-stability) for TS models with uncertainties, noise, time-delays ... exist in the literature. Nevertheless, it is illusive thinking to solve every nonlinear problem of stability using a simple quadratic Lyapunov function. Moreover, most of the nonlinear systems only have properties of local stability. This work follows the idea of using non quadratic Lyapunov functions for continuous Takagi-Sugeno models restricting the global asymptotic stability to a local one. Therefore, it tries to estimate the best stabilization domain possible. A rather “simple” LMI constraints problem is derived to answer to this question.

Index Terms — Takagi-Sugeno (TS) models, Linear Matrix Inequality (LMI), Fuzzy Lyapunov Function (FLF), Local Stabilization.

I. INTRODUCTION

The so-called Takagi-Sugeno models [1] have reached a very big attention in the control community. The results for stability, stabilization, estimation [2, 3, 4] are now converging towards quasi-LPV models results [5]. A global view of the last works can give the conclusion that more or less, lots of new results just represent outperforming previous ones, using the same techniques. Of course, some of these improvements are particularly important and allows dealing with problems that were unfeasible before. Among these nice improvements, we can cite the Polya’s approach that gives sufficient and necessary conditions for the copositivity problem [6], piecewise Lyapunov functions when restricted to specific TS models [7, 8], non quadratic Lyapunov functions for discrete TS models with important results [9, 10, 11, 12].

The paper focuses on non quadratic stabilization for continuous TS models where the results are not satisfactory, especially when compared with the discrete case. Two families of results have to be described. As the derivative of the Lyapunov function will involve the derivative of the membership function of the TS model, several results propose just to bound them a priori [13, 14, 15]. This way of doing is not satisfactory because the verification of these bounds can only be done a posteriori on a case by case approach. On

another side, independent path Lyapunov functions were used in [16] that present a fine result but restricted to specific TS models. Notice also that for stabilization, the problem remains BMI. All these approaches consider the problem of global stability which is far to be the general rule for nonlinear systems. Therefore, some works proposed to reduce this goal to local stability and estimate a domain of convergence [17, 18]. A tentative for stabilization has been presented in [19] which introduced bounds on the control law and complicated treatments of the problem. The work presented therein allows solving the stabilization problem in a local stabilization framework without this drawback.

The work is organized as follows. Part 1 recalls some properties; part 2 presents the problem of NQ stabilization and part 3 introduces the main result of the paper. An example illustrates the interest of the results and a conclusion ends the paper.

II. PRELIMINARIES

First, we will present some notations and properties helpful to derive the main result of the work.

As usual, a star (*) in a symmetric matrix denotes the transpose element. When suitable, the next notations are used. Consider nonlinear functions $h_i(z) \geq 0$ sharing the convex

sum property, i.e. $\sum_{i=1}^r h_i(z) = 1$, then:

$$\Upsilon_z = \sum_{i=1}^r h_i(z(t)) \Upsilon_i, \quad \Upsilon_z^{-1} = \left(\sum_{i=1}^r h_i(z(t)) \Upsilon_i \right)^{-1} \quad \text{and} \quad \text{for}$$

$$\text{multiple sums: } \Upsilon_{zz} = \sum_{i=1}^r \sum_{j=1}^r h_i(z(t)) h_j(z(t)) \Upsilon_{ij}.$$

Property 1: for $y \in \mathbb{R}^n$ and $\beta > 0$ the following equivalence holds:

$$y^T y - \beta < 0 \Leftrightarrow yy^T - \beta I < 0 \quad (1)$$

Proof: it follows directly using the Schur complement:

$$y^T y - \beta < 0 \Leftrightarrow \begin{bmatrix} \beta & y^T \\ y & I \end{bmatrix} > 0 \Leftrightarrow \beta^{-1} yy^T - I < 0 \Leftrightarrow yy^T < \beta I.$$

Property 2: with $P = P^T > 0$ and $\delta \in \mathbb{R}$, $\delta > 0$ the following property holds:

$$P_z \geq \delta I \Rightarrow P_z^2 \geq \delta P_z. \quad (2)$$

Proof: consider $x^T P_z x \geq \delta x^T x$ and the change of variable: $x = P_z^{1/2} y$, it follows directly: $y^T P_z^2 y \geq \delta y^T P_z y$.

Property 3 [18]: with X, Y two matrices of appropriate dimension $\delta, \lambda \in \mathbb{R}$, $\lambda > 0$ and $|\delta| \leq \lambda$ the following implication holds:

$$S = S^T > 0, \quad Y + \frac{1}{2}(\lambda^2 S + X S^{-1} X) \leq 0 \Rightarrow Y + \delta X \leq 0. \quad (3)$$

Proof: use the completion of squares, i.e. with $S = S^T > 0$, $U^T V + V^T U \leq U^T S U + V^T S^{-1} V^T$. Thus, with $U = \delta I$ and $V = X$ it is direct.

Property 4 [20]: (relaxation lemma).

Let Υ_{ij} , $i, j \in \{1, \dots, r\}$ be matrices of appropriate dimensions.

$$\Upsilon_{zz} = \sum_{i=1}^r \sum_{j=1}^r h_i(z(t)) h_j(z(t)) \Upsilon_{ij} \geq 0 \text{ holds if}$$

$$\frac{2}{r-1} \Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} > 0, \quad i, j \in \{1, \dots, r\}. \quad (4)$$

Remark: conditions (4) represent one of the numerous possible relaxations, see [6] for example. Notice that once the quantities Υ_{ij} defined and being LMI constraints, any relaxation can be used.

Recall that the open loop TS model [1] writes:

$$\dot{x}(t) = A_z x(t) + B_z u(t), \quad (5)$$

with $x(t) \in \mathbb{R}^n$ the state vector, $u(t) \in \mathbb{R}^m$ the input vector, $z(t) \in \mathbb{R}^p$ the premise vector. This latter is linked to the state vector $x(t)$ via a linear or a nonlinear mapping.

Consider also the validity domain of the TS model [21] which is a compact set of the state space:

$$C = \{x \in \mathbb{R}^n, \underline{x}_i \leq x \leq \bar{x}_i, i = 1, \dots, n\}; \quad (6)$$

and $0 \in C$, as the equilibrium point is 0 without lack of generality. It is supposed that the functions $h_i(z) \geq 0$ are obtained via the sector nonlinearity approach [2]. Therefore, we can write [18]:

$$h_i = h_{1+i_1+i_2 \times 2 + \dots + i_p \times 2^{p-1}} = \prod_{j=1}^p w_{i_j}^j(z_j), \quad (7)$$

with $i \in \{1, \dots, 2^p\}$, $i_j \in \{0, 1\}$ and $w_1^j(\cdot) = 1 - w_0^j(\cdot)$.

Remark: other possible compact sets are possible, such as for example: $C = \{x \in \mathbb{R}^n, \|x\| \leq \lambda_x\}$, the form (6) is classically used because of the sector nonlinearity approach [2].

Hypothesis 1: In this work the functions $h_i(z)$ and the mapping between the premise vector $z(t)$ and the state vector $x(t)$ are of class C^1 on the defined compact set C ; i.e. the partial derivatives are well-defined.

III. NON QUADRATIC STABILIZATION

This section recalls some basics on NQ stabilization for continuous TS models. First of all, a non-PDC control law is used:

$$u(t) = F_z P_z^{-1} x(t). \quad (8)$$

Therefore, the closed loop writes:

$$\dot{x}(t) = (A_z + B_z F_z P_z^{-1}) x(t), \quad (9)$$

whose stability properties will be investigated through the following non-quadratic fuzzy Lyapunov function candidate:

$$V(x(t)) = x^T(t) P_z^{-1} x(t), \quad (10)$$

where $P_z > 0$. Its derivative along the trajectories of the closed-loop TS model (9) is:

$$\dot{V}(x) = x^T \left(P_z^{-1} (A_z + B_z F_z P_z^{-1}) + (A_z + B_z F_z P_z^{-1})^T P_z^{-1} + \dot{P}_z^{-1} \right) x$$

Since elementary matrix manipulations show that is verified if

$$A_z P_z + B_z F_z + (A_z P_z + B_z F_z)^T - \dot{P}_z < 0. \quad (11)$$

Recall now that $\dot{P}_z$ writes:

$$\dot{P}_z = \sum_{i=1}^r \dot{h}_i P_i = \sum_{i=1}^r \left(\frac{\partial h_i(z)}{\partial z} \right)^T \dot{z} P_i. \quad (12)$$

Using the definition of the functions $h_i(z)$ and w_0^j equation (7) and after some manipulations see [17, 18], (12) can be written as:

$$\begin{aligned} \dot{P}_z &= \sum_{i=1}^r \sum_{k=1}^p h_i \frac{\partial w_0^k}{\partial z_k} \dot{z}_k (P_{g_1(i,k)} - P_{g_2(i,k)}) \\ &= \sum_{k=1}^p \frac{\partial w_0^k}{\partial z_k} \dot{z}_k (P_{g_1(z,k)} - P_{g_2(z,k)}) \end{aligned} \quad (13)$$

with $g_1(i, k) = \lfloor (i-1)/2^{p+1-k} \rfloor \times 2^{p+1-k} + 1 + (i-1) \bmod 2^{p-k}$ and $g_2(i, k) = g_1(i, k) + 2^{p-k}$, $\lfloor \cdot \rfloor$ being the floor function.

Remark: it is *very important* to notice that the expression (13) shows that if a quadratic Lyapunov function exists than $\dot{P}_z = 0$ is guaranteed via $P_{g_1(\alpha, k)} = P_{g_2(\alpha, k)}$ for each $i \in \{1, \dots, r\}$ and $k \in \{1, \dots, p\}$. This remark will hold also for the LMI constraints thereafter. Therefore, every conditions derived in this work are *always* including the quadratic case.

Theorem 1 (Local stabilizability): If there exist matrices of the proper size $P_i = P_i^T > 0$, F_i , $i \in \{1, \dots, r\}$ such that $A_z P_z + B_z F_z + P_z A_z^T + F_z^T B_z^T < 0$, then there exists a domain D , $0 \in D$, such that TS model (5) is locally asymptotically stabilizable under control law (8).

Proof: The NQLF candidate (10) satisfies $V(0)=0$ and $V(x) \geq 0$ in $\mathbb{R}^n$. Its time-derivative (11) holds $\dot{V}(0)=0$. Provided that $A_z P_z + B_z F_z + P_z A_z^T + F_z^T B_z^T < 0$ it is implied that there exists a sufficiently small $\lambda > 0$ such that $A_z P_z + B_z F_z + P_z A_z^T + F_z^T B_z^T + \lambda I < 0$ which can be used to define $D = \{x : x \in B, \|\dot{P}_z\| < \lambda\}$. The origin belongs to domain D since

$$\begin{aligned} \dot{P}_z &= \sum_{i=1}^r \dot{h}_i P_i = \sum_{i=1}^r \left(\frac{\partial h_i}{\partial z} \right)^T \frac{\partial z}{\partial x} \dot{x} P_i \\ &= \sum_{i=1}^r \left[\left(\frac{\partial h_i}{\partial z} \right)^T \frac{\partial z}{\partial x} (A_z + B_z F_z P_z^{-1}) x(t) \right] P_i \end{aligned} \quad (14)$$

depends on the state vector $x(t)$. Since $V(x) > 0$ and $\dot{V}(x) < 0$ in $D - \{0\}$, the equilibrium point $x=0$ is locally asymptotically stable, thus concluding the proof.

Remark: conditions for theorem 1 can be ensured for example with property 4, i.e. define $\Upsilon_{ij} = A_i P_j + B_i F_j + P_j A_i^T + F_j^T B_i^T$, find $P_i = P_i^T > 0$, F_i , $i \in \{1, \dots, r\}$ such that:

$$\frac{2}{r-1} \Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} > 0, \quad i, j \in \{1, \dots, r\}. \quad (15)$$

IV. MAIN RESULT

A precedent work [19] used a bound on the control law to derive conditions of stabilization. Therein a more interesting way to cope with the stabilization problem is used in order to remove this drawback. Let us focus on the real $\dot{z}_k \in \mathbb{R}$. In a general framework, the premise vector $z(t)$ depends linearly or not on the state vector $x(t)$. Thus:

$$\dot{z}_k = \left(\frac{\partial z_k}{\partial x} \right)^T \dot{x} = \left(\frac{\partial z_k}{\partial x} \right)^T (A_z + B_z F_z P_z^{-1}) x. \quad (16)$$

Therefore the goal is to find a “nice” bound $\beta_k > 0$ for:

$$\left| \frac{\partial w_0^k}{\partial z_k} \dot{z}_k \right| \leq \beta_k, \quad (17)$$

which is equivalent to:

$$\left| \frac{\partial w_0^k}{\partial z_k} \left(\frac{\partial z_k}{\partial x} \right)^T (A_z P_z + B_z F_z) P_z^{-1} x \right| \leq \beta_k. \quad (18)$$

Remark: it is also important to note that this is *not equivalent* to other approaches found in the literature [13, 14, 15] where the required condition is in the form of: $|\dot{h}_i(z(t))| \leq \phi_i$. In this later case, ϕ_i is given a priori and can only be checked a posteriori. In the presented case, the bounds β_k are *included* in the problem to solve.

In order for (18) to hold, we will introduce (19):

$$\left(\frac{\partial w_0^k}{\partial z_k} \left(\frac{\partial z_k}{\partial x} \right)^T (A_z P_z + B_z F_z) P_z^{-1} x \right)^2 \leq \beta_k^2. \quad (19)$$

Or equivalently:

$$\left(\frac{\partial w_0^k}{\partial z_k} \right)^2 \left(\frac{\partial z_k}{\partial x} \right)^T (A_z P_z + B_z F_z) P_z^{-1} x x^T P_z^{-1} (A_z P_z + B_z F_z)^T \frac{\partial z_k}{\partial x} - \beta_k^2 \leq 0 \quad (20)$$

Knowing that $\|x\|^2 \leq \lambda_x^2 \Leftrightarrow x x^T \leq \lambda_x^2 I$ (property 1 (1)), (20) holds if:

$$\lambda_x^2 \left(\frac{\partial w_0^k}{\partial z_k} \right)^2 \left(\frac{\partial z_k}{\partial x} \right)^T (A_z P_z + B_z F_z) P_z^{-1} P_z^{-1} (A_z P_z + B_z F_z)^T \frac{\partial z_k}{\partial x} - \beta_k^2 \leq 0 \quad (21)$$

Applying again property 1 to (21) gives:

$$\lambda_x^2 \left(\frac{\partial w_0^k}{\partial z_k} \right)^2 P_z^{-1} (A_z P_z + B_z F_z)^T \frac{\partial z_k}{\partial x} \left(\frac{\partial z_k}{\partial x} \right)^T (A_z P_z + B_z F_z) P_z^{-1} - \beta_k^2 I \leq 0 \quad (22)$$

At last, as the functions w_0^k are known explicitly, thus

bounding $\frac{\partial w_0^k}{\partial z_k}$ is direct. On the other side, $\frac{\partial z_k}{\partial x}$ represents the

linear or nonlinear mapping between $z(t)$ and $x(t)$, and it is also known. Therefore we can compute easily the bounds:

$$\left(\frac{\partial w_0^k}{\partial z_k} \right)^2 \left\| \frac{\partial z_k}{\partial x} \right\|^2 \leq \lambda_k^2 \Leftrightarrow \left(\frac{\partial w_0^k}{\partial z_k} \right)^2 \frac{\partial z_k}{\partial x} \left(\frac{\partial z_k}{\partial x} \right)^T \leq \lambda_k^2 I.$$

These bounds define regions in the state space:

$$R_k = \left\{ x \in \mathbb{R}^n, \left(\frac{\partial w_0^k}{\partial z_k} \right)^2 \left\| \frac{\partial z_k}{\partial x} \right\|^2 \leq \lambda_k^2 \right\}, \quad (23)$$

and it follows that:

$$\lambda_x^2 \lambda_k^2 P_z^{-1} (A_z P_z + B_z F_z)^T (A_z P_z + B_z F_z) P_z^{-1} - \beta_k^2 I \leq 0 \quad (24)$$

Then, multiplying left and right with P_z gives:

$$\lambda_x^2 \lambda_k^2 (A_z P_z + B_z F_z)^T (A_z P_z + B_z F_z) - \beta_k^2 P_z^2 \leq 0. \quad (25)$$

Considering now that $P_z \geq \delta I$ and using property 2 (2), (17) is satisfied if:

$$\lambda_x^2 \lambda_k^2 (A_z P_z + B_z F_z)^T (A_z P_z + B_z F_z) - \beta_k^2 \delta P_z \leq 0.$$

Thus with the help of the Schur complement:

$$\begin{aligned} &\lambda_x^2 \lambda_k^2 (A_z P_z + B_z F_z)^T \beta_k^{-2} \delta^{-1} (A_z P_z + B_z F_z) - P_z \leq 0 \\ \Leftrightarrow &\begin{bmatrix} P_z & (*) \\ \lambda_x \lambda_k (A_z P_z + B_z F_z) & \beta_k^2 \delta I \end{bmatrix} > 0 \end{aligned} \quad (26)$$

Now, coming back to (11) we can use

$$A_z P_z + B_z F_z + (A_z P_z + B_z F_z)^T - \sum_{k=1}^p \frac{\partial w_0^k}{\partial z_k} \dot{z}_k (P_{g_1(z,k)} - P_{g_2(z,k)}) < 0 \quad (27)$$

And we have the bound: $\left| \frac{\partial w_0^k}{\partial z_k} \dot{z}_k \right| \leq \beta_k$.

Thus using property 3 second part (3) we can derive the theorem 2. Note that introducing matrices $S_{zz}^k > 0$ we can write:

$$\begin{aligned} & \frac{\partial w_0^k}{\partial z_k} \dot{z}_k (P_{g_1(z,k)} - P_{g_2(z,k)}) \\ & \leq \frac{1}{2} \left(\beta_k^2 S_{zz}^k + (P_{g_1(z,k)} - P_{g_2(z,k)}) (S_{zz}^k)^{-1} (P_{g_1(z,k)} - P_{g_2(z,k)}) \right) \end{aligned} \quad (28)$$

Therefore, let us consider the following quantities:

$$\Gamma_{ij} = \begin{bmatrix} A_i P_j + B_i F_j + P_j A_i^T + F_j^T B_i^T + \frac{1}{2} \sum_{k=1}^p \beta_k^2 S_{ij}^k & (*) & \dots & (*) \\ P_{g_1(i,1)} - P_{g_2(i,1)} & -2S_{ij}^1 & 0 & \dots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ P_{g_1(i,p)} - P_{g_2(i,p)} & 0 & \dots & 0 & -2S_{ij}^p \end{bmatrix} \quad (29)$$

$$\Upsilon_{ij} = \begin{bmatrix} P_i & (*) \\ \lambda_i \lambda_j (A_i P_j + B_i F_j) & \beta_k^2 \delta I \end{bmatrix} \quad (30)$$

Theorem 2: consider the TS closed loop model (9) and expressions defined in (30) and (29). If there exist matrices P_i , F_i , $S_{ij}^k > 0$, $i, j \in \{1, \dots, r\}$, $k \in \{1, \dots, p\}$ such that the following LMI are satisfied:

$$P_i \geq \delta I, i \in \{1, \dots, r\} \quad (31)$$

$$\frac{2}{r-1} \Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} > 0, i, j \in \{1, \dots, r\} \quad (32)$$

$$\frac{2}{r-1} \Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0, i, j \in \{1, \dots, r\} \quad (33)$$

Then, the control law (8) stabilizes the TS open loop model (5) in a local domain. An estimation of this domain is given by

$$R = C \cap \left\{ \bigcap_{k=1}^p R_k \right\}$$

with C defined in (6) and R_k in (23).

Proof: from (27), repeating (28) for each term of the sum, gives and applying the Shur complement gives directly $\Gamma_{zz} < 0$.

(26) represents $\Upsilon_{zz} > 0$. Thus, conditions (32) and (33) just expresses the sum relaxation (4) to $\Upsilon_{zz} > 0$ and $\Gamma_{zz} < 0$.

Remark: notice that (31)~(33) are LMI constraints only if the scalars β_k^2 and δ are known.

A procedure to determine these values is possible, inspired from [22] for static output feedback. Recall that if the conditions of theorem 1 are satisfied it ensures the existence of a local stability domain for theorem 2. Therefore, the procedure is based on this remark.

A first step consists in obtaining a solution to theorem 1 conditions (15). The obtained gains give a control law that can be seen as stabilizing the “frozen” time-invariant continuous TS model. Thus bounds β_k^2 and δ can be directly obtained from this first step.

The second step is the solution to the theorem 2 conditions (31)~(33).

V. EXAMPLE

Example 1: Consider the following TS model [23]:

$$\dot{x}(t) = A_z x(t) = \sum_{i=1}^2 h_i(z(t)) (A_i x(t) + B_i u(t)) \quad (34)$$

with $A_1 = \begin{bmatrix} 3.6 & -1.6 \\ 6.2 & -4.3 \end{bmatrix}$, $A_2 = \begin{bmatrix} -0.35 & -1.6 \\ 6.2 & -4.3 \end{bmatrix}$, $B_1 = \begin{bmatrix} -0.45 \\ -3 \end{bmatrix}$,

$B_2 = \begin{bmatrix} -0.5 \\ -3 \end{bmatrix}$, $z_1(t) = x_1(t)$, $h_1 = w_0^1 = \frac{1 - \sin x_1}{2}$ and

$h_2 = w_1^1 = \frac{1 + \sin x_1}{2}$ defined in the compact set

$$C = \{x : |x_i| \leq \pi/2, i = 1, 2\}.$$

Fig. 1 shows the stabilization region (region $R_0 = \{x : x^T P_z^{-1} x \leq r\}$ enclosed by a solid line).

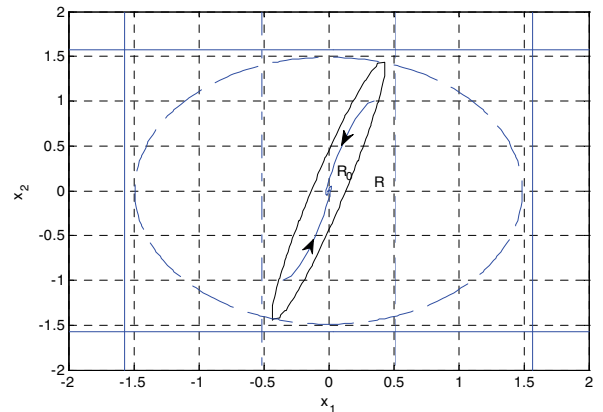


Figure 1: Stabilization region for TS model

The gains and the matrices involved in the control law are :

$$F_1 = [-30.9367 \quad -64.6603], F_2 = [-7.9679 \quad 0.8826]$$

$$P_1 = \begin{bmatrix} 14.1439 & 44.8202 \\ 44.8202 & 156.7774 \end{bmatrix}, P_2 = \begin{bmatrix} 14.0601 & 44.4733 \\ 44.4733 & 155.5855 \end{bmatrix}$$

$$\lambda_k = 0.19, \gamma = 1, \beta_k = 370 \text{ and } \lambda_x = 2.2214.$$

The control law evolution for initial states $x_0 = [0.35 \quad 1]$ can be seen in Fig. 2.

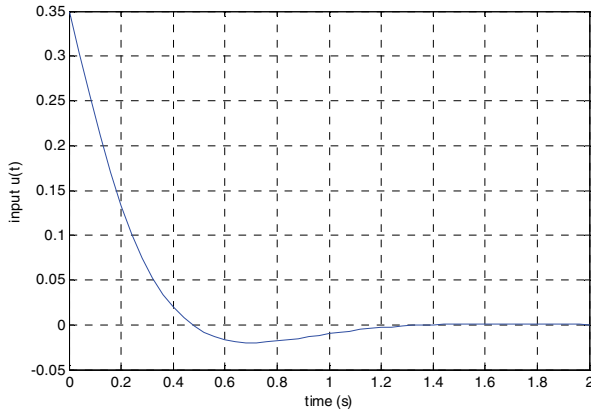


Fig. 2: Control law evolution.

Acknowledgments

The present paper has been supported by the International Campus on Safety and Intermodality in Transportation, the Nord-Pas-de-Calais Region, the European Community, the Regional Delegation for Research and Technology, the Ministry of Higher Education and Research, and the National Center for Scientific Research. The authors gratefully acknowledge the support of these institutions.

This work has been done during the one year PhD stay of Juntao Pan at LAMIH, University of Valenciennes Hainaut-Cambresis (France) under the China Scholarship Council Postgraduate scholarship program.

References

- [1]. T. Takagi and M. Sugeno, "Fuzzy identification of systems and its application to modeling and control", IEEE Trans. on System Man and Cybernetics, Vol.15(1), pp116-132. 1985.
- [2]. K. Tanaka and H.O. Wang, "Fuzzy control systems design and analysis. A linear matrix inequality approach". John Wiley and Sons, New York, USA. 2001.
- [3]. Zs. Lendek, T.M. Guerra, R. Babuska, B. De Schutter, "Stability analysis and nonlinear observer design using Takagi-Sugeno fuzzy models" – Studies in Computational Intelligence, Springer-Verlag. 2010
- [4]. G. Feng, "A Survey on Analysis and Design of Model-Based Fuzzy Control Systems", IEEE T. Fuzzy Systems, 14 (5), 676-697. 2006
- [5]. C. Scherer, S. Weiland, "Linear Matrix Inequalities in Control" (lecture notes). 2004
- [6]. A. Sala, C. Ariño, "Asymptotically necessary and sufficient conditions for stability and performance in fuzzy control: Applications of Poly's theorem" Fuzzy Sets and Systems, Vol. 158, (24), pp 2671-2686, 2007.
- [7]. M. Johansson, A. Rantzer, K. Arzen, "Piecewise quadratic stability of fuzzy systems", IEEE Trans. on Fuzzy Systems, Vol.7, pp713-722,1999.
- [8]. G. Feng, C.L. Chen, D. Sun, Y. Zhu, "H ∞ controller synthesis of fuzzy dynamic systems based on piecewise Lyapunov functions and bilinear

- matrix inequalities". IEEE Trans. On Fuzzy Systems, Vol. 13 (1), pp 94-103, 2005.
- [9]. T.M. Guerra and L. Vermeiren, "LMI-based relaxed non-quadratic stabilization conditions for nonlinear systems in Takagi-Sugeno's form", Automatica, Vol. 40(5), pp823-829. 2004.
- [10]. B.C. Ding, H.X. Sun, P. Yang, "Further studies on LMI-based relaxed stabilization conditions for nonlinear systems in Takagi-Sugeno's form". Automatica Vol. 42, pp503-508, 2006.
- [11]. A. Kruszewski, R. Wang, T.M. Guerra, "Non-quadratic stabilization conditions for a class of uncertain non linear discrete-time T-S fuzzy models: a new approach". IEEE Trans. on Automatic Control, 53 (2), pp. 606-611, 2008.
- [12]. T.M. Guerra, A. Kruszewski, M. Bernal, "Control Law Proposition for the Stabilization of Discrete Takagi-Sugeno Models". IEEE Trans. on Fuzzy Systems, 17 (3), 724-731, 2009.
- [13]. K. Tanaka, T. Hori, H.O. Wang, "A multiple Lyapunov function approach to stabilization of fuzzy control systems", IEEE Trans. on Fuzzy Systems, Vol. 11 (4), pp 582-589, 2003.
- [14]. M. Bernal, P. Hušek, V. Kučera, "Non quadratic stabilization of continuous-time systems in the Takagi-Sugeno form", Kybernetika, vol. 42 (6), pp. 665-672, 2006.
- [15]. L. Mozelli, R. Palhares, G.S.C. Avellar, "A systematic approach to improve multiple Lyapunov function stability and stabilization conditions for fuzzy systems", Information Sciences, Vol. 179(8), pp 1149-1162.
- [16]. B.J. Rhee, S. Won, "A new fuzzy Lyapunov function approach for a Takagi-Sugeno fuzzy control system design", Fuzzy Sets & Systems, Vol. 157(9), pp1211-1228, 2006.
- [17]. T.M. Guerra and M. Bernal, "A way to escape from the quadratic framework", in Proc. FUZZ-IEEE Conference, Jeju, Korea, 2009
- [18]. M. Bernal, T.M. Guerra, "Generalized non-quadratic stability of nonlinear models in the Takagi-Sugeno form", IEEE T. on Fuzzy Systems 18 (4), 815-822. 2010
- [19]. M. Bernal, T.M. Guerra, A. Jaadari, "Non-quadratic stabilization of Takagi-Sugeno models: a local point of view", Fuzz'IEEE Barcelona, Spain. 2010
- [20]. H.D. Tuan, P. Apkarian, T. Narikiyo, and Y. Yamamoto. "Parameterized linear matrix inequality techniques in fuzzy control system design". IEEE Trans. on Fuzzy Systems, 9(2), 324-332. 2001
- [21]. T. Taniguchi, K. Tanaka, H.O. Wang, "Model construction, rule reduction and robust compensation for generalized form of Takagi-Sugeno fuzzy systems", IEEE T. Fuzzy Systems, 9 (4), 525-537. 2001
- [22]. D. Mehdi, E. K. Boukas, O. Bachelier, "Static output feedback design for uncertain linear discrete time systems," IMA J. Mathematical Control Information, 21 (1), 1-13, 2004.
- [23]. L. Mozelli, R. Palhares, G.S.C. Avellar, "A systematic approach to improve multiple Lyapunov function stability and stabilization conditions for fuzzy systems", Information Sciences, Vol. 179(8), pp 1149-1162.