

Actuator Delayed Active Vehicle Suspension Control: A T-S Fuzzy Approach

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Abstract—This paper focuses on fuzzy H_∞ controller design for uncertain active suspension systems with actuator delay based on Takagi-Sugeno (T-S) model approach. This dynamic system is presented by taking into account the sprung and unsprung mass variations, the actuator delay, and the suspension performance. The fuzzy H_∞ controller is designed such that the resulting T-S fuzzy system is asymptotically stable and guarantees H_∞ performance, and simultaneous satisfying the constraint performance. The existence condition of fuzzy H_∞ control is obtained in terms of linear matrix inequalities (LMIs) and can be solved by using the standard software. A quarter-car suspension model is provided to validate the effectiveness of the proposed design procedures.

Keywords: Fuzzy control, T-S model, H_∞ control, actuator delay, uncertain active suspension systems.

I. INTRODUCTION

The vehicle suspension is a crucial component that is used to support the vehicle weight, to provide effective isolation of the chassis from road excitations, to keep tyre contact with the ground, and to maintain the wheels in appropriate position on the road surface. Recently, considerable attention has been paid to the hot issue how to achieve the required suspension performances, namely, ride comfort, road handling, and suspension deflection [1]. Recently, many active suspension control approaches have been presented to handle the tradeoff [2]–[7]. It has been proved that H_∞ control method is feasible to achieve a compromised vehicle performance in [6], [7].

For active suspension systems, the vehicle sprung and unsprung masses may vary according to the different loading conditions, such as the payload and number of vehicle passengers. If we do not take into account the varying vehicle sprung and unsprung masses in the control design process, the performance of the vehicle suspension systems will be affected. More recently, the authors in [8], [9] used the polytopic parameter uncertainties to model the varying vehicle sprung or unsprung masses. Among [8], [10], the parameter-dependent controllers has been proposed for the quarter-car suspension systems with sprung mass variation. The parameter-independent sampled-data H_∞ controller design strategy has been provided in [9] for quarter-car suspension systems with both sprung and unsprung mass variations. In particular, the authors in [11] presented T-S model-based fuzzy control design

approach for electrohydraulic active suspension systems with input constraint.

This paper is concerned with the problem of fuzzy H_∞ control for uncertain active suspension systems with actuator delay based on T-S model approach. The parallel-distributed compensation (PDC) scheme is used to develop fuzzy H_∞ performance analysis condition for the proposed T-S fuzzy systems presenting the uncertain active suspension systems. Then, the fuzzy H_∞ controller is designed to ensure the resulting T-S fuzzy system is asymptotically stable and satisfy H_∞ performance, and guarantee the suspension performance. The linear matrix inequality (LMI)-based condition of fuzzy H_∞ controller is derived and can be solved by using the standard software. Finally, a quarter-car suspension model is provided to demonstrate the effectiveness of the proposed design approach.

II. PROBLEM FORMULATION

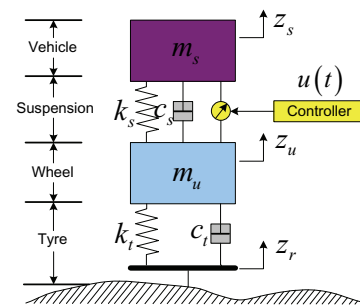


Fig. 1. Quarter-car model

Over the past years, many researchers became to investigate the modeling and control design for active suspension systems since these systems play an important role in guaranteeing the suspension performance, such as ride comfort, road holding, and suspension deflection. A quarter-car model has been used widely for the research in the literature and is shown in Fig. 1, in which m_s is used to denote the sprung mass, which represents the car chassis; m_u is the unsprung mass, which represents mass of the wheel assembly; $u(t)$ is used to stand for the active input of the suspension system; z_s and z_u are used to

denote the displacements of the sprung and unsprung masses, respectively; z_r is used to denote the road displacement input; c_s and k_s are damping and stiffness of the suspension system, respectively; k_t and c_t stand for compressibility and damping of the pneumatic tyre, respectively. Then, the dynamic equation of the suspension model is established as:

$$\begin{aligned} & m_u \ddot{z}_u(t) + c_s [\dot{z}_u(t) - \dot{z}_s(t)] + k_s [z_u(t) - z_s(t)] \\ & + k_t [z_u(t) - z_r(t)] + c_t [\dot{z}_u(t) - \dot{z}_r(t)] \\ = & -u(t), \\ & m_s \ddot{z}_s(t) + c_s [\dot{z}_s(t) - \dot{z}_u(t)] + k_s [z_s(t) - z_u(t)] \\ = & u(t). \end{aligned} \quad (1)$$

Denote $x_1(t) = z_s(t) - z_u(t)$ as the suspension deflection, $x_2(t) = z_u(t) - z_r(t)$ as the tire deflection, $x_3(t) = \dot{z}_s(t)$ as the sprung mass speed, $x_4(t) = \dot{z}_u(t)$ as the unsprung mass speed, and $w(t) = \dot{z}_r(t)$ as the disturbance input, respectively. The equations in (1) can be rewritten as:

$$\dot{x}(t) = A(t)x(t) + B_1(t)w(t) + B(t)u(t), \quad (2)$$

where

$$\begin{aligned} A(t) &= \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & -\frac{k_t}{m_u} & \frac{c_s}{m_u} & -\frac{c_s+c_t}{m_u} \end{bmatrix}, \\ B(t) &= \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s} \\ -\frac{1}{m_u} \end{bmatrix}, \quad B_1(t) = \begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{c_t}{m_u} \end{bmatrix}, \\ x(t) &= [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^T. \end{aligned}$$

For suspension systems control design problems, their performance, such as ride comfort and suspension deflection, road holding should be taken into account. For quarter car suspension model showed in Fig. 1, it is extensively accepted that ride comfort can be generally quantified by the body acceleration in the vertical direction, hence it is practically acceptable to choose body acceleration, $\ddot{z}_s(t)$, as the first control output. Therefore, one of our main objectives is to minimize the vertical acceleration $\ddot{z}_s(t)$, which means that the passengers feel comfortable. Here we recall H_∞ control method since the value of H_∞ norm is defined as an upper bound of the root-mean-square gain. Our main goal is to minimize the H_∞ norm of the transfer function from the disturbance $w(t)$ to the control output $z_1(t) = \ddot{z}_s(t)$ with an emphasis on ride comfort improving. Meanwhile the following required performance should be taken into account:

I) The suspension deflection cannot exceed a maximum value constrained due to mechanical structure, that is,

$$|z_s(t) - z_u(t)| \leq z_{\max}, \quad (3)$$

where $z_{\max}$ is the maximum suspension deflection.

II) The dynamic tyre load has to be less than the static tyre load in order to ensure a firm uninterrupted contact of the

wheels on the road,

$$k_t(z_u(t) - z_r(t)) < (m_s + m_u)g. \quad (4)$$

Based on the above discussion, we choose the body acceleration $\ddot{z}_s(t)$ as performance control output, the suspension stroke $z_s(t) - z_u(t)$ and relative dynamic tire load $k_t(z_u(t) - z_r(t)) / (m_s + m_u)g$ as constrained control output $z_2(t)$. Therefore, we can use the following system to present the active vehicle suspension system:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B_1(t)w(t) + B(t)u(t), \\ z_1(t) &= C_1(t)x(t) + D_1(t)u(t), \\ z_2(t) &= C_2(t)x(t), \end{aligned} \quad (5)$$

where $A(t)$, $B_1(t)$ and $B(t)$ are defined in (2), and

$$\begin{aligned} C_1(t) &= \begin{bmatrix} -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \end{bmatrix}, \\ D_1(t) &= \frac{1}{m_s}, \\ C_2(t) &= \begin{bmatrix} \frac{1}{z_{\max}} & 0 & 0 & 0 \\ 0 & \frac{k_t}{(m_s+m_u)g} & 0 & 0 \end{bmatrix}. \end{aligned}$$

It can be found that the suspension suspension system in (5) is an uncertain model in that the sprung mass m_s and the unsprung mass m_u vary in given ranges. When carrying out the modeling the suspension systems, the actuator time delay should be taken into account, which can be expressed as:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B_1(t)w(t) + B(t)u(t-d(t)), \\ z_1(t) &= C_1(t)x(t) + D_1(t)u(t-d(t)), \\ z_2(t) &= C_2(t)x(t), \\ x(t) &= \phi(t), \quad t \in [-\bar{d}, 0], \end{aligned} \quad (6)$$

where $\phi(t)$ is a vector-valued initial continuous function defined on $t \in [-\bar{d}, 0]$. $d(t)$ denotes the time-varying delay satisfying

$$0 \leq d(t) \leq \bar{d}, \quad d(t) \leq \mu. \quad (7)$$

The sprung mass m_s and the unsprung mass m_u are assumed to be uncertainties and replaced by $m_s(t)$ and $m_u(t)$, which vary in a given range, i.e. $m_s(t) \in [m_{s\min}, m_{s\max}]$ and $m_u(t) \in [m_{u\min}, m_{u\max}]$. This means the uncertain mass $m_s(t)$ is bounded by its minimum value $m_{s\min}$ and its maximum value $m_{s\max}$. In addition, the uncertain mass $m_u(t)$ is bounded by its minimum value $m_{u\min}$ and its maximum value $m_{u\max}$. Next, we can obtain the values of $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ from $m_s(t) \in [m_{s\min}, m_{s\max}]$ and $m_u(t) \in [m_{u\min}, m_{u\max}]$. Then we have

$$\begin{aligned} \text{maximum} \frac{1}{m_s(t)} &= \frac{1}{m_{s\min}} =: \hat{m}_s, \\ \text{minimum} \frac{1}{m_s(t)} &= \frac{1}{m_{s\max}} =: \check{m}_s, \\ \text{maximum} \frac{1}{m_u(t)} &= \frac{1}{m_{u\min}} =: \hat{m}_u, \\ \text{minimum} \frac{1}{m_u(t)} &= \frac{1}{m_{u\max}} =: \check{m}_u. \end{aligned}$$

We can represent $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ by

$$\begin{aligned}\frac{1}{m_s(t)} &= M_1(\xi_1(t))\hat{m}_s + M_2(\xi_1(t))\check{m}_s, \\ \frac{1}{m_u(t)} &= N_1(\xi_2(t))\hat{m}_u + N_2(\xi_2(t))\check{m}_u,\end{aligned}$$

where $\xi_1(t) = \frac{1}{m_s(t)}$ and $\xi_2(t) = \frac{1}{m_u(t)}$,

$$\begin{aligned}M_1(\xi_1(t)) + M_2(\xi_1(t)) &= 1, \\ N_1(\xi_2(t)) + N_2(\xi_2(t)) &= 1.\end{aligned}$$

The membership functions can be calculated as

$$\begin{aligned}M_1(\xi_1(t)) &= \frac{\frac{1}{m_s(t)} - \check{m}_s}{\hat{m}_s - \check{m}_s}, & M_2(\xi_1(t)) &= \frac{\hat{m}_s - \frac{1}{m_s(t)}}{\hat{m}_s - \check{m}_s}, \\ N_1(\xi_2(t)) &= \frac{\frac{1}{m_u(t)} - \check{m}_u}{\hat{m}_u - \check{m}_u}, & N_2(\xi_2(t)) &= \frac{\hat{m}_u - \frac{1}{m_u(t)}}{\hat{m}_u - \check{m}_u}.\end{aligned}$$

We name the member functions **Heavy**, **Light**, **Heavy** and **Light**. Then, the uncertain systems in (6) is represented by the following fuzzy model.

Model Rule 1: IF $\xi_1(t)$ is **Heavy** and $\xi_2(t)$ is **Heavy**, THEN

$$\begin{aligned}\dot{x}(t) &= A_1x(t) + B_1u(t-d(t)) + B_{11}w(t), \\ z_1(t) &= C_{11}x(t) + D_{11}u(t-d(t)), \\ z_2(t) &= C_{21}x(t),\end{aligned}$$

matrices $A_1, B_1, B_{11}, C_{11}, D_{11}$ and C_{21} are obtained by replacing $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ with matrices $A(t), B(t), B_1(t), C_1(t), D_1(t)$ and $C_2(t)$ with $\hat{m}_s$ and $\hat{m}_u$ respectively.

Model Rule 2: IF $\xi_1(t)$ is **Heavy** and $\xi_2(t)$ is **Light**, THEN

$$\begin{aligned}\dot{x}(t) &= A_2x(t) + B_2u(t-d(t)) + B_{12}w(t), \\ z_1(t) &= C_{12}x(t) + D_{12}u(t-d(t)), \\ z_2(t) &= C_{22}x(t),\end{aligned}$$

matrices $A_2, B_2, B_{12}, C_{12}, D_{12}$ and C_{22} are obtained by replacing $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ with matrices $A(t), B(t), B_1(t), C_1(t), D_1(t)$ and $C_2(t)$ with $\hat{m}_s$ and $\hat{m}_u$ respectively.

Model Rule 3: IF $\xi_1(t)$ is **Light** and $\xi_2(t)$ is **Heavy**, THEN

$$\begin{aligned}\dot{x}(t) &= A_3x(t) + B_3u(t-d(t)) + B_{13}w(t), \\ z_1(t) &= C_{13}x(t) + D_{13}u(t-d(t)), \\ z_2(t) &= C_{23}x(t),\end{aligned}$$

matrices $A_3, B_3, B_{13}, C_{13}, D_{13}$ and C_{23} are obtained by replacing $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ with matrices $A(t), B(t), B_1(t), C_1(t), D_1(t)$ and $C_2(t)$ with $\check{m}_s$ and $\check{m}_u$ respectively.

Model Rule 4: IF $\xi_1(t)$ is **Light** and $\xi_2(t)$ is **Light**, THEN

$$\begin{aligned}\dot{x}(t) &= A_4x(t) + B_4u(t-d(t)) + B_{14}w(t), \\ z_1(t) &= C_{14}x(t) + D_{14}u(t-d(t)), \\ z_2(t) &= C_{24}x(t),\end{aligned}$$

matrices $A_4, B_4, B_{14}, C_{14}, D_{14}$ and C_{24} are obtained by replacing $\frac{1}{m_s(t)}$ and $\frac{1}{m_u(t)}$ with matrices $A(t), B(t), B_1(t), C_1(t), D_1(t)$ and $C_2(t)$ with $\check{m}_s$ and $\check{m}_u$ respectively.

By fuzzy blending, the overall fuzzy model is inferred as follows:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^4 h_i(\xi(t)) [A_i x(t) + B_i u(t-d(t)) + B_{1i} w(t)], \\ z_1(t) &= \sum_{i=1}^4 h_i(\xi(t)) [C_{1i} x(t) + D_{1i} u(t-d(t))], \\ z_2(t) &= \sum_{i=1}^4 h_i(\xi(t)) C_{2i} x(t),\end{aligned}\tag{8}$$

where

$$\begin{aligned}h_1(\xi(t)) &= M_1(\xi_1(t)) \times N_1(\xi_2(t)), \\ h_2(\xi(t)) &= M_1(\xi_1(t)) \times N_2(\xi_2(t)), \\ h_3(\xi(t)) &= M_2(\xi_1(t)) \times N_1(\xi_2(t)), \\ h_4(\xi(t)) &= M_2(\xi_1(t)) \times N_2(\xi_2(t)).\end{aligned}$$

It is obvious that the fuzzy weighting functions $h_i(\xi(t))$ satisfy

$$h_i(\xi(t)) \geq 0, \quad \sum_{i=1}^4 h_i(\xi(t)) = 1.$$

In this study, we consider the design of fuzzy controllers. By using parallel distributed compensation, we suppose the following fuzzy controller:

Control Rule j: IF $\xi_1(t)$ is **Heavy (Light)** and $\xi_2(t)$ is **Heavy (Light)**,

THEN $u(t) = K_j x(t) (j = 1, 2, 3, 4)$.

Hence, the overall fuzzy control law is represented by

$$u(t) = \sum_{j=1}^4 h_j(\xi(t)) K_j x(t)\tag{9}$$

where $K_j (j = 1, 2, 3, 4)$ are the local control gains and $u(t-d(t)) = \sum_{j=1}^4 h_j(\xi(t-d(t))) K_j x(t-d(t))$. Therefore, in this paper, we assume that $h_j(\xi(t-d(t)))$ is well defined for $t \in [-d, 0]$, and $h_j(\xi(t-d(t))) \geq 0, (j = 1, 2, 3, 4)$ $\sum_{j=1}^4 h_j(\xi(t-d(t))) = 1$. For simplicity, the following notations will be used:

$$h_i =: h_i(\xi(t)), \quad h_j^d =: h_j(\xi(t-d(t))).$$

Applying the fuzzy controller (9) to system (8) yields the

closed-loop system:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^4 \sum_{j=1}^4 h_i h_j^d [A_i x(t) + B_i K_j x(t-d(t)) + B_{1i} w(t)], \\ z_1(t) &= \sum_{i=1}^4 \sum_{j=1}^4 h_i h_j^d [C_{1i} x(t) + D_{1i} K_j x(t-d(t))], \\ z_2(t) &= \sum_{i=1}^4 h_i C_{2i} x(t).\end{aligned}\tag{10}$$

It should be pointed out that the T-S fuzzy system in (10) can be established due to the sprung $m_s(t)$ and unsprung $m_u(t)$ can be measured in practice. In our study, the sector nonlinearity method [12] is used to analysis the variations of the sprung $m_s(t)$ and unsprung $m_u(t)$ to present the T-S fuzzy system in (10).

Without loss of generality, It is assumed, $w \in L_2[0, \infty)$, and $\|w\|_2^2 \leq w_{\max} < \infty$. The main object of this subsection is just to design the feedback gain matrices K_j ($j = 1, 2, 3, 4$) such that the following requirements are satisfied: the closed-loop system is asymptotically stable; under zero initial condition, the closed-loop system guarantees that $\|z_1\|_2 < \gamma \|w\|_2$ for all nonzero $w \in L_2[0, \infty)$, where $\gamma > 0$ is a prescribed scalar; the following control output constraints are guaranteed:

$$|\{z_2(t)\}_q| \leq 1, \quad q = 1, 2. \quad (11)$$

III. FUZZY CONTROLLER DESIGN

In this section, fuzzy H_∞ state-feedback controller is derived for the active suspension system with actuator delay such that the closed-loop system in (10) is asymptotically stable and can also guarantee a prescribed gain from disturbance $w(t)$ to performance output z_1 while keeping the suspension stroke and tire deflection constraints satisfied. Based on the Lyapunov stability, we have the following theorem.

Theorem 1: Consider the closed-loop system in (10). For given scalars $\bar{d} > 0$, μ and matrices K_j , if there exist matrices $P > 0$, $Q > 0$, $S > 0$, $R > 0$, N_j , and M_j with appropriate dimensions ($j = 1, 2, 3, 4$) such that the following LMIs hold for $q = 1, 2$:

$$\begin{bmatrix} \Phi_{11}^{ij} & \Phi_{12}^{1ij} & \Phi_{13}^{ij} & \Phi_{14}^{ij} \\ 0 & -R & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -R \end{bmatrix} < 0, \quad (12)$$

$$\begin{bmatrix} \Phi_{11}^{ij} & \Phi_{12}^{2ij} & \Phi_{13}^{ij} & \Phi_{14}^{ij} \\ 0 & -R & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -R \end{bmatrix} < 0, \quad (13)$$

$$\begin{bmatrix} -P & \sqrt{\rho}\{C_{2i}\}_q^T \\ * & -I \end{bmatrix} < 0, \quad (14)$$

where

$$\begin{aligned} \Phi_{11}^{ij} &= \begin{bmatrix} \Xi_{11}^{ij} & \Xi_{12}^{ij} & \Xi_{13}^{ij} & \Xi_{14}^{ij} \\ * & \Xi_{22}^{ij} & \Xi_{23}^{ij} & \Xi_{24}^{ij} \\ * & * & \Xi_{33}^{ij} & -N_4^T \\ * & * & * & -\gamma^2 I \end{bmatrix}, \\ \Phi_{12}^{1ij} &= \begin{bmatrix} \sqrt{\bar{d}}M_1 \\ \sqrt{\bar{d}}M_2 \\ \sqrt{\bar{d}}M_3 \\ \sqrt{\bar{d}}M_4 \end{bmatrix}, \Phi_{12}^{2ij} = \begin{bmatrix} \sqrt{\bar{d}}N_1 \\ \sqrt{\bar{d}}N_2 \\ \sqrt{\bar{d}}N_3 \\ \sqrt{\bar{d}}N_4 \end{bmatrix}, \\ \Phi_{13}^{ij} &= \begin{bmatrix} C_{1i}^T \\ K_j^T D_{1i}^T \\ 0 \\ 0 \end{bmatrix}, \Phi_{14}^{ij} = \begin{bmatrix} \sqrt{\bar{d}}A_i^T R \\ \sqrt{\bar{d}}K_j^T B_i^T R \\ 0 \\ \sqrt{\bar{d}}B_{1i}^T R \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \Xi_{11}^{ij} &= PA_i + A_i^T P + Q + S + M_1 + M_1^T, \\ \Xi_{12}^{ij} &= PB_i K_j - M_1 + M_2^T + N_1, \\ \Xi_{13}^{ij} &= M_3^T - N_1, \Xi_{14}^{ij} = PB_{1i} + M_4^T, \\ \Xi_{22}^{ij} &= -(1 - \mu)S + N_2 + N_2^T - M_2 - M_2^T, \\ \Xi_{23}^{ij} &= N_3^T - N_2 - M_3^T, \Xi_{24}^{ij} = N_4^T - M_4^T, \\ \Xi_{33}^{ij} &= -Q - N_3 - N_3^T. \end{aligned}$$

Then,

(1) the closed-loop system is asymptotically stable;

(2) the performance $\|T_{z_1 w}\|_\infty < \gamma$ is minimized subject to output constraints (11) with the disturbance energy under the bound $w_{\max} = (\rho - V(0))/\gamma^2$, where $T_{z_1 w}$ denotes the closed-loop transfer function from the road disturbance $w(t)$ to the control output $z_1(t)$.

Proof. Considering the Lyapunov-Krasovskii functional as follows:

$$\begin{aligned} V(t) &= x^T(t)Px(t) + \int_{t-\bar{d}}^t x^T(s)Qx(s)ds \\ &\quad + \int_{t-d(t)}^t x^T(s)Sx(s)ds \\ &\quad + \int_{-\bar{d}}^0 \int_{t+\alpha}^t \dot{x}^T(s)R\dot{x}(s)dsd\alpha. \end{aligned} \quad (15)$$

For any appropriately dimensioned matrices M and N , the following equalities hold directly according to Newton-Leibniz formula:

$$\begin{aligned} \eta_1(t) &= 2\bar{\xi}^T(t)M \\ &\quad \left(x(t) - x(t-d(t)) - \int_{t-d(t)}^t \dot{x}(s)ds \right) = 0, \\ \eta_2(t) &= 2\bar{\xi}^T(t)N \\ &\quad \left(x(t-d(t)) - x(t-\bar{d}) - \int_{t-\bar{d}}^{t-d(t)} \dot{x}(s)ds \right) = 0, \end{aligned}$$

where

$$\begin{aligned} M &= [M_1^T \ M_2^T \ M_3^T \ M_4^T]^T, \\ N &= [N_1^T \ N_2^T \ N_3^T \ N_4^T]^T, \\ \bar{\xi}^T(t) &= [x^T(t) \ x^T(t-d(t)) \ x^T(t-\bar{d}) \ w^T(t)]. \end{aligned}$$

We first establish the H_∞ performance of the system in (10) under zero initial conditions. Adding $\eta_1(t)$ and $\eta_2(t)$ into $\dot{V}(t)$, we have

$$\begin{aligned} &\dot{V}(t) + z_1^T(t)z_1(t) - \gamma^2 w^T(t)w(t) \\ &\leq \sum_{i=1}^4 \sum_{j=1}^4 h_i h_j^d \bar{\xi}^T(t) \left[\frac{d(t)}{\bar{d}} (\Xi_{ij} + \bar{d}MR^{-1}M^T) \right. \\ &\quad \left. + \frac{\bar{d}-d(t)}{\bar{d}} (\Xi_{ij} + \bar{d}NR^{-1}N^T) \right] \bar{\xi}(t), \end{aligned}$$

where

$$\begin{aligned} \tilde{\Xi}_{ij} = & \begin{bmatrix} \Xi_{11}^{ij} & \hat{\Xi}_{12}^{ij} & \Xi_{13}^{ij} & \Xi_{14}^{ij} \\ * & \Xi_{22}^{ij} & \Xi_{23}^{ij} & \Xi_{24}^{ij} \\ * & * & \Xi_{33}^{ij} & -N_4^T \\ * & * & * & -\gamma^2 I \end{bmatrix} \\ & + \begin{bmatrix} C_{1i}^T \\ K_j^T D_{1i}^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} C_{1i}^T \\ K_j^T D_{1i}^T \\ 0 \\ 0 \end{bmatrix}^T \\ & + \begin{bmatrix} A_i^T \\ K_j^T B_i^T \\ 0 \\ B_{1i}^T \end{bmatrix} \bar{d} R \begin{bmatrix} A_i^T \\ K_j^T B_i^T \\ 0 \\ B_{1i}^T \end{bmatrix}^T. \end{aligned}$$

According to Schur complement and the above method, we develop

$$\dot{V}(t) + z_1^T(t)z_1(t) - \gamma^2 w^T(t)w(t) < 0, \quad (16)$$

for all nonzero $w \in L_2[0, \infty)$. Under zero initial conditions, we have $V(0) = 0$ and $V(\infty) \geq 0$. Integrating both sides of (16) yields $\|z_1\|_2 < \gamma \|w\|_2$ for all nonzero $w \in L_2[0, \infty)$, and the H_∞ performance is established. Then, we can show that system (10) is stable with $w(t) = 0$ after using above similar proof.

The rest proof of Theorem 1 is cut off due to space limitation. Please refer to the proof of [9].

In what follows, the fuzzy H_∞ controller existence condition for the active suspension system in (10) is presented based on fuzzy H_∞ performance analysis criterion in Theorem 1.

Theorem 2: Consider the closed-loop system in (10). For given scalars $\bar{d} > 0$ and μ , if there exist matrices $\bar{P} > 0$, $\bar{Q} > 0$, $\bar{S} > 0$, $\bar{R} > 0$, Y_j , $\bar{N}_j$, and $\bar{M}_j$ with appropriate dimensions such that the following LMIs hold for $q = 1, 2$:

$$\begin{bmatrix} \bar{\Phi}_{11}^{ij} & \bar{\Phi}_{12}^{ij} & \bar{\Phi}_{13}^{ij} & \bar{\Phi}_{14}^{ij} \\ 0 & \bar{R} - 2\bar{P} & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -R \end{bmatrix} < 0, \quad (17)$$

$$\begin{bmatrix} \bar{\Phi}_{11}^{ij} & \bar{\Phi}_{12}^{2ij} & \bar{\Phi}_{13}^{ij} & \bar{\Phi}_{14}^{ij} \\ 0 & \bar{R} - 2\bar{P} & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -R \end{bmatrix} < 0, \quad (18)$$

$$\begin{bmatrix} -\bar{P} & \sqrt{\rho}\bar{P}\{C_{2i}\}_q^T \\ * & -I \end{bmatrix} < 0, \quad (19)$$

where

$$\begin{aligned} \bar{\Phi}_{11}^{ij} = & \begin{bmatrix} \Xi_{11}^{ij} & \Xi_{12}^{ij} & \Xi_{13}^{ij} & \Xi_{14}^{ij} \\ * & \Xi_{22}^{ij} & \Xi_{23}^{ij} & \Xi_{24}^{ij} \\ * & * & \Xi_{33}^{ij} & -N_4^T \\ * & * & * & -\gamma^2 I \end{bmatrix}, \\ \bar{\Phi}_{12}^{1ij} = & \begin{bmatrix} \sqrt{\bar{d}}\bar{M}_1 \\ \sqrt{\bar{d}}\bar{M}_2 \\ \sqrt{\bar{d}}\bar{M}_3 \\ \sqrt{\bar{d}}\bar{M}_4 \end{bmatrix}, \bar{\Phi}_{12}^{2ij} = \begin{bmatrix} \sqrt{\bar{d}}\bar{N}_1 \\ \sqrt{\bar{d}}\bar{N}_2 \\ \sqrt{\bar{d}}\bar{N}_3 \\ \sqrt{\bar{d}}\bar{N}_4 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \bar{\Phi}_{13}^{ij} = & \begin{bmatrix} \bar{P}C_{1i}^T \\ Y_j^T D_{1i}^T \\ 0 \\ 0 \end{bmatrix} \bar{\Phi}_{14}^{ij} = \begin{bmatrix} \sqrt{\bar{d}}A_i^T \\ \sqrt{\bar{d}}Y_j^T B_i^T \\ 0 \\ \sqrt{\bar{d}}B_{1i}^T \end{bmatrix}, \\ \bar{\Xi}_{11}^{ij} = & A_i \bar{P} + \bar{P} A_i^T + \bar{Q} + \bar{S} + \bar{M}_1 + \bar{M}_1^T, \\ \bar{\Xi}_{12}^{ij} = & B_i Y_j - \bar{M}_1 + \bar{M}_2^T + \bar{N}_1, \\ \bar{\Xi}_{13}^{ij} = & \bar{M}_3^T - \bar{N}_1, \bar{\Xi}_{14}^{ij} = B_{1i} + \bar{M}_4^T, \\ \bar{\Xi}_{22}^{ij} = & -(1 - \mu) \bar{S} + \bar{N}_2 + \bar{N}_2^T - \bar{M}_2 - \bar{M}_2^T, \\ \bar{\Xi}_{23}^{ij} = & \bar{N}_3^T - \bar{N}_2 - \bar{M}_3^T, \bar{\Xi}_{24}^{ij} = \bar{N}_4^T - \bar{M}_4^T, \\ \bar{\Xi}_{33}^{ij} = & -Q - \bar{N}_3 - \bar{N}_3^T. \end{aligned}$$

Then a controller in the form of (9) exists, such that

- (1) the closed-loop system is asymptotically stable;
- (2) the performance $\|T_{z_1 w}\|_\infty < \gamma$ is minimized subject to output constraint (11) with the disturbance energy under the bound $w_{\max} = (\rho - V(0))/\gamma^2$.

Moreover, if inequalities (17)–(19) have a feasible solution, then the control gain K_j in (9) is given by $K_j = Y_j \bar{P}^{-1}$.

The proof is cut off due to space limitation. Contact the authors for the detailed proof.

IV. SIMULATION RESULTS

In this section, a quarter vehicle active suspension system is exploited to demonstrate the effectiveness of the proposed approach. The quarter-car suspension model parameters are listed in Table I for the following controller design. The sprung

TABLE I
QUARTER-CAR MODEL PARAMETERS

k_s	k_t	c_s	c_t
42720N/m	101115N/m	1095Ns/m	14.6Ns/m

mass $m_s(t)$ is assumed to belong to the range [873kg, 1073kg] and the unsprung mass $m_u(t)$ is assumed to belong to [104kg, 124kg]. In this study, the maximum allowable suspension stroke is assumed as $z_{\max} = 0.08$ m with $\rho = 1$. For the actuator delay $d(t) = 5 + 5 \sin(\frac{1}{50})$ ms satisfying $\bar{d} = 10$ ms and $\mu = 0.1$, we consider fuzzy H_∞ controller design for the uncertain active suspension systems in (10). By using the convex optimization, it is found that the minimum guaranteed closed-loop H_∞ performance index $\gamma_{\min}$ is 5.9805 and the fuzzy controller gain matrices

$$K_j = 10^4 \times \begin{bmatrix} -8.0250 & 4.0130 & -3.3667 & 0.2256 \end{bmatrix}, \quad (20)$$

where $j = 1, 2, 3, 4$.

It is expected that the desired the fuzzy H_∞ controller with the parameters in (20) can be designed such that: 1) the sprung mass acceleration $z_1(t)$ is as small as possible; 2) the suspension deflection is below the maximum allowable suspension stroke $z_{\max} = 0.08$ m, which means that $x_1(t)/z_{\max}$ below 1; 3) the relation dynamic tire load $k_t x_2(t)/(m_s(t) + m_u(t))g < 1$. We first consider the following test road disturbance as

$$z_r(t) = 0.0254 \sin 2\pi t + 0.005 \sin 10.5\pi t + 0.001 \sin 21.5\pi t \text{ (m)} \quad (21)$$

As discussed in [11], this road disturbance is close to the car body resonance frequency (1Hz), with high-frequency disturbance added to simulate the rough road surface. In order to carry out the simulation for the fuzzy H_∞ controller for the system in (10), the variational sprung mass $m_s(t)$ and the variational unsprung mass $m_u(t)$ are regarded as $m_s(t) = 973 + 100 \sin(t)$ kg and $m_u(t) = 114 + 10 \cos(t)$ kg for deriving the fuzzy membership functional $h_i(\xi(t))$ ($i = 1, 2, 3, 4$). By using the fuzzy H_∞ controller with the parameters in (20), we derive the corresponding closed-loop fuzzy system. Fig. 2 (a) depicts the responses of body vertical accelerations for the open- (passive) and closed-loop (active) systems and Fig. 2 (b) plots the responses of suspension stroke and tire deflection constraint for the passive and active systems. It is observed from Fig. 2 (a) that the proposed fuzzy H_∞ control strategy reduces the sprung mass acceleration significantly compared to the passive suspension under the same road disturbance. The designed fuzzy H_∞ controller can achieve the less value of the maximum body acceleration for the active suspension system than the passive system, and passenger acceleration in the active suspension system is reduced significantly, which guarantee better ride comfort. In addition, we can see from Fig. 2 (b) that the suspension deflection constrain $x_1(t)/z_{\max} < 1$ and the relation dynamic tire load $k_t x_2(t)/(m_s(t) + m_u(t))g < 1$ are guaranteed, which implies the road holding capability is ensured by the desired fuzzy controller. These two figures confirm that the designed standard state-feedback fuzzy H_∞ controller can achieve much better ride comfort and road handling, guarantee constrain suspension deflection for the active suspension system.

V. CONCLUSIONS

This paper has investigated the problem of fuzzy H_∞ control for T-S fuzzy-model-based active suspension systems with actuator delay. The sprung and unsprung mass variations, the actuator delay, and the suspension performance have been taken into account to establish the T-S fuzzy system for the control design objective. Based on the PDC scheme and stability theory, the fuzzy H_∞ performance analysis condition has been presented for the proposed T-S fuzzy system presenting the uncertain active suspension system. Then, the fuzzy H_∞ controller has been designed such that the resulting closed-loop T-S fuzzy system is asymptotically stable with H_∞ performance, and simultaneously satisfy the constraint suspension performance. A quarter-car suspension model has been given to validate the effectiveness of the proposed design method.

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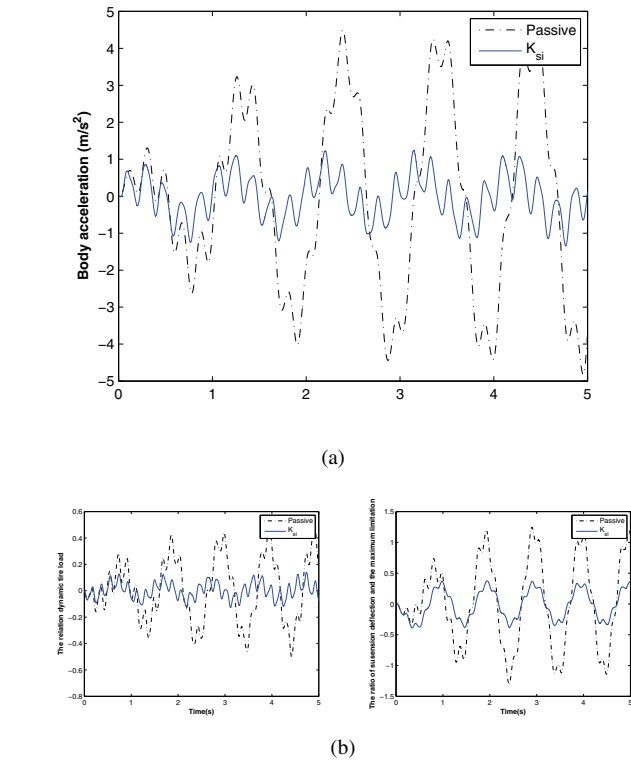


Fig. 2. (a) Responses of body vertical acceleration (b) Responses of suspension deflection constrain; Responses of tire stroke constrain.

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