

Interdependent Multiple Objective Programming- A Monte Carlo Method

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Abstract—Although multiple objective programming (MOP) skills have been extensively studied in various issues, the problem of MOP with interdependence has received little attention. In this paper, we overcome the problem of Carlsson and Fullér's method and propose a novel index to measure the interdependence grade between objectives by using Monte Carlo simulation and regression analysis. Then, an interdependent multiple objective programming (IMOP) model is proposed. In addition, we give three numerical examples to demonstrate the proposed method. From the numerical results, we can conclude that the proposed method can rationally deal with the problem of MOP with interdependence.

Keywords- Multiple objective programming (MOP); interdependence; interdependent multiple objective programming (IMOP).

I. INTRODUCTION

Since Kuhn and Tucker [1] proposed the concepts of vector optimization, and Koopmans [2] applied the efficient vector method to derive necessary conditions of efficient solution, more and more researchers engaged in studies about multiple objective programming (MOP) problems. The common characteristics of MOP can be described as: (1) a set of quantifiable objectives; (2) a set of well-defined constraints; (3) a process of obtaining some trade-off information, implicit or explicit, between the stated quantifiable objectives and also between stated or unstated non-quantifiable objectives [3]. The general form of MOP can be represented as solving the following mathematical programming problem:

$$\begin{aligned} \max/\min \quad & \{f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_n(\mathbf{x})\}, \\ & \text{over } \mathbf{x} \in X \\ \text{s.t. } \quad & \mathbf{x} = \{\mathbf{x} \in \mathbf{R}^n \mid g_k(\mathbf{x}) \geq 0, \quad k = 1, \dots, m\}. \end{aligned} \quad (1)$$

Although MOP have been widely extended for many issues, such as global optimization [4, 5], multistage [6], and multilevel [7,8,9,10,11], the assumption of independent objectives should be released. That is, supportive and conflicting objectives usually happen in realistic multiple criteria decision making (MCDM) problems, and we should

not ignore the problem of MOP with interdependence between objectives under the viewpoint of optimization.

Carlsson and Fullér [12-14] first proposed two methods to consider the problem of MOP with interdependence by estimating the interdependent grade between objectives. Thereafter, several issues have been proposed to consider further situations, such as uncertainty environment [13,14] and temporal interdependence [15]. However, several shortcomings of their methods should be modified so that we can employ interdependent multiple objective programming (IMOP) in practice. Firstly, their first method [12] does not precisely measure the supportive or conflicting grade between objectives and can only deal with the one-dimensional decision problem. In contrast, their second method [14] can only be suitable for symmetric linear interdependent multiple objective programming (SLIMOP). However, it can be seen that since real-life problems are usually complex and intricate, a general method should be given to handle all kinds of IMOP problems.

In this paper, Monte Carlo simulation and regression analysis are used to measure the grade of interdependence between objectives. The proposed method is suitable not only for the one-dimensional decision problem but also for the high-dimensional decision problem. In addition, the proposed index can also be suitable for asymmetric nonlinear interdependent multiple objective programming (ANIMOP). Three numerical examples are used to demonstrate the proposed method. The results are compared with the conventional MOP and Carlsson and Fullér's model. On the basis of the numerical results, we can conclude that the proposed method can deal with the IMOP problems and overcome the problem in [12-15].

The remainder of this paper is organized as follows. The problem of MOP is discussed in Section 2. The method to solve IMOP is proposed in Section 3. In Section 4, three numerical examples are used to demonstrate the proposed method and compared with the conventional MOP and

Carlsson and Fullér's method. Discussions are presented in Section 5 and the conclusion is in the last section.

II. THE PROBLEM OF IMOP

The main problem of conventional MOP models is the unrealistic assumption of independence between objectives. In order to show the impact, we can transform the MOP model into the single-objective programming (SOP) model by using the following compromise programming [16]:

$$\begin{aligned} \min \quad & r(\mathbf{y}; p) = \|\mathbf{y} - \mathbf{y}^*\|_p, \\ \text{s.t.} \quad & \mathbf{x} = \{\mathbf{x} \in \mathbf{R}^n \mid g_k(\mathbf{x}) \leq 0, \quad k = 1, \dots, m\}, \end{aligned}$$

where $r(\mathbf{y}; p)$ is a measurement of regret from $\mathbf{y}$ to $\mathbf{y}^*$ according to the l_p -norm, $\mathbf{y}$ denotes the objective vector, and $\mathbf{y}^*$ denotes the ideal-point vector.

Assume a biobjective problem and let $p = \infty$. Assume a biobjective problem and let $p = \infty$. Consider a case depicted in Figure 1. Then the optimal solution should be $\mathbf{y}^\infty$ if and only if $f_1(\mathbf{x})$ is independent of $f_2(\mathbf{x})$. However, if $f_1(\mathbf{x})$ supports $\hat{f}_2(\mathbf{x})$, the optimal solution should transfer from $\mathbf{y}^\infty$ to $\hat{\mathbf{y}}_\infty$.

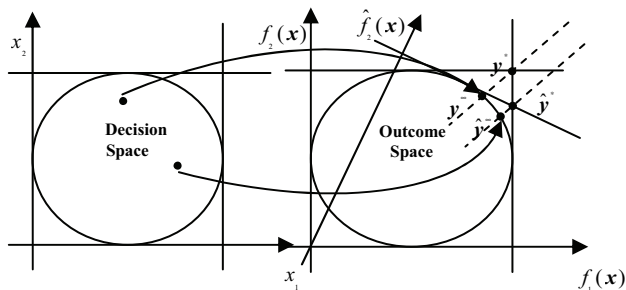


Figure 1. The optimal solution between independent and interdependent objectives

Therefore, if we want to extend MOP to IMOP, we should consider the interdependence between objectives as shown in Figure 2.

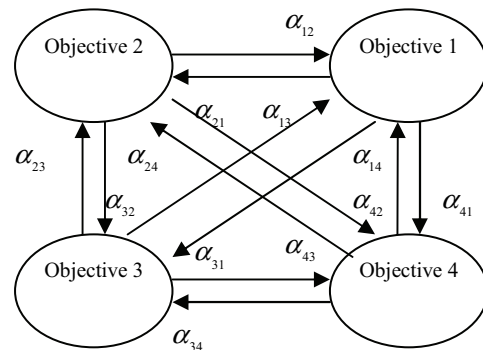


Figure 2. The interdependence with multiple objectives

From Figure 2, it can be seen that the objectives in MOP should be modified [12-14] as:

$$\hat{f}_i(\mathbf{x}) = f_i(\mathbf{x}) + \sum_{j=1, j \neq i}^n \alpha_{ij} f_j(\mathbf{x}), \quad \forall \quad 1 \leq i, j \leq n \quad (3)$$

where α_{ij} denotes the estimated grade of interdependence from the j th objective to the i th objective. Note that from Eq. (3), it can be seen that MOP is a special case while $\alpha_{ij} = 0$ in IMOP.

Therefore, the MOP problem should be reformulated as:

$$\begin{aligned} & \max_{\mathbf{x} \in X} / \min_{\mathbf{x} \in X} \quad \{\hat{f}_1(\mathbf{x}), \hat{f}_2(\mathbf{x}), \dots, \hat{f}_n(\mathbf{x})\} \\ & \text{s.t.} \quad \mathbf{x} = \{\mathbf{x} \in R^n \mid g_k(\mathbf{x}) \leq 0, \quad k = 1, \dots, m\}. \end{aligned}$$

In order to solve the IMOP problem above, the grade of interdependence should be derived first. In [12-14], they defined the grade of interdependence of the i th objective using the following equation:

$$\Delta(f_i) = \sum_{j=1, j \neq i}^n \text{sign}(\alpha_{ji}) \quad (5)$$

where $\Delta(f_i)$ is the number of objectives supported by the i th objective minus the number of objectives hindered by the i th objective.

Although using Eq. (5), we can reformulate conventional MOP to IMOP, two main shortcomings should be overcome so that we can solve IMOP problems more precisely. Firstly, it can be seen that Eq. (5) does not precisely reflect the supportive or conflicting grade between objectives because of using the sign operation. Furthermore, their method can only deal with the one-dimensional decision problem. To overcome the above problem, [4] proposed another index to measure the

grade of interdependence between objectives in the high-dimensional decision space as follows.

Assume two linear objectives, $f_i(\mathbf{x}) = \langle \mathbf{c}_i, \mathbf{x} \rangle$ and $f_j(\mathbf{x}) = \langle \mathbf{c}_j, \mathbf{x} \rangle$ where $\|\mathbf{c}_i\|, \|\mathbf{c}_j\| = 1$, then the grade of interdependence between $f_i(\mathbf{x})$ and $f_j(\mathbf{x})$ can be defined by:

$$k(f_i(\mathbf{x}), f_j(\mathbf{x})) = k(f_j(\mathbf{x}), f_i(\mathbf{x})) = \frac{1 - \langle \mathbf{c}_i, \mathbf{c}_j \rangle}{2}, \quad (6)$$

where $\langle \rangle$ denotes the dot product, $\|\cdot\|$ denotes the length of the specific vector, and $\alpha_{ji} = -2 \times k(f_i(\mathbf{x}), f_j(\mathbf{x})) + 1$.

According to Eq. (6), however, it can be seen the above index can only deal with the linear case. In addition, it is clear that α_{ji} is not always equal to α_{ji} in the realistic problems. In order to overcome the above problems, in this paper, we propose a novel index to measure the interdependent grade between objectives by using Monte Carlo simulation and regression analysis.

III. INTERDEPENDENT MULTIPLE OBJECTIVE PROGRAMMING

As mentioned above, the crucial point of dealing with IMOP is to obtain the grade of interdependence between objectives so that we can reformulate the conventional MOP model. To overcome the problem of interdependence in MOP, we first define the notations as follows.

Let $f_i(\mathbf{x})$ and $f_j(\mathbf{x})$ be two objectives. We say that

- (i) $f_i(\mathbf{x})$ supports $f_j(\mathbf{x})$ on X (denoted by $f_i(\mathbf{x}) \uparrow f_j(\mathbf{x})$) if $f_i(\mathbf{x}') \geq f_j(\mathbf{x})$ entails $f_j(\mathbf{x}') \geq f_i(\mathbf{x})$, for all $\mathbf{x}', \mathbf{x} \in X$; $\partial f_j(\mathbf{x}) / \partial f_i(\mathbf{x}) > 0, \forall \mathbf{x} \in \mathbf{R}^n$;
- (ii) $f_i(\mathbf{x})$ is in conflict with $f_j(\mathbf{x})$ on X (denoted by $f_i(\mathbf{x}) \downarrow f_j(\mathbf{x})$) if $f_i(\mathbf{x}') \geq f_j(\mathbf{x})$ entails $f_j(\mathbf{x}') \leq f_i(\mathbf{x})$, for all $\mathbf{x}', \mathbf{x} \in X$; $\partial f_j(\mathbf{x}) / \partial f_i(\mathbf{x}) < 0, \forall \mathbf{x} \in \mathbf{R}^n$;
- (iii) $f_i(\mathbf{x})$ is independent with $f_j(\mathbf{x})$ on X (denoted by $f_i(\mathbf{x}) \perp f_j(\mathbf{x})$), otherwise.

If $\mathbf{x} \in \mathbf{R}^n$ then we say that $f_i(\mathbf{x})$ supports (or is in conflict with) $f_j(\mathbf{x})$ globally.

If the objective functions are differentiable on X , we may define

$$(i) \quad f_i(\mathbf{x}) \uparrow f_j(\mathbf{x}) \quad \text{on} \quad X \quad \Leftrightarrow \quad \partial_e f_i(\mathbf{x}) / \partial_e f_j(\mathbf{x}) \geq 0, \forall e \in \mathbf{R}^n, \mathbf{x} \in X,$$

$$(ii) \quad f_i(\mathbf{x}) \downarrow f_j(\mathbf{x}) \quad \text{on} \quad X \quad \Leftrightarrow \quad \partial_e f_i(\mathbf{x}) / \partial_e f_j(\mathbf{x}) \leq 0, \forall e \in \mathbf{R}^n, \mathbf{x} \in X,$$

where $\partial_e f_i(\mathbf{x})$ denotes the derivative of $f_i(\mathbf{x})$ with respect to the direction $e \in \mathbf{R}^n$ at $\mathbf{x} \in \mathbf{R}^n$. If for a given direction $e \in \mathbf{R}^n$, $\partial_e f_i(\mathbf{x}) / \partial_e f_j(\mathbf{x}) \geq 0$ holds for all $\mathbf{x} \in X$ then we say that $f_i(\mathbf{x})$ supports $f_j(\mathbf{x})$ with respect to the direction e on X .

It is clear that the information of $\partial f_j(\mathbf{x}) / \partial f_i(\mathbf{x})$ is hard to obtain because it may vary with respect to different decision points. Therefore, the compromise is to find a constant estimated value of $\partial f_j(\mathbf{x}) / \partial f_i(\mathbf{x})$ by α_{ji} . Now, we depict Figure 3 to describe the concept of calculating α_{ji} . First, we transform objective value to satisfaction level $\lambda(\cdot)$. From Figure 3, it can be seen that $\partial f_j(\mathbf{x}) / \partial f_i(\mathbf{x})$ is negative because when the satisfaction level of $f_i(\mathbf{x})$ increases, the satisfaction level of $f_j(\mathbf{x})$ decreases, and vice versa. Due to the problem of obtaining the value of $\partial f_j(\mathbf{x}) / \partial f_i(\mathbf{x})$, we can calculate the estimated value of $\partial f_j(\mathbf{x}) / \partial f_i(\mathbf{x})$, i.e., α_{ji} , by using regression analysis to estimate the best fitted line and obtain the regression coefficient as α_{ji} . Note that except for linear regression analysis, we can also use spline regression analysis to calculate the piece wise situation.

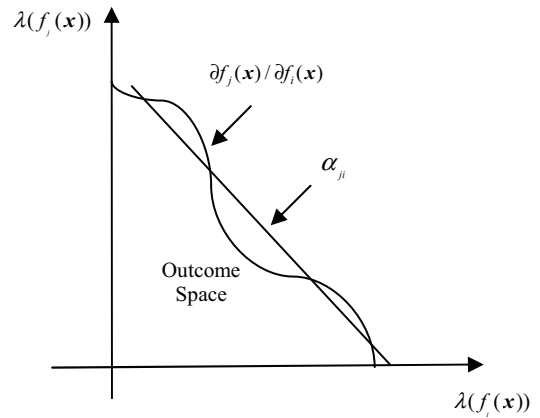


Figure 3. The concept of α_{ji}

According to the above concepts, the procedure of calculating the grade of interdependence between objectives can be developed as follows. First, we employ Monte Carlo simulation to calculate the value of each objective and the

corresponding satisfaction level ($\lambda_i(\mathbf{x})$) which is subject to the specific constraints, as shown in TABLE I.

TABLE I. Monte Carlo Sampling to Calculate Objective and Satisfaction Values.

Monte Carlo Simulation	Calculate						Subject to
	$f_1(\mathbf{x})$	...	$f_n(\mathbf{x})$	$\lambda_1(\mathbf{x})$	...	$\lambda_n(\mathbf{x})$	
1	f_{11}	...	f_{n1}	λ_{11}	...	λ_{n1}	$g_1(\mathbf{x}) \leq 0,$ $\vdots$ $g_m(\mathbf{x}) \leq 0.$
2	f_{12}	...	f_{n2}	λ_{12}	...	λ_{n2}	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
N	f_{1N}	...	f_{nN}	λ_{1N}	...	λ_{nN}	

Note that the satisfaction of each objective can be represented as:

$$\lambda_i(\mathbf{x}) = \begin{cases} 1 & \text{if } f_i(\mathbf{x}) \geq M_i, \\ 1 - \frac{M_i - f_i(\mathbf{x})}{M_i - m_i} & \text{if } m_i < f_i(\mathbf{x}) < M_i, \\ 0 & \text{if } f_i(\mathbf{x}) \leq m_i, \end{cases} \quad (9)$$

where $m_i = \min\{f_{ir}\}, \forall r = 1, \dots, N$ and $M_i = \max\{f_{ir}\}, \forall r = 1, \dots, N$. Next, we give two simple cases to describe the concepts of the proposed method.

Case 1. Consider the following biobjective mathematical programming:

$$\begin{aligned} & \max \{f_1(\mathbf{x}), f_2(\mathbf{x})\}, \\ & \text{where } f_1(\mathbf{x}) = 2x_1^2 - 3x_2^2 \text{ and } f_2(\mathbf{x}) = -3x_1 + x_2^2. \\ & \text{s.t. } x_1 + x_2 \leq 20, \\ & \quad x_1, x_2 \geq 0. \end{aligned}$$

Next, we use Monte Carlo simulation with the uniform distribution and generate 5,000 samples to derive the feasible solutions of the above problem, as shown in Figure 4. It can be seen that Objective 1 and Objective 2 are in conflict with each other and the block line of the feasible solutions indicates the efficient set which represents the grade of interdependence between the objectives. Since the interdependent grade with respect to different point is a non-linear curve, we can estimate the approximate interdependent grade between the objectives by a straight line, as the dash line shown in Figure 4.

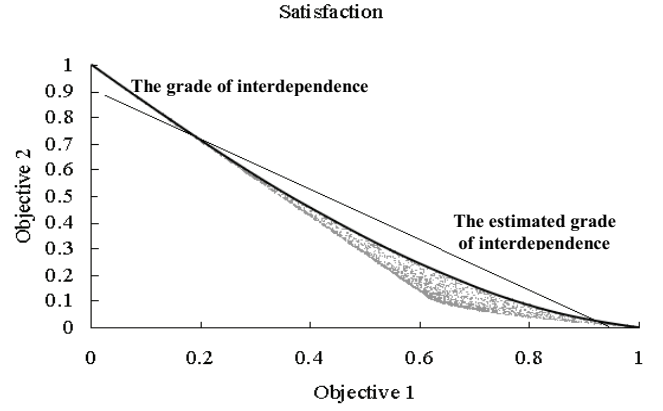


Figure 4. The conflicting case between the objectives.

Case 2. Consider the following biobjective mathematical programming:

$$\begin{aligned} & \max \{f_1(\mathbf{x}), f_2(\mathbf{x})\}, \\ & \text{where } f_1(\mathbf{x}) = x_1^2 + 3x_2^2 \text{ and } f_2(\mathbf{x}) = 2x_1 + 3x_1x_2. \\ & \text{s.t. } 0 \leq x_1, x_2 \leq 2. \end{aligned}$$

With the same procedures in **Case 1**, we can depict Figure 5 to represent the interdependent relationship between Objective 1 and Objective 2. From Figure 5, it can be seen that Objective 1 and Objective 2 are supportive to each other.

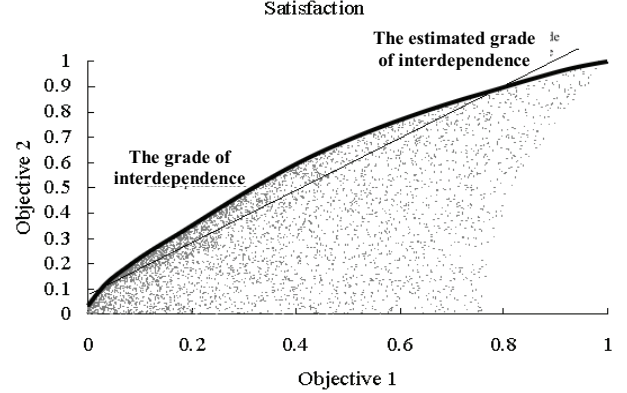


Figure 5. The supportive case between the objectives.

Then, we can calculate the estimated grade of interdependence between the objectives using the following regression equation:

$$\lambda_i(\mathbf{x}) = \mu + \alpha_{ij} \times \lambda_j(\mathbf{x}) + \varepsilon_{ij},$$

where μ denotes the intercept term, α_{ij} is the regression coefficient, and $\varepsilon_{ij} \sim N(0, \sigma^2)$ denotes the error term.

Now, we can incorporate the information about the grade of interdependence between objectives to developing IMOP

by adopting the ideas of Carlsson and Fullér [2-4] to reshape the satisfaction level. The grade of interdependence of the i th objective can be defined as $\Delta(f_i) = \sum_{j=1, j \neq i}^n \alpha_{ji}$. Then, the satisfaction level of the i th objective can be reformulated as:

For $\Delta(f_i) \geq 0$,

$$H_i(\mathbf{x}, \Delta(f_i)) = \begin{cases} 1 & \text{if } f_i(\mathbf{x}) \geq M_i \\ \left(1 - \frac{M_i - f_i(\mathbf{x})}{M_i - m_i}\right)^{1/(\Delta(f_i)+1)} & \text{if } m_i < f_i(\mathbf{x}) < M_i \\ 0 & \text{if } f_i(\mathbf{x}) \leq m_i \end{cases}$$

(12)

For $\Delta(f_i) < 0$,

$$H_i(\mathbf{x}, \Delta(f_i)) = \begin{cases} 1 & \text{if } f_i(\mathbf{x}) \geq M_i \\ \left(1 - \frac{M_i - f_i(\mathbf{x})}{M_i - m_i}\right)^{|\Delta(f_i)|+1} & \text{if } m_i < f_i(\mathbf{x}) < M_i \\ 0 & \text{if } f_i(\mathbf{x}) \leq m_i \end{cases}$$

(13)

The above concepts for reshaping the grade of satisfaction can be depicted by Figure 6:

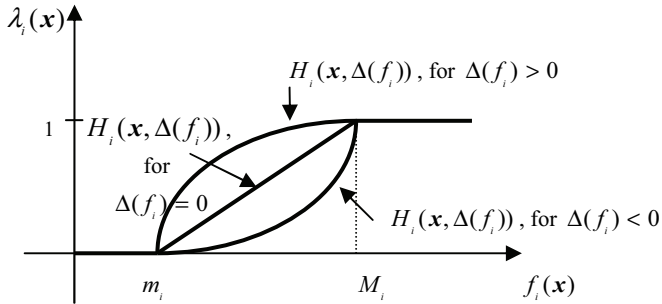


Figure 6. The reshaping grade of satisfaction

Finally, assume the optimal solution of a decision can be represented with “the average, the better.” We can derive the optimal solution and reformulate the conventional MOP model to the following IMOP model:

$$\begin{aligned} \max \quad & T(H_1(\mathbf{x}, \Delta(f_1)), \dots, H_n(\mathbf{x}, \Delta(f_n))), \\ \text{s.t.} \quad & \mathbf{x} = \{\mathbf{x} \in \mathbf{R}^n \mid g_j(\mathbf{x}) \geq 0, \quad j = 1, \dots, m\}, \end{aligned}$$

where $T(\cdot)$ denotes the t -norm operation and indicates the preference structure of a decision maker in this paper. Note that other operations can also be used in the similar way to represent the specific preference structure of a decision maker.

IV. NUMERICAL EXAMPLES

In this section, three numerical examples are used to demonstrate the proposed method for dealing with IMOP problem. In addition, we also employ the conventional MOP model and Carlsson and Fullér's method to compare the results with the proposed method.

Example 1. This example contains five-objective and one-dimensional decision problem and is from Carlsson and Fullér [4].

$$\begin{aligned} \max \quad & \{f_1(x), f_2(x), f_3(x), f_4(x), f_5(x)\}, \\ \text{where,} \quad & f_1(x) = x, \quad f_2(x) = (x+1)^2 - 1, \quad f_3(x) = 2x+1, \\ & f_4(x) = x^4 - 1, \quad \text{and } f_5(x) = -3x+1, \\ \text{s.t.} \quad & 0 \leq x \leq 2. \end{aligned}$$

It is easy to identify the interdependent grade between the objectives using Carlsson and Fullér's method as $\Delta(f_1) = \Delta(f_2) = \Delta(f_3) = \Delta(f_4) = 3 - 1 = 2$, $\Delta(f_5) = 0 - 4 = -4$. On the other hand, we can also use Monte Carlo simulation to calculate the interdependent grade between the objectives, as shown in TABLE II.

TABLE II. The Comparison of Various Methods

α_{21}	α_{31}	α_{41}	α_{51}	α_{12}
0.997	1.000	0.790	-1.000	0.986
α_{32}	α_{42}	α_{52}	α_{13}	α_{23}
0.986	0.836	-0.986	1.000	0.997
α_{43}	α_{53}	α_{14}	α_{24}	α_{34}
0.790	-1.000	0.943	1.000	0.943
α_{54}	α_{15}	α_{25}	α_{35}	α_{45}
-0.943	-1.000	-0.997	-1.000	-0.790

Next, we use MOP, Carlsson and Fullér's method (CF), and the proposed model (i.e., Eq. (11)) to derive the optimal solutions, as shown in TABLE III.

TABLE III. The Comparison of Various Methods

	x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$	$f_5(x)$
MOP	1.76	1.76	6.62	4.52	8.60	-4.28
CF	0.65	0.65	1.72	2.30	-0.73	-0.95
IMOP	0.69	0.69	1.86	2.38	-0.67	-1.07

From TABLE 3, it can be seen that the results of Carlsson and Fullér's method are approximate to the proposed model, because of the similar interdependent grade between the objectives. In addition, compared to MOP, two IMOP models

suggest that we should decrease x to achieve the optimal solution.

Example 2. Consider the following three-objective and three-dimensional linear programming problem:

$$\begin{aligned} & \max \{f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})\}, \\ & \text{where } f_1(\mathbf{x}) = 4x_1 + 2x_2 - 3x_3, \\ & \quad f_2(\mathbf{x}) = 4x_1 + 3x_2 + 2x_3, \\ & \quad f_3(\mathbf{x}) = -7x_1 - 4x_2 + 6x_3. \\ & \text{s.t. } 0 \leq x_1, x_2, x_3 \leq 10. \end{aligned}$$

To calculate the interdependent grade between the objectives, we can use Carlsson and Fullér's method (i.e., Eq.(6)) and Monte Carlo simulation, respectively, as shown in TABLE IV.

TABLE IV. The interdependence grade between objectives of Example 2

CF	α_{21}	α_{31}	α_{12}	α_{32}	α_{13}	α_{23}
	0.552	-0.998	0.552	-0.517	-0.998	-0.517
IMOP	α_{21}	α_{31}	α_{12}	α_{32}	α_{13}	α_{23}
	0.543	-0.988	0.544	-0.503	-1.000	-0.513

By incorporating the information of the interdependent grade between the objectives, we can derive the optimal solutions of **Example 2** using MOP, Carlsson and Fullér's method, and the proposed method, respectively, as shown in TABLE V.

TABLE V. The Comparison of MOP and IMOPs in Example 2

	x_1	x_2	x_3	$f_1(\mathbf{x})$	$f_2(\mathbf{x})$	$f_3(\mathbf{x})$
MOP	2.8571	10.0000	10.0000	1.4286	61.4286	0.0000
CF	7.1987	2.0374	10.0000	2.8694	54.9068	1.4598
IMOP	7.3354	1.7810	10.0000	2.9036	54.6846	1.5282

Similar to Example 1, the proposed method and Carlsson and Fullér's method have the approximate results. Both results show that we should decrease x_2 and increase x_1 to obtain the optimal solution.

Example 3. Consider the following three-objective and three-dimensional nonlinear programming problem:

$$\begin{aligned} & \max \{f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})\}, \\ & \text{where } f_1(\mathbf{x}) = 3x_1^2 + 6x_1x_2 + 2x_2x_3, \\ & \quad f_2(\mathbf{x}) = 4x_1 + 2x_2 + 3x_1x_3, \\ & \quad f_3(\mathbf{x}) = 2x_1x_3 + 3x_2 + 4x_3. \\ & \text{s.t. } 0 \leq x_1 + x_2 + x_3 \leq 10. \end{aligned}$$

Note that since Carlsson and Fullér's method cannot handle this nonlinear problem, only MOP is compared with the proposed model. Next, using Monte Carlo simulation and regression analysis, we can calculate the interdependent grade between the objectives, as shown in Table VI.

TABLE VI. The Interdependence Grade between Objectives of Example 3

α_{21}	α_{31}	α_{12}	α_{32}	α_{13}	α_{23}
-0.5535	-0.2577	-0.4843	0.4915	0.1738	0.5576

Next, we can solve the above problem by using MOP and IMOP to obtain the optimal solution, as shown in TABLE VII.

TABLE VII. The Comparison of MOP and IMOPs in Example 3

	x_1	x_2	x_3	$f_1(\mathbf{x})$	$f_2(\mathbf{x})$	$f_3(\mathbf{x})$
MOP	3.4257	2.2860	4.2883	101.7993	62.34609	53.39206
IMOP	2.7086	3.4074	3.8840	103.8525	49.20918	46.79829

Compared to the results with the conventional MOP, IMOP suggests that we should decrease both x_1 and x_3 and increase to x_2 achieve the optimal solution. Next, we give discussions in Section 5 according to the findings of our examples.

V. DISCUSSIONS

MOP has been widely applied in various areas to deal with MCDM problems. However, the problem of interdependence between objectives seems to be ignored in MOP for dealing well with the realistic decision problems. It can be seen that if we ignore the problem of interdependence between objectives, the result may be huge differences with the optimal solution.

Although Carlsson and Fullér have proposed several papers to deal with the IMOP problems, their method can only deal with the simple IMOP problems and cannot measure the grade of interdependence between the objectives precisely. In this paper, we propose another index by using Monte Carlo simulation and regression analysis to overcome the above problems simultaneously.

From the results of numerical examples, it can be seen that the interdependent grade of the proposed method and Carlsson and Fullér's method have the similar results. Therefore, it enhances the justification of the proposed method. On the other hand, it also shows that the proposed method should be more flexible and useful than Carlsson and Fullér's method because the proposed method can be used in the high-dimensional decision space, linear and nonlinear problems, and asymmetric interdependent grade situations. In addition, all kinds of interdependent grade between objectives can be solved using the standard operating procedures.

VI. CONCLUSIONS

In this paper, the problem of MOP with interdependence between objectives is considered. A novel index is proposed to measure the grade of interdependence between objectives by using Monte Carlo simulation and regression analysis. An IMOP model is developed to solve the IMOP problems. From the numerical results, we can conclude that the proposed method can provide a satisfactory solution to deal with the IMOP problems.

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