

Adaptive Fuzzy Interpolation with Uncertain Observations and Rule Base

Longzhi Yang and Qiang Shen
Department of Computer Science
Aberystwyth University
Aberystwyth, SY23 3DB, UK
Email: {lly07,qqs}@aber.ac.uk

Abstract—Adaptive fuzzy interpolation strengthens the potential of fuzzy interpolative reasoning. It views interpolation procedures as artificially created system components, and identifies all possible sets of faulty components that may each have led to all detected contradictory results. From this, a modification procedure takes place, which tries to modify each of such components, termed candidates, in an effort to remove all the contradictions and thus restore consistency. This approach assumes that the employed interpolation mechanism is the only cause of contradictions, that is all given observations and rules are believed to be true and fixed. However, this may not be the case in certain real situations. It is common in fuzzy systems that each observation or rule is associated with a certainty degree. This paper extends the adaptive approach by taking into consideration both observations and rules also, treating them as diagnosable and modifiable components in addition to interpolation procedures. Accordingly, the modification procedure is extended to cover the cases of modifying observations or rules in a given rule base along with the modification of fuzzy reasoning components. This extension significantly improves the robustness of the existing adaptive approach.

Index Terms—Fuzzy interpolation, uncertain observations, uncertain rules.

I. INTRODUCTION

Fuzzy rule interpolation enables inference in systems with sparse rule bases and helps reduce the complexity of fuzzy models. When given observations have no overlap with any antecedent values, no rule can be fired in classical inference. However, interpolative reasoning through a sparse rule base may still obtain certain conclusions and thus improve the applicability of fuzzy models. Also, with the help of fuzzy interpolation, the complexity of a large rule base can be reduced by omitting those fuzzy rules which may be approximated from their neighboring ones. A number of important interpolation approaches have been presented in the literature, including [1], [2], [3], [7], [8], [9], [12], [13]. In particular, the scale and move transformation-based approach can handle both interpolation and extrapolation, which involve multiple fuzzy rules with each rule consisting of multiple antecedents. This approach also guarantees the uniqueness as well as the normality and convexity of the resulting interpolated fuzzy sets. Yet, it is possible that more than one object value of a single variable may be derived in fuzzy interpolation. This implies that certain inconsistencies may result.

To address this problem, *adaptive fuzzy interpolation* has recently been proposed [14], [15]. This approach is capable

of efficiently detecting inconsistencies, locating possible fault candidates and effectively modifying the candidates in an effort to remove all the inconsistencies. It works by viewing the interpolative inference procedures as artificial system components, and then utilizing an assumption-based truth maintenance system (ATMS) [4] to record the dependencies between an interpolated value and its proceeding interpolative procedures. From this, the classical algorithm of general diagnostic engine (GDE) [5] is employed to hypothesize all possible candidates of defective rules, by manipulating the sets of the artificial (fuzzy reasoning) components that led to the detected contradictions.

The adaptive approach of [14], [15] presumes that the underlying interpolation procedures have caused all the contradictory interpolated results. This limits its application to the reasoning components only. However, in general, observations and rules in a given rule base may also be faulty and be so to a certainty degree. Fortunately this is not fundamentally restricted by the underlying approach. This work extends that of [14], in order to allow that observations and rules are also diagnosable and modifiable. Due to the ability of using more information, this extension will considerably improve the approach in two aspects. First, one interpolative solution may still be derived when the existing approach fails. Second, the solution generated may be more reasonable than that by the existing approach.

The reminder of this paper is structured as follows. Sec. II reviews the adaptive fuzzy interpolation approach. Sec. III extends the candidate generation procedure by taking observations, rules, and fuzzy interpolative procedures as diagnosable components. Sec. IV generalizes the candidate modification procedure in order that all kinds of candidate component are modifiable. Sec. V reconsiders the example given in [14] to illustrate how this extension can improve the original approach. Sec. VI concludes the paper and points out important future research directions.

II. ADAPTIVE FUZZY INTERPOLATION

Adaptive interpolative reasoning [14] provides a way to ensure that inference results remain consistent throughout the fuzzy interpolative process. In this work, the fuzzy interpolation procedure based on each pair of neighboring rules R_i and R_j ($i \neq j$) is defined as a *fuzzy reasoning component*, denoted

as $R_i R_j$. Such a component takes an input, an observation or a previously interpolated result, including that obtained by extrapolation (which are all hereafter referred to as an observation for simplicity), and produces another (the consequent of the interpolated rule) as output. The process of adaptive interpolation is summarized in Fig. 1. Firstly, the interpolator carries out interpolation and passes the interpolated results to the ATMS for dependency-recording. Then, the ATMS relays to the GDE any β_0 -contradiction (i.e. inconsistency between two values for a common variable that at least differ to the degree of a given threshold β_0 , $0 \leq \beta_0 \leq 1$) as well as their dependent fuzzy reasoning components. Next, GDE diagnoses the problem and generates all possible component candidates which may have led to the detected inconsistencies. After that, a modification process takes place to correct a certain candidate in order to restore consistency. A brief description of each of these key methods is given below.

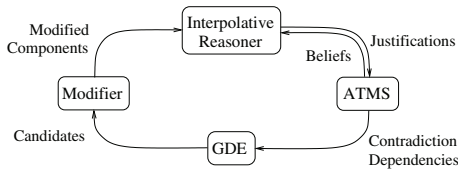


Fig. 1. Adaptive interpolative reasoning

A. Truth maintenance

ATMS is utilized to record the dependency of the interpolated results and that of contradictions, upon the fuzzy reasoning components from which they are inferred. Thus, propositions, contradictions and fuzzy interpolative reasoning components are all represented as ATMS nodes. In addition to the so-called datum field, which trivially denotes the actual meaning of the node, an ATMS node has two other fields: justification and label.

1) *Justification*: Briefly, a justification describes how a node is derivable from other nodes. Any ATMS node with an inferred proposition can be verified by an ATMS justification:

$$O, R_i R_j \Rightarrow C, \quad (1)$$

which means that the outcome C is inferred from observations O through the interpolation procedure $R_i R_j$ ($i \neq j$) by firing rules R_i and R_j .

By definition, any two propositions $P(x_i \text{ is } A_{ij})$ and $P'(x_i \text{ is } A_{ik})$ concerning the same variable x_i are contradictory if the values differ at least by a given degree β_0 , which can be represented as:

$$P, P' \Rightarrow_{\beta_0} \perp. \quad (2)$$

2) *Label and label-updating*: A label is a set of environments, each of which supports their associated node. In particular, an environment is a minimal set of fuzzy reasoning components that jointly entail the supported node, thereby describing how the node ultimately depends on those fuzzy reasoning components. An environment is said to be β_0 -inconsistent if β_0 -contradiction is derivable propositionally by

the environment and a given justification. An environment is said to be $(1 - \beta_0)$ -consistent if it is not β_0 -inconsistent.

The label of each node is guaranteed to be $(1 - \beta_0)$ -consistent, sound, minimal and complete by the label updating algorithm, except that the label of the special “false” node is guaranteed to be β_0 -inconsistent rather than $(1 - \beta_0)$ -consistent. In particular, the label of the special “false” node gathers all minimal β_0 -inconsistent environments. The corresponding label-updating process for this special node is given as follows. Whenever a β_0 -contradiction is detected, each environment in its label is added into the label of “false” node and all such environments and their supersets are removed from the label of every other node. Also, any such environment which is a superset of another is removed from the label of the node “false”.

B. Candidate generation

GDE [5] generates minimal candidates by manipulating the label of the specific “false” node. A candidate is a particular set of nodes or fuzzy reasoning components which may be responsible for the whole set of current contradictions. Because a β_0 -inconsistent environment indicates that at least one of its elements is faulty (contradictory to the extent of β_0), a candidate must have a non-empty intersection with each β_0 -inconsistent environment. Thus, each candidate is constructed by taking one fuzzy reasoning component from each environment in the label of “false” node. Supersets removal then ensures such generated candidates to be minimal. In light of this, a successful correction of any single candidate will remove all the contradictions.

C. Candidate modification

Consistency can be restored by successfully correcting any single candidate because each single candidate explains the entire set of current contradictions. Given a set of candidates, the modification procedure is shown in Fig. 2.

CONSISTENCYRESTORING($\mathbb{Q}$)

$\mathbb{Q}$, the candidate set sorted in descending cardinality, each element of which is a set of fuzzy reasoning components (f); MODIFY(f), the modification procedure for a single fuzzy reasoning component (f), which returns **true** when modification succeeds and **false** otherwise.

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(1) success ← false
(2) do
(3)   C ← Dequeue( $\mathbb{Q}$ )
(4)   foreach f ∈ C
(5)     success ← MODIFY(f)
(6)     if (success == false)
(7)       break
(8) until ((success == true) or ( $\mathbb{Q}$  == ∅))
(9) return success

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Fig. 2. The CONSISTENCYRESTORING procedure

For convenience, in the rest of this paper, A_{ij}^* is used to denote the modified consequence of a culprit interpolated

rule whose consequent value is A_{ij} , and A_{ij}' and λ_{ij}^* are used to denote the corresponding modified intermediate rule consequence and the *relative placement factor* [14] of A_{ij}' , respectively. Also, for simplicity, only rules with a single antecedent are considered here, though extension to multi-antecedent rules is straightforward. Suppose that the neighboring rules $A_{11} \Rightarrow A_{21}$ and $A_{1n} \Rightarrow A_{2n}$ are the two rules used for interpolation by a defective fuzzy reasoning component, that $A_{12}, A_{13}, \dots, A_{1(n-1)}$ are observations or previously interpolated results located in between A_{11} and A_{1n} , and that A_{1j} ($2 \leq j \leq n-1$) is the middle most observation (which informally, is the one that is closest to both antecedent values of the neighboring rules). The modification procedure for a single fuzzy reasoning component is summarized as follows:

1. Find the rule ($A_{1j} \Rightarrow A_{2j}$) whose antecedent is located in the middle most of the neighborhood of the antecedents of any two rules that may be used for interpolation, with respect to their representative values [7]. Assume that the *relative placement factor* of its consequence λ_{2j} is modified to λ_{2j}^* .

2. Calculate the *correction rate pair* according to the *relative placement factor* modification of rule $A_{1j} \Rightarrow A_{2j}$:

$$\begin{cases} c^- = \frac{\lambda_{2j}^*}{\lambda_{2j}} \\ c^+ = \frac{1-\lambda_{2j}^*}{1-\lambda_{2j}} \end{cases} \quad (3)$$

3. Calculate the modified *relative placement factors* of the consequences of all other interpolated rules which are generated from the same defective fuzzy reasoning component as per the *correction rate pair* computed above, where $i \in \{2, 3, \dots, j-1\}$ and $k \in \{j+1, j+2, \dots, n-1\}$:

$$\begin{cases} \lambda_{2i}^* = \lambda_{2i} \cdot c^- \\ 1 - \lambda_{2k}^* = (1 - \lambda_{2k}) \cdot c^+ \end{cases} \quad (4)$$

4. Calculate the modified consequences of all interpolated rules which are generated from the same defective fuzzy reasoning component in accordance with their modified *relative placement factors*:

$$\begin{cases} A_{2x}' = (1 - \lambda_{2x}^*)A_{21} + \lambda_{2x}^*A_{2n} \\ T(A_{1x}', A_{1x}) = T(A_{2x}', A_{2x}), \end{cases} \quad (5)$$

where $x \in \{2, 3, \dots, n-1\}$, and $T(A', A)$ represents the scale and move transformations [7], [8] from fuzzy set A' to A .

5. Restrict the modified consequence such that if m object values $A_{i1}, A_{i2}, \dots, A_{im}$ are obtained for variable x_i then they must satisfy:

$$\bigcap_{j=1}^m (A_{ij})_{\beta_0} \neq \emptyset, \quad (6)$$

where $(A_{ij})_{\beta_0}$ denotes the β_0 -cut of fuzzy set A_{ij} . That is, all derived values for a given variable are at least $(1 - \beta_0)$ -consistent.

6. Restrict the propagations of all modified consequences so that they are mutually consistent to the extent of at least $(1 - \beta_0)$. For simplicity, let function $I(A_{ij}, R_l R_r) = A_{kj}$ denote the standard interpolation from the antecedent fuzzy set

A_{ij} to the consequent value A_{kj} , through fuzzy reasoning component $R_l R_r$. Suppose that m object values $A_{i1}, A_{i2}, \dots, A_{im}$ of variable x_i , located between the antecedent values of rules R_l and R_r , are modified, that the corresponding modified object values of variable x_k are A_{kj}^* , $j \in \{1, 2, \dots, m\}$, and that n object values A_{kl} , $l \in \{1, 2, \dots, n\}$, of variable x_k are already derived previously. If the modified consequences A_{kj}^* are all $(1 - \beta_0)$ -consistent, then they must satisfy:

$$\begin{cases} A_{kl}^* = I(A_{ij}^*, R_l R_r) \\ \left(\bigcap_{j=1}^m (A_{kj}^*)_{\beta_0} \right) \cap \left(\bigcap_{l=1}^n (A_{kl})_{\beta_0} \right) \neq \emptyset. \end{cases} \quad (7)$$

7. Solve all these simultaneous equations. The result is the modified solution which ensures β_0 -inconsistency-free.

III. GENERALIZING CANDIDATE GENERATION

The approach described above assumes that observations and rules in a given rule base are true and fixed and thus, inference procedures are the only possible cause of any inconsistencies. Consequently, only inference procedures (i.e. fuzzy reasoning components) are treated as diagnosable and modifiable components. However, this may not be true in real-world problems as observations and rules in a given rule base may also be faulty to a certain degree. Therefore, observations and rules in a rule base may also need to be diagnosed and modified. A generalization of the existing approach is described below.

A. Certainty degrees of observations and rules

Conceptually, inexact information may be classified into the following four general categories [6]: i) vagueness, which arises due to lack of sharp distinctions or boundaries between pieces of information and is usually modeled by a fuzzy set that identifies a soft constraint on a set of elements; ii) uncertainty, which depicts the reliability or confidential weight of a given piece of information stated in a proposition and can be captured by a numerical value or a fuzzy set; iii) both vagueness and uncertainty, which means that information of type i) and type ii) coexists but the uncertainties are expressed by numerical values; iv) Both vagueness and uncertainty with the latter also expressed in vague terms (i.e. fuzzy sets). The previous work [14] involves in type i information only. This work extends that by introducing type ii information into the system, but using numerical uncertainty representation only. This results in the use of type iii information. How to further extend this work to deal with type iv information remains active research.

In particular, an observation in this work is of the form:

$$O: x_i \text{ is } A_{ij} (c), \quad (8)$$

where c , a crisp number between 0 and 1, represents the certainty degree of observation O . According to the aforementioned categories of inexactness, an object value is modeled as a fuzzy set because no clear boundary between pieces of information is available; the certainty degree of the observation is expressed as a crisp number because the current description

of the object value (i.e. the fuzzy set) may not be of full confidence or fully reliable.

Since c represents the certainty degree, $1 - c$ then naturally describes the uncertainty degree of the same piece of information. Thus, an uncertainty degree value as given in Eq. 8 may be interpreted being the extent to which the factual object value of O is located on the left side or right side of its current position. From this, intuitively, the modifiable range of object value O is bounded to the proportion of $1 - c$ in reference to the entire variable domain. That is, it can shift leftwards or rightwards to a maximal distance of $\frac{1-c}{2}(max_i - min_i)$, where max_i and min_i are the maximum and minimum of the domain values of variable x_i . Note that the shifting of a fuzzy set does not affect the vague term (i.e. the shape and area of the fuzzy set). It is equivalent to add a crisp number to the representative value of the original fuzzy set [10]. In summary, the modified value A_{ij}^* of observation O as given in Eq. 8 must satisfy:

$$\begin{cases} A_{ij}^* \geq A_{ij} - \frac{1-c}{2}(max_i - min_i) \\ A_{ij}^* \leq A_{ij} + \frac{1-c}{2}(max_i - min_i). \end{cases} \quad (9)$$

It is possible that the shifting may be out of the variable domain. In order to keep the shifting result within the underlying domain, the shifting also needs to satisfy:

$$\begin{cases} min(supp(A_{ij}^*)) \geq min_i \\ max(supp(A_{ij}^*)) \leq max_i, \end{cases} \quad (10)$$

where $supp(A_{ij}^*)$ is the support of fuzzy set A_{ij}^* .

A uncertain rule is of the form as follows:

$$R: \text{ If } x_k \text{ is } A_{kj}, \text{ then } x_i \text{ is } A_{ij} (c), \quad (11)$$

which indicates that rule R is certain to the degree of c . Similar to the meaning of the certainty degree of an observation, this is interpreted as that the factual consequence object value A_{ij} may be located in the left or right side of the current position. Again, the shifting must satisfy Eqs. 9 and 10.

B. Dependency recording

In adaptive fuzzy interpolation, ATMS is used to record the dependencies of the interpolated results and contradictions upon the artificial (fuzzy reasoning) components. In this extended work, such dependencies can also be upon observations and rules in a given rule base. That is, propositions (including observations), contradictions, fuzzy reasoning components, and rules in a rule base are all represented as ATMS nodes.

Recall that a justification describes how a node is derivable from other nodes. Observations, rules in a given rule base and fuzzy reasoning components are assumed to be initially true and may be established to be false subsequently. For each of such nodes (i.e. assumptions in classical ATMS terms [4]), its justification simply assumes itself to be true. Any ATMS node with an inferred proposition (i.e. a derived node in [4]), which is obtained through the scale and move transformation-based interpolation, may now be verified by an ATMS justification:

$$O, R_i, R_j, R_i R_j \Rightarrow C. \quad (12)$$

Note that when rules R_i and R_j ($i \neq j$) are fixed and true, there is no point to keep these dependencies and then Eq. 12 degenerates to Eq. 1. The justification of a β_0 -contradiction in this extended work is the same as that in the existing work.

Note that if a consequence that is justified by a new interpolation does not already exist in the ATMS network, a new node will be created. Then, the label of every node will be updated, which is achieved by the so called label-updating algorithm. In order to avoid duplications, the label-updating algorithm is omitted here because it is basically the same as that used in the existing approach to adaptive fuzzy interpolation [14].

C. Candidate generation

From recorded dependencies of all contradictions within the ATMS network, GDE is employed to generate the candidate set. In particular, each candidate is formed by taking one element from each label environment of each contradiction. Of course, duplication and superset removal takes place for each candidate to ensure that the generated candidate set is minimal. Although candidate generation procedure in this work is similar to that in [14], the constituent components of the generated candidates hence may be different from those in the existing approach. This is obvious because an element of a certain candidate may now be an observation, a rule in a given rule base or a fuzzy reasoning component rather than just a fuzzy reasoning component.

To facilitate the modification of a candidate, it is necessary to determine whether multiple related components with respect to a single step of interpolation can appear within one candidate. If any candidate contains at most one element with respect to a single interpolation step, the modification of the candidate can be decomposed to the modification of its individual elements. Otherwise, the modification of related components needs to be implemented jointly.

Suppose that a step of interpolation $O, R_i, R_j, R_i R_j \Rightarrow C$ is given. Let $N_O, N_{R_i}, N_{R_j}, N_{R_i R_j}$ and N_C be the identity nodes, regarding $O, R_i, R_j, R_i R_j$ and C , respectively. As stated previously, each observation, rule or fuzzy reasoning component has an environment containing only one node which represents itself. According to the label updating algorithm, each label environment of node N_O and those of nodes $\{N_{R_i}, N_{R_j}, N_{R_i R_j}\}$ jointly form a label environment of node N_C . Without losing generality, assume that N_C contributes to a certain contradiction directly or indirectly. Then, if any of its label environments contains $N_{R_i R_j}$, it must also contain N_{R_i} and N_{R_j} , and vice versa. Therefore, it is impossible that $\{N_{R_i}, N_{R_i R_j}\}$ or $\{N_{R_j}, N_{R_i R_j}\}$ is contained within one candidate in the minimal candidate set. This is because a candidate is generated by taking one element from each label environment of each contradiction and any candidate which is a superset of any other candidate is removed.

Similarly, if O is an observation, $\{N_O, N_{R_i}\}$, $\{N_O, N_{R_j}\}$, or $\{N_O, N_{R_i R_j}\}$ cannot be contained in any one candidate in the minimal candidate set. However, it is possible that N_{R_i} and N_{R_j} are contained within the same candidate. This is because

N_{R_i} may also be used in conjunction with another rule to carry out interpolation rather than N_{R_j} , and vice versa. Thus, one label environment of the special “false” node may only contains N_{R_i} but not N_{R_j} while another may only contains N_{R_j} but not N_{R_i} . Therefore, it is possible to generate a candidate in the minimal candidate set such that it contains both N_{R_i} and N_{R_j} .

IV. GENERALIZING CANDIDATE MODIFICATION

Having generated the minimal candidate set, a certain candidate needs to be selected for modification in an effort to remove all the contradictions and thus to restore consistency. The principle underlying the consistency-restoring algorithm as outlined in Fig. 2 can be extended for use here. However, the extension is not straightforward due to the involvement of vague and uncertain information. This is because the constituent components of a fault candidate can now also be observations or rules given in a rule base apart from fuzzy reasoning components. However, as indicated in Sec. III-C, for the particular application of GDE in the present problem, elements contained within any minimal candidate returned by the GDE are independent amongst themselves except that a pair of rules that were used for interpolation may be contained within one certain candidate. Thus, their modification can be carried out independently except that the modification of a pair of neighboring rules should be carried out jointly. The modification of fuzzy reasoning components has been outlined in Sec. II-C. The modifications of observations and rules in a given rule base are described in the rest of this section.

A. Observation modification

For a given defective observation associated with certainty degree c , what the modification needs to do is to shift the fuzzy set along the variable axis while fixing its shape and area. This follows a naturally appealing interpretation of certainty degrees in the present context. In particular, the shifting must satisfy the three constraints below:

- 1) The shifting must be bounded by Eqs. 9 and 10, with respect to the given c .
- 2) The shifted observation needs to be consistent with the rest observations (or interpolated results), which corresponds to point 5 of the modification procedure for fuzzy reasoning components as outlined in Sec. II-C.
- 3) The propagation of the shifted observation needs to be mutually consistent with other consequences, which corresponds to point 6 of the modification procedure for fuzzy reasoning components outlined in Sec. II-C.

All three constraints can be satisfied by posing and then solving a set of simultaneous equations and inequations. The solution guarantees that the interpolated results using the modified observation are at least $(1 - \beta_0)$ -consistent throughout.

B. Rule modification

The situation that only one of two neighboring rules is defective is considered first. Similar to the modification of an observation, given a defective rule with certainty degree c ,

what the modification needs to do is to shift the consequence of the rule along with the consequent axis by satisfying Eqs. 9 and 10. However, because the defective rule has been used for interpolation, all the interpolated results which have been generated by employing this defective rule also need to be modified accordingly. There are two cases to consider in order to modify the defective rule, depending on whether the antecedent of the other rule that was used together with it to carry out interpolation is on which side of its own antecedent.

Without losing generality, suppose that $x_1 = A_{1i} \Rightarrow x_2 = A_{2i}$ and $x_1 = A_{1d} \Rightarrow x_2 = A_{2d}$ are the two neighboring rules which flank a given observation A_{1j} and that the second rule is defective, where $1 \leq i, d \leq N$ with N being the cardinality of the rule base, and $1 \leq j \leq M$ with M being the number of all given observations (including derived values) for x_1 . Assume that A_{1i} is on the left hand side of A_{1d} , the antecedent of the defective rule.

Note that in implementing the scale and move transformation-based fuzzy interpolation, given an observation and two neighboring rules which flank the observation, the logical consequence of the observation is derived from the consequent fuzzy set of an artificially created intermediate rule via the scale and move transformations. This is achieved by minimizing the area and shape differences between the observation and the antecedent of the intermediate rule. Therefore, the area and shape of the logical consequence ultimately depend on the value of the relative placement factor λ (see [14] for details), the areas and shapes of the observation, and all the fuzzy sets defined in the rules used for interpolation. Thus, to maintain the interpretability of the fuzzy models and reasoning, it is desirable to modify a defective rule without changing the shape and area of its consequence. The only aspect that is modifiable is its location.

The overall location of a fuzzy set may be approximately captured by a crisp number, termed representative value [7]. In carrying out interpolation, the location relation between A_{1j} and its corresponding interpolated consequence A_{2j} can be mapped by a line which is determined by the locations of the the antecedents and consequences of two neighboring rules used for interpolation. This corresponds to the line P_1P_3 shown in Fig. 3. Suppose that the consequence of the defective rule

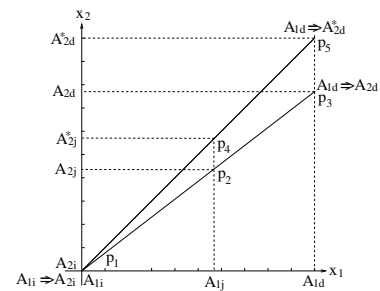


Fig. 3. Rule modification propagation

is modified to A_{2d}^* , then the location relation line P_1P_3 is accordingly shifted to line P_1P_5 . In order to measure the extent

of such shifting, the following *correction rate* c^- is introduced:

$$c^- = \frac{d(A_{2i}, A_{2d}^*)}{d(A_{2i}, A_{2d})}, \quad (13)$$

where $d(A, A')$ stands for the distance between the locations of the two fuzzy sets A and A' (shorted as the distance between fuzzy sets A and A' hereafter for simplicity). Denote the modified result of A_{2j} as A_{2j}^* , then the distance between A_{2i} and A_{2j}^* can be calculated by applying the correction rate c^- to the distance between A_{2i} and A_{2j} . Having known the locations of A_{2i} and A_{2j} , the location of A_{2j}^* can be calculated. Given only location is modified, A_{2j}^* is then obtained. For any other given interpolated results, if their corresponding observations are flanked by the same pair of original neighboring rules, their locations are corrected in a similar manner.

Of course, such modified results need to be consistent with the rest, so do their propagations. The procedure required is the same as points 5 and 6 of the modification procedure of a fuzzy reasoning component as outlined in Sec. II-C. Putting the above together, a set of constraints in terms of equations and inequations is constructed. The solution of these equations and inequations in conjunction with those imposed by all other elements of the same candidate in question guarantees the modified result of the candidate to be β_0 -contradiction-free.

The case discussed so far covers the situation where all the observations which have invoked the use of the defective rule for interpolation are located on the left side of its antecedent. For the case where observations are located on the right side of its antecedent, the modification follows a mirrored procedure, with a different *correction rate* c^+ . Suppose that a given observation A_{1k} , $1 \leq k \leq M$ is flanked by the defective rule and its right neighboring rule, $x_1 = A_{1l} \Rightarrow x_2 = A_{2l}$, $1 \leq l \leq N$, then c^+ is calculated by:

$$c^+ = \frac{d(A_{2d}^*, A_{2l})}{d(A_{2d}, A_{2l})}. \quad (14)$$

Similar to the left hand side case, the modified result of observation A_{1k} can then be computed by utilizing this correction rate. From this, the rest of the modification procedure is the same as the previous case.

The above has addressed the situations where only one of the two neighboring rules is defective. For the situation where both neighboring rules are defective, the modification can be done by modifying the two individual defective rules separately in a sequence. This is because the modified result is independent of the order of modifications.

The independence of the order of modifications carried out is proven below. Without losing generality, following the above description, suppose that rule $x_1 = A_{1l} \Rightarrow x_2 = A_{2l}$ is also defective, that the modified consequence of the left neighboring rule is A_{2d}^* , and that the modified consequence of the right neighboring rule is A_{2l}^* . Then, the final modification corresponds to the replacement of the location mapping line P_1P_3 by line P_6P_5 as illustrated in Fig. 4(b). This is because if the left defective rule is modified first as shown in Fig. 4(a), the modification of the right defective rule will be performed using

the result of the modification of the left rule as illustrated in the final result of Fig. 4(b). If, however, the modification starts from the right defective rule, then the modification process is illustrated in Fig. 5, which also leads to the final result that is the same as the one given in the line P_6P_5 of Fig. 4(b). Thus, the modification in this case is simply a combination of the two procedures previously introduced.

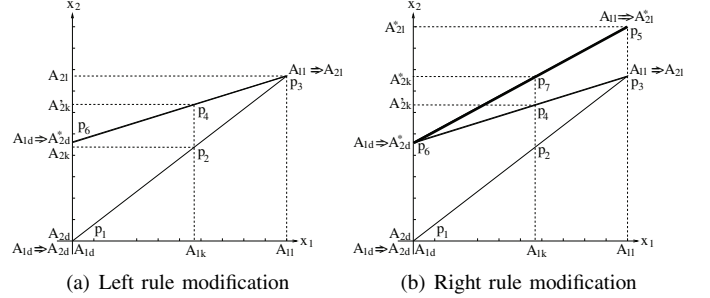


Fig. 4. The left defective rule is modified first

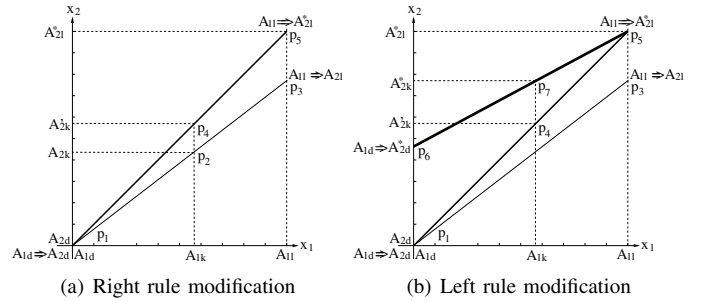


Fig. 5. The right defective rule is modified first

V. AN ILLUSTRATIVE EXAMPLE

To illustrate the potential of this extended adaptive approach, the problem given in [14] is reconsidered, with information on certainty degrees added. The rule base is given as follows:

- R_1 : If x_1 is A_{11} , then x_2 is A_{21} (0.82);
- R_2 : If x_1 is A_{12} , then x_2 is A_{22} (0.79);
- R_3 : If x_2 is A_{23} , then x_3 is A_{31} (0.93);
- R_4 : If x_2 is A_{24} , then x_3 is A_{32} (0.64);
- R_5 : If x_2 is A_{25} , then x_4 is A_{41} (0.75);
- R_6 : If x_2 is A_{26} , then x_4 is A_{42} (0.88);
- R_7 : If x_3 is A_{33} , then x_5 is A_{51} (0.82);
- R_8 : If x_3 is A_{34} , then x_5 is A_{52} (0.86);
- R_9 : If x_4 is A_{43} , then x_5 is A_{53} (0.90);
- R_{10} : If x_4 is A_{44} , then x_5 is A_{54} (0.62).

Given $\beta_0 = 0.5$ and three observations, $x_1 = A_{13} = (7.0, 8.0, 9.0)$ (0.95), $x_1 = A_{14} = (7.6, 8.6, 9.6)$ (0.83) and $x_4 = A_{45} = (12.0, 13.0, 14.0)$ (0.92), the interpolation procedures are illustrated in Fig. 6 and the original observations as well as the results obtained by scale and move transformation-based interpolation are presented in Fig. 7. Note that in this example, to keep the illustration simple, it has been assumed that the variable x_1 takes two different object values A_{13} and A_{14} with different certainty degrees. In practice, this may be caused by the fact that the observations are taken by different

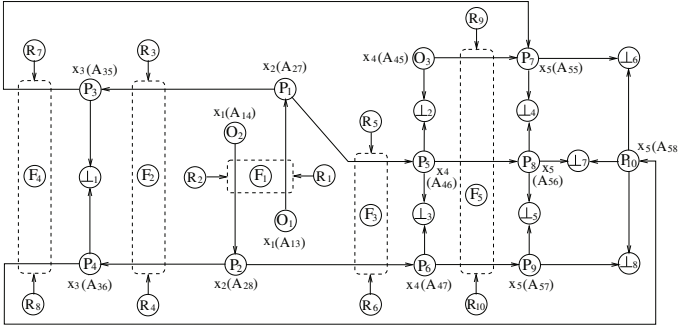


Fig. 6. Discrepancy records in ATMS

agents or that one is a real observation and the other may be produced by another inference mechanism.

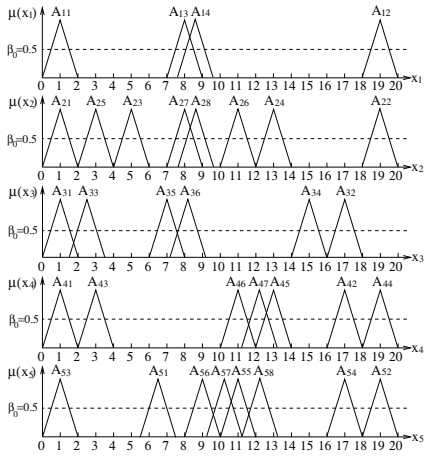


Fig. 7. Fuzzy sets used in the example

A. Dependency recording

In Fig. 6, ATMS nodes and contradictions are represented by circles. Particularly, each of O_g , $g \in \{1, 2, 3\}$, is a node denoting an observation; each of R_h , $h \in \{1, 2, \dots, 10\}$, is a node denoting a rule in the rule base; each of F_i , $i \in \{1, 2, \dots, 5\}$, is a node denoting a fuzzy reasoning component; each of P_j , $j \in \{1, 2, \dots, 10\}$, is a node denoting a proposition; and each of $\perp_k$, $k \in \{1, 2, \dots, 8\}$, denotes a β_0 -contradiction. These ATMS nodes and contradictions are listed as follows, with all justifications omitted:

- | | |
|---|--|
| $R_1 : \langle x_1 = A_{11} \Rightarrow x_2 = A_{21}, \{\{R_1\}\} \rangle;$ | $O_1 : \langle x_1 = A_{13}, \{\{O_1\}\} \rangle;$ |
| $R_2 : \langle x_1 = A_{12} \Rightarrow x_2 = A_{22}, \{\{R_2\}\} \rangle;$ | $O_2 : \langle x_1 = A_{14}, \{\{O_2\}\} \rangle;$ |
| $R_3 : \langle x_2 = A_{23} \Rightarrow x_3 = A_{31}, \{\{R_3\}\} \rangle;$ | $O_3 : \langle x_4 = A_{45}, \{\{O_3\}\} \rangle;$ |
| $R_4 : \langle x_2 = A_{24} \Rightarrow x_3 = A_{32}, \{\{R_4\}\} \rangle;$ | $F_1 : \langle R_1 R_2, \{\{F_1\}\} \rangle;$ |
| $R_5 : \langle x_2 = A_{25} \Rightarrow x_4 = A_{41}, \{\{R_5\}\} \rangle;$ | $F_2 : \langle R_3 R_4, \{\{F_2\}\} \rangle;$ |
| $R_6 : \langle x_2 = A_{26} \Rightarrow x_4 = A_{42}, \{\{R_6\}\} \rangle;$ | $F_3 : \langle R_5 R_6, \{\{F_3\}\} \rangle;$ |
| $R_7 : \langle x_3 = A_{33} \Rightarrow x_5 = A_{51}, \{\{R_7\}\} \rangle;$ | $F_4 : \langle R_7 R_8, \{\{F_4\}\} \rangle;$ |
| $R_8 : \langle x_3 = A_{34} \Rightarrow x_5 = A_{52}, \{\{R_8\}\} \rangle;$ | $F_5 : \langle R_9 R_{10}, \{\{F_5\}\} \rangle;$ |
| $R_9 : \langle x_4 = A_{43} \Rightarrow x_5 = A_{53}, \{\{R_9\}\} \rangle;$ | |
| $R_{10} : \langle x_4 = A_{44} \Rightarrow x_5 = A_{54}, \{\{R_{10}\}\} \rangle;$ | |
| $P_1 : \langle x_2 = A_{27}, \{\{O_1, R_1, R_2, F_1\}\} \rangle;$ | |
| $P_2 : \langle x_2 = A_{28}, \{\{O_2, R_1, R_2, F_1\}\} \rangle;$ | |
| $P_3 : \langle x_3 = A_{35}, \{\{O_1, R_1, R_2, F_1, R_3, R_4, F_2\}\} \rangle;$ | |
| $P_4 : \langle x_3 = A_{36}, \{\{O_2, R_1, R_2, F_1, R_3, R_4, F_2\}\} \rangle;$ | |
| $P_5 : \langle x_4 = A_{46}, \{\{O_1, R_1, R_2, F_1, R_5, R_6, F_3\}\} \rangle;$ | |

- $P_6 : \langle x_4 = A_{47}, \{\{O_2, R_1, R_2, F_1, R_5, R_6, F_3\}\} \rangle;$
 $P_7 : \langle x_5 = A_{55}, \{\{O_1, R_1, R_2, F_1, R_3, R_4, F_2, R_7, R_8, F_4\}, \{O_3, R_9, R_{10}, F_5\}\} \rangle;$
 $P_8 : \langle x_5 = A_{56}, \{\{O_1, R_1, R_2, F_1, R_5, R_6, F_3, R_9, R_{10}, F_5\}\} \rangle;$
 $P_9 : \langle x_5 = A_{57}, \{\{O_2, R_1, R_2, F_1, R_5, R_6, F_3, R_9, R_{10}, F_5\}\} \rangle;$
 $P_{10} : \langle x_5 = A_{58}, \{\{O_2, R_1, R_2, F_1, R_3, R_4, F_2, R_7, R_8, F_4\}\} \rangle;$
 $\perp_1 : \langle \perp, \{\{O_1, O_2, R_1, R_2, F_1, R_3, R_4, F_2\}\} \rangle;$
 $\perp_2 : \langle \perp, \{\{O_1, O_3, R_1, R_2, F_1, R_5, R_6, F_3\}\} \rangle;$
 $\perp_3 : \langle \perp, \{\{O_1, O_2, R_1, R_2, F_1, R_5, R_6, F_3\}\} \rangle;$
 $\perp_4 : \langle \perp, \{\{O_1, O_3, R_1, R_2, F_1, R_5, R_6, F_3, R_9, R_{10}, F_5\}\} \rangle;$
 $\perp_5 : \langle \perp, \{\{O_1, O_2, R_1, R_2, F_1, R_5, R_6, F_3, R_9, R_{10}, F_5\}\} \rangle;$
 $\perp_6 : \langle \perp, \{\{O_2, O_3, R_1, R_2, F_1, R_3, R_4, F_2, R_7, R_8, F_4\}\} \rangle;$
 $\perp_7 : \langle \perp, \{\{O_1, O_2, R_1, R_2, F_1, R_3, R_4, F_2, R_5, R_6, F_3, R_7, R_8, F_4, R_9, R_{10}, F_5\}\} \rangle;$
 $\perp_8 : \langle \perp, \{\{O_2, R_1, R_2, F_1, R_3, R_4, F_2, R_5, R_6, F_3, R_7, R_8, F_4, R_9, R_{10}, F_5\}\} \rangle.$

In particular, a specific ATMS node “false”, denoted by $P_{\perp}$, which collectively represents all the contradictions listed above from $\perp_1$ to $\perp_8$, is given as follows:

- $P_{\perp} : \langle \perp, \{\{O_1, O_2, R_1, R_2, F_1, R_3, R_4, F_2\}, \{O_1, O_3, R_1, R_2, F_1, R_5, R_6, F_3\}, \{O_1, O_2, R_1, R_2, F_1, R_5, R_6, F_3\}, \{O_2, O_3, R_1, R_2, F_1, R_3, R_4, F_2, R_7, R_8, F_4\}\} \rangle.$

B. Candidate generation

After removing all duplications in each candidate and all candidates which are supersets of any other candidate, twenty four minimal candidates are generated, each of which is formed by taking one element from each environment in the label of the “false” node:

- $C_1 = [R_1]; C_2 = [R_2]; C_3 = [F_1]; C_4 = [O_1, O_2]; C_5 = [O_1, O_3];$
 $C_6 = [O_1, R_3]; C_7 = [O_1, R_4]; C_8 = [O_1, F_2]; C_9 = [O_1, R_7];$
 $C_{10} = [O_1, R_8]; C_{11} = [O_1, F_4]; C_{12} = [O_2, R_5]; C_{13} = [O_2, O_3];$
 $C_{14} = [O_2, R_6]; C_{15} = [O_2, F_3]; C_{16} = [R_3, R_5]; C_{17} = [R_3, R_6];$
 $C_{18} = [R_3, F_3]; C_{19} = [R_4, R_5]; C_{20} = [F_2, R_5]; C_{21} = [R_4, F_3];$
 $C_{22} = [R_4, R_6]; C_{23} = [F_2, R_6]; C_{24} = [F_2, F_3].$

C. Candidate modification

As with the existing approach, candidates of the smallest cardinality are examined first. Those candidates which only contain one element, i.e. candidates C_1 , C_2 and C_3 in this example, are therefore chosen to be modified first. Particularly, C_1 is randomly chosen as the first to be modified in this case. According to Eqs. 9 and 10, the modified consequence of rule R_1 must satisfy:

$$\begin{cases} A_{21}^* \geq A_{21} - \frac{1-0.82}{2}(20-0) \\ A_{21}^* \leq A_{21} + \frac{1-0.82}{2}(20-0) \\ \min(\text{supp}(A_{21}^*)) \geq 0 \\ \max(\text{supp}(A_{21}^*)) \leq 20. \end{cases}$$

Note that each of the following two rules has been used to form the neighboring rules in conjunction with the defective rule R_1 to carry out interpolation:

- $IR_1 : \text{If } x_1 \text{ is } A_{13}, \text{ then } x_2 \text{ is } A_{27};$
 $IR_2 : \text{If } x_1 \text{ is } A_{14}, \text{ then } x_2 \text{ is } A_{28}.$

Since both the antecedent of IR_1 and that of IR_2 are on the right hand side of their counterpart of the defective rule, the following correction rate is used:

$$c^+ = \frac{d(A_{21}^*, A_{22})}{d(A_{21}, A_{22})}$$

Then, the locations of the modified interpolated results must satisfy:

$$\begin{cases} d(A_{27}^*, A_{22}) = d(A_{27}, A_{22}) \cdot c^+, \\ d(A_{28}^*, A_{22}) = d(A_{28}, A_{22}) \cdot c^+. \end{cases}$$

These are in turn, used to calculate the modified interpolated results as follows:

$$\begin{cases} A_{27}^* = A_{27} + (d(A_{27}^*, A_{22}) - d(A_{27}, A_{22})) \\ A_{28}^* = A_{28} + (d(A_{28}^*, A_{22}) - d(A_{28}, A_{22})). \end{cases}$$

From this, the rest of the modification is to pose a set of equations and inequations to ensure that the modified results and their propagations are consistent with the rest, which is the same as that of the modification of a fuzzy reasoning component and thus omitted here. Unfortunately, solving all these simultaneous equations and inequations does not lead to any solution. This is also the case when the single candidates C_2 and C_3 are examined.

According to the general candidate modification procedure given in Fig. 2, a candidate of a cardinality of 2 is then randomly taken for modification. In this particular case, suppose that candidate C_4 is taken first, which is comprised of two observations O_1 and O_2 . By satisfying the first constraint of a modified observation as given in Sec. IV-A, the modified result of O_1 must satisfy:

$$\begin{cases} A_{13}^* \geq A_{13} - \frac{1-0.95}{2}(20-0) \\ A_{13}^* \leq A_{13} + \frac{1-0.95}{2}(20-0) \\ \min(\text{supp}(A_{13}^*)) \geq 0 \\ \max(\text{supp}(A_{13}^*)) \leq 20. \end{cases}$$

Similarly, a set of constraining inequations can be posed on the modified observation O_2 . From this, the rest of the modification, which correspond to the second and third constraints outlined in Sec. IV-A, is the same as that of the modification of a fuzzy reasoning component and thus omitted. Again, this leads to no solution.

The tentative modification continues until one modification succeeds. In this case, candidate C_{12} leads to one solution as illustrated in Fig. 8. It is clear from this figure that there is no β_0 -contradiction any more and thus consistency has been successfully restored. This means that the original inconsistent interpolation process has been corrected with consistent interpolated results throughout.

VI. CONCLUSIONS

This paper has extended the recent work on adaptive fuzzy interpolation by allowing observations and rules in a given rule base to be diagnosable and modifiable. In particular, it first represents the concepts of generalized adaptive fuzzy interpolation in ATMS terms. Then, GDE is employed to generate the minimal candidates by making use of the label of the specific “false” node within the ATMS. The consequence of this is that a candidate may consist of observations, rules and/or fuzzy reasoning components. While the modification method for individual fuzzy reasoning components has been given previously, this work proposes an extended approach to modifying individual observations and individual rules. The working of this method is illustrated with an example, though scale-up applications remain as active research. Theoretical analysis of the general properties of the work however, requires future investigation.

Further improvements may enhance the potential of the proposed research. Firstly, the extended approach can only be

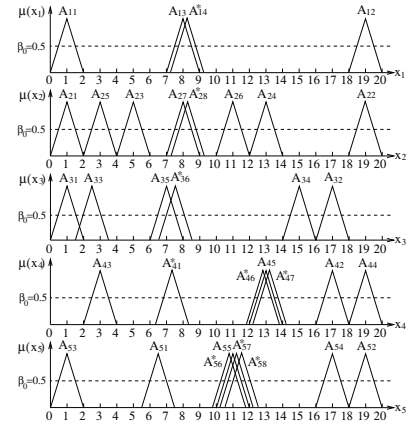


Fig. 8. Solution for the running example

applied to fuzzy interpolation with two single-antecedent rules. It is worthwhile to generalize the approach to cope with fuzzy interpolation with multi-antecedent rules and fuzzy extrapolation. Also, due to more complex and uncertain information has been introduced into the underlying knowledge representative scheme used, the number of candidates grows dramatically. Thus, how to reduce the candidate set size and prioritize these candidates require further research. Use of an uncertainty-based ATMS such as that of [11] may help addressing this issue.

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