

Local Non-Quadratic H-Infinity Control For Continuous-Time Takagi-Sugeno Models

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Abstract — This paper extends to H_∞ performance design a recent local approach for non-quadratic stabilization of continuous-time Takagi-Sugeno (TS) models based on the fuzzy Lyapunov function (FLF). The main obstacle for using FLFs altogether with TS models consists in handling the time-derivatives of the membership functions (MFs). By reducing global stability goals via local considerations, the aforementioned impediment has been removed leading to stabilization domains where H_∞ disturbance rejection is guaranteed. Conditions are expressed as linear matrix inequalities (LMIs), thus enabling systematic and efficient solutions which are solved by convex optimization techniques.

Keywords — Fuzzy Lyapunov Function (FLF), Linear Matrix Inequality (LMI), Local Stabilization, H_∞ disturbance rejection, Takagi-Sugeno (TS) models.

I. INTRODUCTION

SINCE its appearance, Takagi-Sugeno models (TS) [1] have attracted the interest of researchers in control because of their systematic construction methodology which leads to exact convex representations of nonlinear models in a compact set of the state variables [2]. These convex representations facilitate system analysis and control design [3], [4] via the direct Lyapunov method with quadratic Lyapunov functions [5]. Parallel distributed compensation (PDC) is widely used among quadratic schemes because it naturally fits with the structure of TS models providing a fuzzy controller that allows performance design via linear matrix inequalities (LMIs) [6].

Despite its success, there are several sources of pessimism for stabilization of TS models: model construction is not unique [7], membership functions (MFs) can only be partially taken into account through different and not necessarily compatible approaches [8], [9], [10], [11], quadratic Lyapunov

functions are a subclass of more complex and powerful choices that may not be easy to implement. This work extends the existing results in the latter direction since it uses a fuzzy Lyapunov function which includes the quadratic case. Nevertheless, an important feature of the proposed approach lies on its local nature which exploits the fact that TS models are only valid in a compact set of the state space. Global stabilization results as those available in the quadratic case prove to be more restrictive than local results in a non-quadratic framework.

Otherwise than the quadratic framework, piecewise [12], [13], integral [14], and fuzzy Lyapunov functions [15] have been proposed. Among the latter, continuous-time results have not been as powerful as those for the discrete-time case [16], [17], [18]. This unevenness is caused by the presence of time-derivatives of the MFs in the analysis, which destroys the convex structure leading to LMIs. Most of the palliative solutions to this problem assume the knowledge of bounds for the time-derivatives of the MFs, which is an unrealistic assumption especially concerning algebraic loops in control design [19]. Other solutions lead to bilinear matrix inequalities, which cannot be guaranteed to be optimally solved [14].

The approach first appeared in [20] and further extended in [21] for local stability analysis of continuous-time TS models is formulated in LMIs and does not require more than a-priori-known bounds from the TS model construction. Based on these works, a paper proposing a control law for local stabilization has been proposed in [22]. This paper extends the former results for local stabilization of a TS model with H_∞ disturbance rejection.

The paper is organized as follows: section II introduces notation, the considered TS model, the Lyapunov function candidate, and the proposed control law; section III extends to H_∞ disturbance rejection the results in [22]; in section IV some illustrative examples are shown; finally section V closes the paper providing some conclusions and perspectives.

II. PRELIMINARIES

Consider the following nonlinear affine-in-control model:

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$$\begin{aligned}\dot{x}(t) &= f_1(z)x(t) + f_2(z)u(t) + f_3(z)\omega(t) \\ y(t) &= f_4(z)x(t) + f_5(z)u(t) + f_6(z)\omega(t)\end{aligned}\quad (1)$$

with input vector $u(t) \in \mathbb{R}^m$, disturbance vector $\omega(t) \in \mathbb{R}^q$, $\|\omega(t)\|_2 \leq \bar{\omega}$, state vector $x(t) \in \mathbb{R}^n$, premise vector $z(x(t)) \in \mathbb{R}^p$ bounded in a compact region $\Delta \supset \mathbf{0}$, output vector $y(t) \in \mathbb{R}^o$, and $f_1(\cdot), \dots, f_6(\cdot)$ nonlinear functions depending on $z(x(t))$. Let $nl_j(\cdot) \in [\underline{nl}_j, \overline{nl}_j]$, $j \in \{1, \dots, p\}$ be the bounded nonlinearities of (1) in the compact region Δ . By sector nonlinearity approach [3] the following weighting functions are defined to capture nonlinearities in (1):

$$w_0^j(\cdot) = \frac{\overline{nl}_j - nl_j(\cdot)}{\overline{nl}_j - \underline{nl}_j}, \quad w_1^j(\cdot) = 1 - w_0^j(\cdot), \quad j \in \{1, \dots, p\} \quad (2)$$

from which membership functions (MFs)

$$h_i = h_{1+i_1 \times 2 + \dots + i_p \times 2^{p-1}} = \prod_{j=1}^p w_{i_j}^j(z_j), \quad (3)$$

$i \in \{1, \dots, 2^p\}$, $i_j \in \{0, 1\}$, are defined satisfying the convex-sum property $\sum_{i=1}^r h_i(\cdot) = 1$, $h_i(\cdot) \geq 0$ in Δ .

Notation: Arguments are omitted where convenient. An asterisk (*) inside a block matrix represents the transpose of

the symmetric element. Sums $\Upsilon_z = \sum_{i=1}^r h_i \Upsilon_i$, $\Upsilon_z^{-1} = \left(\sum_{i=1}^r h_i \Upsilon_i \right)^{-1}$

will be used to simplify expressions. Thus, an exact representation of nonlinear model (1) in Δ can be written as the following TS model:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^r h_i(z(t)) (A_i x(t) + B_i u(t) + E_i \omega(t)) \\ &= A_z x(t) + B_z u(t) + E_z \omega(t)\end{aligned}\quad (4)$$

$$y(t) = \sum_{i=1}^r h_i(z(t)) (C_i x(t) + D_i u(t) + F_i \omega(t))$$

$$= C_z x(t) + D_z u(t) + F_z \omega(t)$$

where $A_i, B_i, C_i, D_i, E_i, F_i$, $i = 1, \dots, r$ are matrices of the proper size and $r = 2^p \in \mathbb{N}$ is the number of rules (linear models) of the fuzzy interpretation of (4).

Double convex sums involving MFs will appear in the sequel. In order to obtain LMI conditions these MFs must be removed. The following result will be applied to perform this task since it combines simplicity and a good solutions' quality:

Relaxation Lemma [8]: Let Υ_{ij} , $i, j \in \{1, \dots, r\}$ be matrices of certain dimension. Condition

$$\sum_{i=1}^r \sum_{j=1}^r h_i(z(t)) h_j(z(t)) \Upsilon_{ij} < 0 \quad (5)$$

is verified if:

$$\begin{aligned}\Upsilon_{ii} &< 0, \quad \forall i \in \{1, \dots, r\} \\ \frac{2}{r-1} \Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} &< 0, \quad \forall (i, j) \in \{1, \dots, r\}^2, i \neq j.\end{aligned}\quad (6)$$

A non-quadratic control law taken from [16] will be used:

$$u(t) = G_z P_z^{-1} x(t) = \left(\sum_{i=1}^r h_i(z) G_i \right) \left(\sum_{i=1}^r h_i(z) P_i \right)^{-1} x(t) \quad (7)$$

where $G_i \in \mathbb{R}^{m \times n}$ are gains and $P_i \in \mathbb{R}^{n \times n}$, $P_i = P_i^T > 0$ are matrices associated with a fuzzy Lyapunov function. Thus, the closed-loop TS model is:

$$\begin{aligned}\dot{x}(t) &= (A_z + B_z G_z P_z^{-1}) x(t) + E_z \omega(t) \\ y(t) &= (C_z + D_z G_z P_z^{-1}) x(t) + F_z \omega(t)\end{aligned}\quad (8)$$

The following candidate fuzzy Lyapunov function will be adopted [15]:

$$V(x(t)) = x^T(t) P_z^{-1} x(t) = x^T(t) \left(\sum_{i=1}^r h_i(z(t)) P_i \right)^{-1} x(t). \quad (9)$$

III. H_∞ CONTROL

The nonlinear model (1), or its equivalent TS model (4), satisfy the H_∞ attenuation criterion in Δ if disturbances $\omega(t)$ are bounded at the output by:

$$\sup_{\|\omega(t)\|_2 \neq 0} \frac{\|y(t)\|_2}{\|\omega(t)\|_2} \leq \gamma \quad (10)$$

where $\|\cdot\|_2$ stands for the L_2 -norm.

As shown in [3], condition (10) is satisfied if:

$$\dot{V}(x(t)) + y^T(t) y(t) - \gamma^2 \omega^T(t) \omega(t) \leq 0. \quad (11)$$

Taking into account (4) and (9), (11) can be rewritten by developing the time-derivative of $V(x(t))$ along the trajectories of (8):

$$\begin{aligned}\dot{V}(x(t)) + y^T(t) y(t) - \gamma^2 \omega^T(t) \omega(t) &= x^T(t) P_z^{-1} \dot{x}(t) + \dot{x}^T(t) P_z^{-1} x(t) + x^T(t) (\dot{P}_z^{-1}) x(t) - \gamma^2 \omega^T(t) \omega(t) \\ &\quad + \left((C_z + D_z G_z P_z^{-1}) x(t) + F_z \omega(t) \right)^T \left((C_z + D_z G_z P_z^{-1}) x(t) + F_z \omega(t) \right) \\ &= x^T \left(P_z^{-1} (A_z + B_z G_z P_z^{-1}) + (A_z + B_z G_z P_z^{-1})^T P_z^{-1} + \dot{P}_z^{-1} \right. \\ &\quad \left. + (C_z + D_z G_z P_z^{-1})^T (C_z + D_z G_z P_z^{-1}) \right) x + \omega^T (F_z^T F_z - \gamma^2 I) \omega \\ &\quad + x^T \left((C_z + D_z G_z P_z^{-1})^T F_z + P_z^{-1} E_z \right) \omega + \omega^T (F_z (C_z + D_z G_z P_z^{-1}) + E_z^T P_z^{-1}) x \\ &= \begin{bmatrix} x \\ \omega \end{bmatrix}^T \begin{bmatrix} \Gamma_1 & (*) \\ F_z^T (C_z + D_z G_z P_z^{-1}) + E_z^T P_z^{-1} & F_z^T F_z - \gamma^2 I \end{bmatrix} \begin{bmatrix} x \\ \omega \end{bmatrix} \leq 0\end{aligned}\quad (12)$$

with $\Gamma_1 = P_z^{-1} (A_z + B_z G_z P_z^{-1}) + (A_z + B_z G_z P_z^{-1})^T P_z^{-1} + \dot{P}_z^{-1} + (C_z + D_z G_z P_z^{-1})^T (C_z + D_z G_z P_z^{-1})$.

Obviously, (12) is satisfied if the block-matrix therein is not positive-definite. Pre- and post-multiplying the referred matrix

in (12) by full-rank matrix $\begin{bmatrix} P_z & 0 \\ 0 & I \end{bmatrix}$ as well as using the

$$\text{property } P_z \dot{P}_z^{-1} P_z = -\dot{P}_z, \text{ it is possible to guarantee (12) if} \quad (13)$$

$$\begin{bmatrix} \Gamma_2 & P_z C_z^T F_z + G_z^T D_z^T F_z + E_z^T \\ F_z^T C_z P_z + F_z^T D_z G_z + E_z^T & F_z^T F_z - \gamma^2 I \end{bmatrix} \leq 0$$

with $\Gamma_2 = A_z P_z + B_z G_z + P_z A_z^T + G_z^T B_z^T - \dot{P}_z + P_z C_z^T C_z P_z + P_z C_z^T D_z G_z + G_z^T D_z^T C_z P_z + G_z^T D_z^T D_z G_z = A_z P_z + B_z G_z + P_z A_z^T + G_z^T B_z^T - \dot{P}_z + (P_z C_z^T + G_z^T D_z^T)(C_z P_z + D_z G_z)$.

Applying Schur complement to expression (13) gives

$$\begin{bmatrix} A_z P_z + B_z G_z + P_z A_z^T + G_z^T B_z^T - \dot{P}_z & (*) & (*) \\ C_z P_z + D_z G_z & -I & \mathbf{0} \\ F_z^T C_z P_z + F_z^T D_z G_z + E_z^T & \mathbf{0} & F_z^T F_z - \gamma^2 I \end{bmatrix} \leq 0. \quad (14)$$

Any term in inequality (14) is an LMI except for the term $\dot{P}_z = \sum_{i=1}^r \dot{h}_i(z(t)) P_i$ which includes the time-derivatives of the MFs. To deal with this problem, most of the published works on the subject propose a bound of the form $\|\dot{P}_z\| < \phi_z$ [23], [24]. Nevertheless, these bounds cannot be established a priori and can create algebraic loops when applied to stabilization problems. The approach here employed overcomes the aforementioned problem by focusing on local stabilization.

In [20] it has been shown that

$$\begin{aligned} \dot{P}_z &= \sum_{k=1}^p \frac{\partial w_0^k}{\partial z_k} (P_{g_1(z,k)} - P_{g_2(z,k)}) \dot{z}_k \\ &= \sum_{j=1}^r \sum_{k=1}^p h_j \frac{\partial w_0^k}{\partial z_k} (P_{g_1(j,k)} - P_{g_2(j,k)}) \dot{z}_k \end{aligned} \quad (15)$$

with

$$\begin{aligned} g_1(j,k) &= \lfloor (j-1)/2^{p+1-k} \rfloor \times 2^{p+1-k} + 1 + (j-1) \bmod 2^{p-k} \\ g_2(j,k) &= g_1(j,k) + 2^{p-k} \end{aligned} \quad (16)$$

and $\lfloor \cdot \rfloor$ standing for the floor function. Note that

$$\dot{z}_k = \left(\frac{\partial z_k(t)}{\partial x(t)} \right)^T \dot{x}(t) = \left(\frac{\partial z_k(t)}{\partial x(t)} \right)^T (A_z x(t) + B_z u(t) + E_z \omega(t)), \quad (17)$$

so expression (15) can be rewritten as:

$$\begin{aligned} \dot{P}_z &= \sum_{k=1}^p \frac{\partial w_0^k}{\partial z_k} (P_{g_1(z,k)} - P_{g_2(z,k)}) \left(\frac{\partial z_k(t)}{\partial x(t)} \right)^T (A_z x(t) + B_z u(t) + E_z \omega(t)) \\ &= \sum_{k=1}^p \sum_{v=1}^n \sum_{s=1}^n \frac{\partial w_0^k}{\partial x_v} x_s (A_z)_{vs} (P_{g_1(z,k)} - P_{g_2(z,k)}) \\ &\quad + \sum_{k=1}^p \sum_{v=1}^n \sum_{e=1}^m \frac{\partial w_0^k}{\partial x_v} u_e (B_z)_{ve} (P_{g_1(z,k)} - P_{g_2(z,k)}) \\ &\quad + \sum_{k=1}^p \sum_{v=1}^n \sum_{d=1}^q \frac{\partial w_0^k}{\partial x_v} \omega_d (E_z)_{vd} (P_{g_1(z,k)} - P_{g_2(z,k)}). \end{aligned} \quad (18)$$

The terms $\left| \frac{\partial w_0^k}{\partial x_v} x_s \right|$, $\left| \frac{\partial w_0^k}{\partial x_v} u_e \right|$, and $\left| \frac{\partial w_0^k}{\partial x_v} \omega_d \right|$ can now be bounded since their components are known *a priori* or can be designed to be bounded.

In order to bound the input $\|u\|_2 < \mu$, consider $|x(0)| < c$ and $P_z \geq c^2 I$ (or $P_z^{-1} \leq c^{-2} I$); this provided, since the Lyapunov function is a monotonous decreasing function, it can be shown that $V(x(t)) \leq x^T(t) P_z^{-1} x(t) \leq x^T(0) P_z^{-1} x(0) \leq 1$, from which

$\|u\|_2 < \mu$ if $\frac{1}{\mu^2} \|u\|_2^2 \leq x^T(t) P_z^{-1} x(t) \leq 1$, i.e.:

$$\frac{1}{\mu^2} x^T(t) P_z^{-1} G_z^T G_z P_z^{-1} x(t) \leq x^T(t) P_z^{-1} x(t) \quad (19)$$

which by Schur complement can be written as:

$$\begin{bmatrix} P_z & G_z^T \\ G_z & \mu^2 I_m \end{bmatrix} > 0 \quad (20)$$

Then, the aforementioned bounds can be written as:

$$\left| \frac{\partial w_0^k}{\partial x_v} x_s \right| \leq \lambda_{kvs}, \quad \left| \frac{\partial w_0^k}{\partial x_v} u_e \right| \leq \mu \eta_{kv}, \quad \left| \frac{\partial w_0^k}{\partial x_v} \omega_d \right| \leq \bar{\omega} \eta_{kv} \quad (21)$$

In order to use (21) taking into account (18) as to obtain LMI conditions implying (14), expression

$$W + \left(\frac{\partial w_0^k}{\partial x_v} x_s \right) X + \left(\frac{\partial w_0^k}{\partial x_v} u_e \right) Y + \left(\frac{\partial w_0^k}{\partial x_v} \omega_d \right) Z \leq 0 \quad (22)$$

is going to be guaranteed by

$$W \pm \lambda_{kvs} \times X \pm \eta_{kv} \times \mu Y \pm \bar{\omega} \eta_{kv} \times Z \leq 0, \quad (23)$$

which represents all the possible sign combinations for every bounded term.

Theorem: If there are matrices of proper size $P_i = P_i^T > c^2 I$, G_i , satisfying the following LMI conditions for $i, j, k \in \{1, \dots, r\}$, $\alpha \in \{1, \dots, 2^{p \times n \times (n+m+q)}\}$:

$$\begin{bmatrix} P_j & G_j^T \\ G_j & \mu^2 I \end{bmatrix} > 0 \quad (24)$$

$$\frac{2}{r-1} \Upsilon_{iik}^\alpha + \Upsilon_{ijk}^\alpha + \Upsilon_{jik}^\alpha < 0, \quad (25)$$

$$\text{with } \Upsilon_{ijk}^\alpha = \begin{bmatrix} A_i P_j + B_i G_j + P_j A_i^T + G_j^T B_i^T - \bar{P}_{ij}^\alpha & (*) & (*) \\ C_i P_j + D_i G_j & -I & \mathbf{0} \\ F_i^T C_i P_j + F_i^T D_i G_j + E_i^T & \mathbf{0} & F_i^T F_j - \gamma^2 I \end{bmatrix},$$

$$\begin{aligned} \bar{P}_{ij}^\alpha &= \sum_{k=1}^p \sum_{v=1}^n \sum_{s=1}^n (-1)^{b_{kvs}^\alpha} \lambda_{kvs} (A_i)_{vs} (P_{g_1(j,k)} - P_{g_2(j,k)}) \\ &\quad + \sum_{k=1}^p \sum_{v=1}^n \sum_{e=1}^m (-1)^{b_{kve}^\alpha} \eta_{kv} \mu (B_i)_{ve} (P_{g_1(j,k)} - P_{g_2(j,k)}) \\ &\quad + \sum_{k=1}^p \sum_{v=1}^n \sum_{d=1}^q (-1)^{b_{kvd}^\alpha} \eta_{kv} \bar{\omega} (E_i)_{vd} (P_{g_1(j,k)} - P_{g_2(j,k)}), \end{aligned}$$

b_{kvl}^α defined from the binary representation of $\alpha - 1 = b_{pn(n+m+q)}^\alpha + b_{pn(n+m+q-1)}^\alpha \times 2 + \dots + b_{111}^\alpha \times 2^{pn(n+m+q)-1}$, and $g_1(j,k)$, $g_2(j,k)$ defined as in (16), then $x(t)$ in (8) is locally asymptotically stable for every initial condition $x(0)$, $|x(0)| < c$, inside the biggest equipotential R_o contained in

$$R = \bigcap_{e,k,s,v} \left\{ x : \left| \frac{\partial w_0^k}{\partial x_v} x_s \right| \leq \lambda_{kvs}, \left| \frac{\partial w_0^k}{\partial x_v} u_e \right| \leq \mu \eta_{kv} \right\};$$

moreover, the model output $y(t)$ satisfies the H_∞ criterion in (10) for disturbances holding the property $\|\omega(t)\|_2 \leq \bar{\omega}$.

Proof. By relaxation lemma in (6), LMI (25) implies:

$$\Upsilon_{zz}^\alpha = \begin{bmatrix} A_z P_z + B_z G_z + P_z A_z^T + G_z^T B_z^T - \bar{P}_{ij}^\alpha & (*) & (*) \\ C_z P_z + D_z G_z & -I & \mathbf{0} \\ F_z^T C_z P_z + F_z^T D_z G_z + E_z^T & \mathbf{0} & F_z^T F_z - \gamma^2 I \end{bmatrix} \leq 0 \quad (26)$$

Since all the possible sign combinations in $\bar{P}_{ij}^\alpha$ are taken into account and inequalities in (21) bound the time-derivative $\dot{P}_z$, it follows that (26) is a simple extension of property (22); in other words, (26) implies (14), which guarantees the H_∞ criterion in (10) to be satisfied.

Note that bounds in (21) are essential to guarantee implication (26) $\Rightarrow$ (14); these bounds stem from a) the fact that $x(t)$ belongs to a compact set Δ in which the TS model is defined, b) the bounds on the input which are designed via LMIs (24), and c) from the disturbances bound $\bar{\omega}$.

Remark 1: Writing $\dot{P}_z$ in the form described in (18) plays a fundamental role in the proposed local approach. The interested reader is referred to [20] for more details.

Remark 2: Besides the region on which the TS model has been defined, other design parameters intervene in the control law: the bound on the input, the bound on the initial conditions, and the attenuation factor $\gamma > 0$. Bounding the disturbance vector is also a reasonable requirement in this kind of problems.

Remark 3: The theorem above guarantees H_∞ performance in the region $R_o \subseteq R$. Should the biggest region R_o be of interest, a simple algorithm can be implemented by linear programming to find it [20]: a) initialize η_{kv} , λ_{kvs} to any initial feasible bounds η_{kv}^0 , λ_{kvs}^0 ; b) search the biggest value $\varepsilon > 0$ such that conditions in the theorem above are satisfied with $\eta_{kv} = \varepsilon \eta_{kv}^0$ and $\lambda_{kvs} = \varepsilon \lambda_{kvs}^0$. Consider $R_o \subseteq R$ defined from the last values thus found.

Remark 4: The previous algorithm is compatible with the minimization of the attenuation factor γ . Once ε is established, the minimum attenuation factor γ guaranteeing H_∞ performance design in the region thus defined can be searched as an LMI problem.

IV. EXAMPLES

Two examples are shown in this section to illustrate the advantages of the proposed methodology. The first one is stabilized in the absence of disturbances to show the influence of certain design parameters; the second one shows the effectiveness of the H_∞ control design for different regions and parameters.

Example 1: Consider the following TS model:

$$\dot{x}(t) = \sum_{i=1}^2 h_i(z(t)) (A_i x(t) + B_i u(t)) \quad (27)$$

with system matrices

$$A_1 = \begin{bmatrix} 2 & -10 \\ 2 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -20 & -5 \\ 1 & 2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix},$$

and MFs

$$h_1(x_1) = \frac{1 - \sin(x_1)}{2}, \quad h_2(x_1) = \frac{1 + \sin(x_1)}{2},$$

$$\text{defined in } \Delta = \left\{ x \in \mathbb{R}^2, |x_i| < \frac{\pi}{2} \right\}.$$

When no perturbations are considered, the proposed approach depends upon the parameter μ (bound on the input $|u| < \mu$) and parameter c (bound on the initial conditions $|x(0)| < c$). In practice, these bounds are directly derived from the model, so LMI conditions in the theorem above are fully determined.

Fig. 1 shows the influence of parameter μ for the model (27) with $|x(0)| < c = \sqrt{2}\pi/2$. As expected, the bigger the values of μ , the bigger the stabilization region R_o . To determine the biggest stabilization region, the algorithm in Remark 3 has been used. Four state trajectories are shown to illustrate the effectiveness of the stabilization method with $|u| < \mu = 40$ (solid trajectories with arrows pointing towards the origin).

In Fig. 2 the effects of changing the modelling region of the TS model (27) for $\Delta = \{x \in \mathbb{R}^2, |x_i| < \pi/4\}$ are studied. In both regions, bounds $|x(0)| < c = \pi/2$ and $|u| < \mu = 30$ are assumed. Dotted lines show the borders of R for the original case.

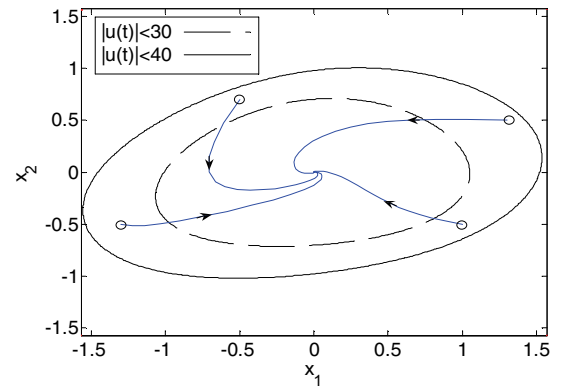


Figure 1: Effects of the input bound μ .

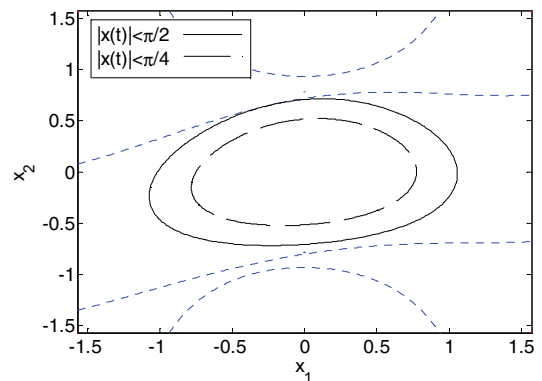


Figure 2: Effects of the region Δ .

It is important to notice that augmenting the modelling region of the TS model do not necessarily mean that a bigger stabilization region will be found. Bigger modelling regions imply that the initial conditions can be further from the origin, thus reducing feasibility of LMIs in (24) since $P_i = P_i^T > c_i^2 I$, $|x(0)| < c$.

Example 2: Consider the following TS model subject to scalar perturbations $\omega(t) = e^{-t} \cos(t)$, $|\omega(t)| \leq 1$:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^2 h_i(z(t)) (A_i x(t) + B_i u(t) + E_i \omega(t)) \\ y(t) &= \sum_{i=1}^r h_i(z(t)) C_i x(t) \end{aligned} \quad (28)$$

with system matrices

$$\begin{aligned} A_1 &= \begin{bmatrix} -5 & -4 \\ -1 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & -4 \\ 20 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 10 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \\ C_1 &= C_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T, \quad E_1 = E_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \end{aligned}$$

and MFs $h_1(x_1) = \frac{1 - \sin(x_1)}{2}$, $h_2(x_1) = \frac{1 + \sin(x_1)}{2}$ defined in

$$\Delta = \left\{ x \in \mathbb{R}^2, |x_i| < \frac{\pi}{2} \right\}.$$

As mentioned in Remark 4, given a specific region

$$R = \bigcap_{e,k,s,v} \left\{ x : \left| \frac{\partial w_0^k}{\partial x_v} x_s \right| \leq \lambda_{kvs}, \left| \frac{\partial w_0^k}{\partial x_v} u_e \right| \leq \mu \eta_{kv} \right\}, \quad \text{the theorem}$$

above can be used to minimize the attenuation H_∞ factor $\gamma > 0$.

Since for TS model (28) in Δ the following bounds hold $\left| \frac{\partial w_0^k}{\partial x_1} x_s \right| \leq \left| \frac{\cos(x_1)}{2} \cdot \frac{\pi}{2} \right| \leq \frac{\pi}{4}$, and $\left| \frac{\partial w_0^k}{\partial x_2} x_s \right| = 0$, let's define

$$R_{l,\mu} = \bigcap_{e,k,s,v} \left\{ x : \left| \frac{\partial w_0^k}{\partial x_v} x_s \right| \leq \frac{\pi}{4} l, \left| \frac{\partial w_0^k}{\partial x_v} u_e \right| \leq \frac{\pi}{2} \mu l \right\} \quad \text{in order to}$$

check the behaviour of the minimum γ found with the aforementioned algorithm under the testing values $(l, \mu) = ((11-i)/10, 1)$ (different sizes of R) and $(l, \mu) = (1, 10^{i-1})$ (different bounds for μ), $i = 1, \dots, 9$.

Fig. 3 shows the behaviour of the minimum γ found via the proposed theorem when parameter l varies, thus changing the modelling region of the TS model. As expected, since this is a local approach, the smaller the modelling region, the smaller minimum attenuation factor γ . In other words, restricting the state space it is easier to satisfy the constraints for H_∞ disturbance rejection.

Fig. 4 shows the behaviour of the minimum attenuation factor γ found through the theorem above when the input bound μ varies. Logically, bigger values of μ allow smaller values of γ , since bigger control gains provide higher flexibility in (7).

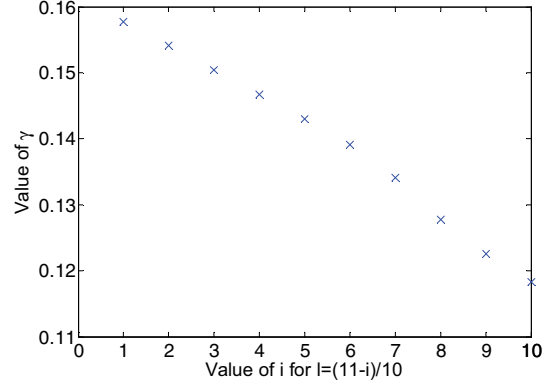


Figure 3: Attenuation factor with different regions (parameter l) and fixed input bound $\mu = 1$.

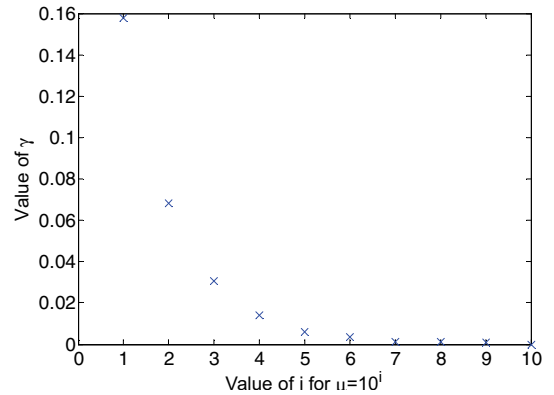


Figure 4: Attenuation factor with different input bounds μ and fixed region size $l = 1$.

V. CONCLUSIONS AND PERSPECTIVES

This paper presented a novel non-quadratic approach for stabilization of continuous-time TS models with H_∞ disturbance rejection. Based on a recently appeared local approach that solves the problem of handling time-derivatives of the MFs, fuzzy Lyapunov functions have been used to design a non-quadratic control law which stabilizes and performs H_∞ disturbance rejection for nonlinear models locally exactly represented via TS models. Two examples are included to illustrate the effectiveness of the approach and to show the effect of the different design parameters.

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