

Fuzzy Active Contour Models

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Abstract—This paper presents a Fuzzy Active Contour Model for image segmentation using three variations. The proposed models are based on the Fuzzy Energy-Based Active Contour model introduced by Krinidis and Chatzis. First, an update criteria that changes only localized membership values at each iteration is introduced. Second, the model is extended to a type-2 fuzzy logic. And finally, a multiple object segmentation schema is applied to the original model. We present some experimental results, showing the performance for each modification and some of its advantages.

Keywords—Image segmentation, active contour models, fuzzy logic

I. INTRODUCTION

In problems related to image processing and computer vision, image segmentation is an important task that can be critical for the success of any image related procedure. Image segmentation is defined as the process to localize or isolate regions or object of interest in a particular image. Several different procedures for image segmentation have been proposed [1]. However, the segmentation procedure is still considered an essential and a difficult process, specially due to the complexity presented by several images.

Active contour models (ACM) are a well known technique used in image segmentation. Due to its capabilities, ACM are successfully applied in object detection and motion tracking [2], [3]. The idea behind active contour models (or snakes) is a curve that evolves subject to restrictions imposed by characteristics of the images and the curve itself. This evolution is guided by a minimization of an energy function.

The classical active contour models use energy functions based on edge information. Therefore they can only detect objects defined by explicit edges. Objects with smooth or discontinuous edges may not be properly detected. Many authors developed different techniques to overcome this limitation [3], [4], [5]. Chan and Vese proposed a well-known model called Active Contours Without Edges. This model uses an energy function based on the Mumford-Shah segmentation technique and is able to detect contour both with or without gradient information [3].

In Fuzzy Energy-Based Active Contour model [6], proposed by Krinidis and Chatzis, the clustering capabilities presented by fuzzy systems are combined with active contour methodology. It uses fuzzy logic to define the membership of each pixel in the image to the object. So, the fuzzified energy is able to reject local minima, which is a characteristic of

fuzzy models. Krinidis and Chatzis used a pseudo level-set formulation that allows the minimization not to be computed using the Euler-Lagrange equations. This procedure decreases the computational time compared with other techniques that use Euler-Lagrange approach. The model is also able to detect objects with discontinuous or smooth edges.

The Fuzzy Energy-Based Active Contour model achieves good segmentation results when applied to two phase images, but it is not appropriate when gradual tonality variations appears in the image. Since it fails to find the object contour, it considers as object other elements of the scene. We propose to modify the Fuzzy Energy-Based Active Contour model so that it is able to deal with more complex images. Besides that, we also extend the model to type-2 fuzzy, in which each primary membership value has a correspondent secondary membership. Increasing the fuzziness of the model means increase the ability to handle inexact information [7]. Last but not least, we present a generalization on the base model that allows it to segment images in more than two areas.

This paper is organized as follows. The base model is described in Section 2. In Section 3, we describe the proposed modifications to the base model: the Morphological Fuzzy Active Contour, Morphological Type-2 Fuzzy Active Contour and Fuzzy Multi Active Contour models. Section 4 describes the experimental results and Section 5 presents the conclusions.

II. FUZZY ENERGY-BASED ACTIVE CONTOUR MODEL (FEB-AC)

The Fuzzy Energy-Based Active Contour model consists in an algorithm that aims to segment regions of interest in a two phase image. First, it is provided to the algorithm an initial partition of the image, i.e., an initial contour. This partition defines the curve that will iteratively evolve by a minimization process. This curve defines the boundaries of the segmented region.

Considering C a curve that is able to evolve in the image domain Ω and I an image composed by two regions, the energy function to be minimized is defined by [6]:

$$F(C, c_1, c_2, u) = \mu \cdot \text{Length}(C) + \lambda_1 \int_{\Omega} [u(x, y)]^m |I(x, y) - c_1|^2 dx dy + \lambda_2 \int_{\Omega} [1 - u(x, y)]^m |I(x, y) - c_2|^2 dx dy, \quad (1)$$

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where u is the membership function, m is a weighting exponent to the fuzzy membership, $\mu \geq 0$ and $\lambda_1, \lambda_2 > 0$, are fixed values. c_1 and c_2 are the average intensity of the inside and outside areas of the object, respectively, both taking into account their correspondent membership values. c_1 and c_2 are defined by:

$$c_1 = \frac{\int_{\Omega} [u(x, y)]^m I(x, y) dx dy}{\int_{\Omega} [u(x, y)]^m dx dy}, \quad (2)$$

$$c_2 = \frac{\int_{\Omega} [1 - u(x, y)]^m I(x, y) dx dy}{\int_{\Omega} [1 - u(x, y)]^m dx dy}. \quad (3)$$

Keeping the values of c_1 and c_2 fixed, the membership of each pixel, $u(x, y)$, is then computed using:

$$u(x, y) = \frac{1}{1 + \left(\frac{\lambda_1 (I(x, y) - c_1)^2}{\lambda_2 (I(x, y) - c_2)^2} \right)^{\frac{1}{m-1}}}. \quad (4)$$

In (1), the length term is approximated by:

$$Length(C) = \sum_{i,j} \sqrt{(Q_{i+1,j} - Q_{i,j})^2 + (Q_{i,j+1} - Q_{i,j})^2}, \quad (5)$$

where $Q_{i,j} = H(u(i, j) - 0.5)$, and $H(\cdot)$ is the Heaviside function [6].

Instead of using a Lipschitz function ϕ , Krinidis and Chatizis used a pseudo level-set formulation, based on the membership function u , to represent different areas of the image, i.e., the curve and the inner and outer regions. The pseudo level-set formulation is defined as:

$$\begin{cases} C &= \{(x, y) \in I \mid u(x, y) = 0.5\} \\ inside(C) &= \{(x, y) \in I \mid u(x, y) > 0.5\} \\ outside(C) &= \{(x, y) \in I \mid u(x, y) < 0.5\}. \end{cases}$$

The algorithm that minimizes the fuzzy energy function $F(C, c_1, c_2, u)$ is described as follows [6]:

- 1) Assign an initial partition to the image. For the inner part use $u > 0.5$, for the outer part use $u < 0.5$;
- 2) Compute c_1 and c_2 using equations (2) and (3), respectively;
- 3) Update the membership value u_o of the pixel I_o with u_n , following equation (4), only if the total energy decreases. That is, $\Delta F < 0$, where this variation is defined by:

$$\Delta F = \lambda_1 s_1 \frac{[u_n^m - u_0^m](I_o - c_1)^2}{s_1 + u_n^m - u_0^m} + \lambda_2 s_2 \frac{[(1 - u_n)^m - (1 - u_0)^m](I_o - c_2)^2}{s_2 + (1 - u_n)^m - (1 - u_0)^m}, \quad (6)$$

where $s_1 = \sum_{i,j} [u(i, j)]^m$ and $s_2 = \sum_{i,j} [1 - u(i, j)]^m$;

- 4) Repeat Step 3 for each pixel of the image;
- 5) Repeat Steps 2 to 4 while the total energy keeps changing.

This model achieves good results detecting objects with boundaries that are not defined by gradient and also in the

presence of noise. However, it has some drawbacks when dealing with images that present strong pixel intensity variation in its background. The proposed method tries to overcome this limitation by applying an evolving curve procedure similar to other active contour techniques.

III. PROPOSED MODIFICATIONS

Active contour models that possess a geometric formulation, as in level-sets, use a high order function ϕ to define the curve boundaries and to control its evolution. This kind of model evolves the contour in regions near to the boundaries.

Due to the pseudo level-set formulation used in Fuzzy Energy-Based Active Contour model, the membership of all pixels in the image may be updated in a single iteration. Consequently, the curve can evolve not only in regions near to its boundaries. In this situation the contour is not continuous, leading to incorrect segmentation results.

A. Morphological Fuzzy Active Contour Model (MF-AC)

In this work we propose a modification of the model introduced by Krinidis and Chatizis that enables the contour to evolve gradually. The main idea is to limit the update area in each iteration to those close to the current contour boundaries. Thus, as the membership function is updated, the border moves, therefore, enabling new areas to be updated. To limit the update area it is necessary to modify the membership values update equation (4) to:

$$u'(x, y) = \frac{\delta(u'(x, y) - 0.5)}{1 + \left(\frac{\lambda_1 (I(x, y) - c_1)^2}{\lambda_2 (I(x, y) - c_2)^2} \right)^{\frac{1}{m-1}}}, \quad (7)$$

where $\delta(\cdot)$ is the δ Dirac function. Thus, δ act as a weighting coefficient to control the update of the membership values. In this way only the pixels at the boundaries are allowed to update their membership values. However, due to the expression in the denominator of (7), it is not easy to accurately find the boundaries of the contour using this function.

Figure 1 shows this situation: (a) shows the original histogram of an image and (b) shows the histogram of the membership values after the first iteration. The fuzzy energy pushes the membership values towards the centers c_1 and c_2 , creating a region with low pixel density close to the value 0.5. Due to this behavior, it is hard to define the boundaries using the δ function. To overcome this problem, we applied mathematical morphology to obtain this desired behavior. Therefore, the new update rule is stated as follows:

$$u'(x, y) = \frac{\beta(H(u'(x, y) - 0.5)) \oplus R}{1 + \left(\frac{\lambda_1 (I(x, y) - c_1)^2}{\lambda_2 (I(x, y) - c_2)^2} \right)^{\frac{1}{m-1}}}, \quad (8)$$

where $\beta(A) = A - (A \ominus B)$ is a morphological boundary extraction operation from image A using suitable structuring element B to cause an erosion and R is another structuring element used to expand the boundaries to neighbor areas. We used the structuring elements B and R , respectively, as a 3x3 square and a circle with radius 5.

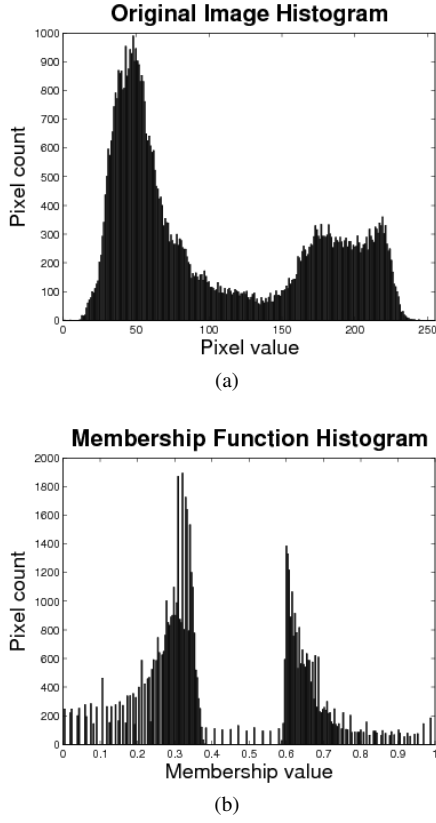


Figure 1: Histogram comparison. (a) Histogram of the original image. (b) Histogram of the membership function after the first iteration.

B. Morphological Type-2 Fuzzy Active Contour Model (MT2F-AC)

Type-2 fuzzy sets are a generalization of common type-1 fuzzy sets. In type-2, the membership value itself is a fuzzy set. In that way, with type-2 fuzzy is possible to model uncertainties because their membership functions are fuzzy themselves. This does not happen with type-1 fuzzy, in which membership functions are totally crisp [8].

In this model we used interval type-2 fuzzy set, that are also called first-order uncertainty fuzzy models. In this case, secondary memberships are either zero or one, reducing the complexity compared when dealing with general type-2 fuzzy [7]. To do so, we extend the type-1 membership values u , calculated using (4), to the type-2 $\tilde{u}$ by assigning a membership function to each membership value of type-1. We have tried four different functions (graphically shown in Figure 2):

- Triangular (Figure 2a):

$$\tilde{u}(x, y) = u(x, y) - \frac{1 - u(x, y)}{2} \quad (9)$$

- Exponential (Figure 2b):

$$\tilde{u}(x, y) = e^{u(x, y)^2} - 1 \quad (10)$$

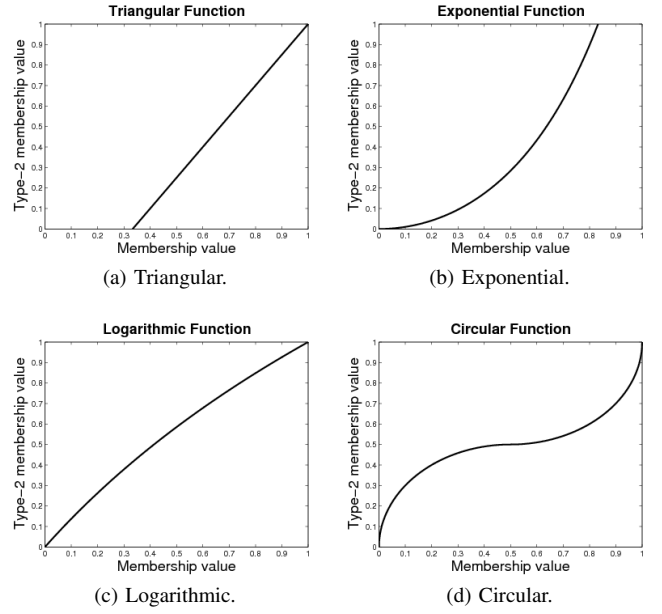


Figure 2: Type-2 membership functions.

- Logarithmic (Figure 2c):

$$\tilde{u}(x, y) = \log_2(u(x, y) + 1) \quad (11)$$

- Circular (Figure 2d):

$$\tilde{u}(x, y) = \begin{cases} \sqrt{u(x, y) - u(x, y)^2}, & \text{if } u(x, y) < 0.5 \\ 1 - \sqrt{u(x, y) - u(x, y)^2}, & \text{else} \end{cases} \quad (12)$$

Then, instead of using u , which is type-1, we applied the type-2 values $\tilde{u}$ in equations (2) and (3), represented now by $\tilde{c}_1$ and $\tilde{c}_2$, respectively. So, the total energy is given by:

$$F(C, \tilde{c}_1, \tilde{c}_2, \tilde{u}) = \mu \cdot \text{Length}(C) + \lambda_1 \int_{\Omega} [\tilde{u}(x, y)]^m |I(x, y) - \tilde{c}_1|^2 dx dy + \lambda_2 \int_{\Omega} [1 - \tilde{u}(x, y)]^m |I(x, y) - \tilde{c}_2|^2 dx dy, \quad (13)$$

Similarly, the length term (5) and the partial energy variation (6) are computed applying new type-2 membership values.

C. Fuzzy Multi Active Contour Model (FM-AC)

The method described in this section extends the Morphological Fuzzy Active Contour Model to use multiple centers, allowing multiple regions be segmented at once. The main difference from the FEB-AC model relies on the pseudo level-set formulation. Instead of using a (similar) distance function to define inner and outer areas (because only two regions are assumed) the new formulation use group indexes to define regions. In this case, the contour is extracted finding the boundaries between groups.

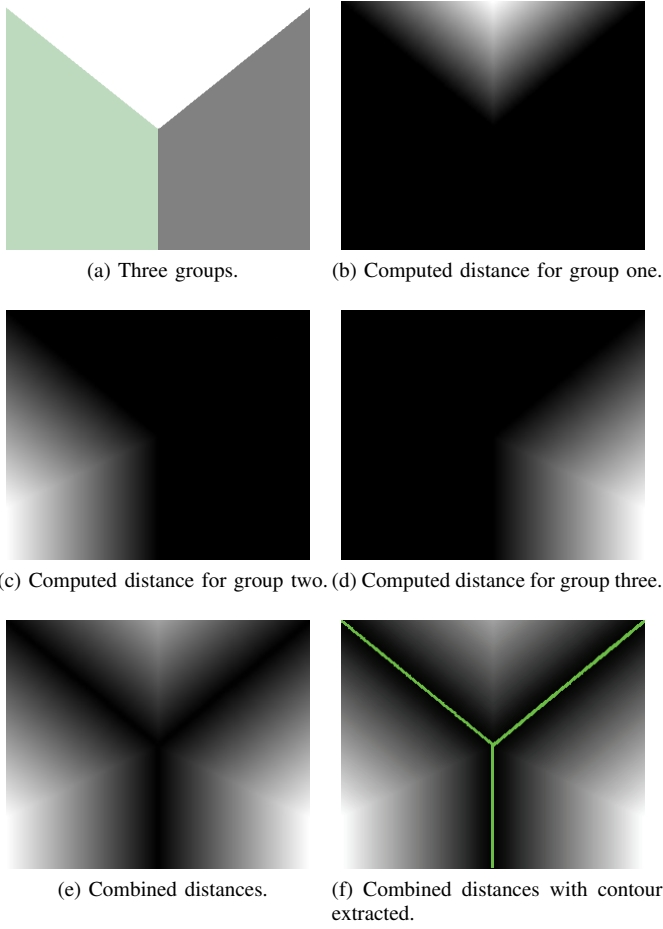


Figure 3: The process used to identify the contour between different groups. The most distant pixels to any other group are displayed in white.

In this proposed modification, the membership function u is used to assign pixels to each of the K disjoint groups ($K > 2$). This process is represented by:

$$I(x, y) \in k \mid u_k = \max(u_j), j = 1, 2, \dots, K.$$

Then, the contour is extracted, finding the borders between regions. To perform this operation we first find all pixels that belongs to a disjoint group k , then we compute the distance d_k from these pixels to the nearest pixel that belongs to any other group. In each group, pixels that are neighbor with other group will have a distance value equals to 1, and the pixels in other groups will have the value equals to zero. This process is repeated for every group and the distances $d_k(x, y)$ are combined (summed). Finally, the contour C is extracted by finding the positions (x, y) with minimal distance (in this case the minimum value is one):

$$C = \left\{ (x, y) \in I \mid \sum_{k=1}^K d_k(x, y) = 1 \right\}. \quad (14)$$

Figure 3 shows an example on how to obtain the contour: (a) a general partition with three groups; (b, c and d) the computed distances d_k for each group; (e) the combined distances; and (f) the combined distances with the contour highlighted.

To allow the use of multiple centers, the fuzzy energy (1) is modified to:

$$F(C) = \mu \cdot \text{Length}(C) + \sum_{k=1}^K \lambda_k \int_{\Omega} [u_k(x, y)]^m |I(x, y) - c_k|^2 dx dy, \quad (15)$$

where $\text{Length}(C)$ is just the number of pixels in the C set.

The centers c_k are now computed using:

$$c_k = \frac{\int_{\Omega} [u_k(x, y)]^m I(x, y) dx dy}{\int_{\Omega} [u_k(x, y)]^m dx dy}, \quad (16)$$

and the membership update rule follows:

$$u_k(x, y) = \frac{1}{\sum_{j=1}^K \left(\frac{\lambda_k (I(x, y) - c_k)^2}{\lambda_j (I(x, y) - c_j)^2} \right)^{\frac{1}{m-1}}}. \quad (17)$$

The algorithm is stated as:

- 1) Provide an initial partition with K disjoint groups and compute the initial membership values. We used:

$$u_k(x, y) = \begin{cases} 0.5 & , \text{if } (x, y) \in k \\ 0.5/(K-1) & , \text{elsewhere} \end{cases} \quad (18)$$

- 2) Compute the centers c_k using (16);
- 3) Update the membership values using (17) where $(x, y) \in I \mid \sum_{k=1}^K d_k(x, y) \leq \alpha$; α defines the thickness around the contour;
- 4) Compute the total energy of the model using (15);
- 5) Repeat Steps 2 to 4 while the total energy keeps changing.

IV. EXPERIMENTAL RESULTS

The experimental results are presented by applying the proposed models to both natural and artificial images. First, we compare the results using the FEB-AC and the MF-AC models. Then, we applied the type-2 fuzzy model (MT2F-AC). Finally, a FM-AC segmentation experiment is also presented. In all experiments we used $\lambda_1 = \lambda_2 = \lambda_k = 1$, and $m = 2$. The initial membership value to the inner region of the contour was set to $u = 0.8$, except for the FM-AC model which follows (18). Finally, the value of μ showed minor changes to the final contour and its value was set empirically to zero.

We present in Figure 4 a comparison of some intermediary steps between FEB-AC and MF-AC models. Note that the original model converges to a wrong configuration that is close to the final solution in early steps of the procedure. On the other hand, with the proposed modifications the contour evolves gradually, correcting the initial partition and approaching to the desired solution. In the FEB-AC model the update of the membership values can occurs on the whole image. In the MF-AC model, only the pixels in a narrow band

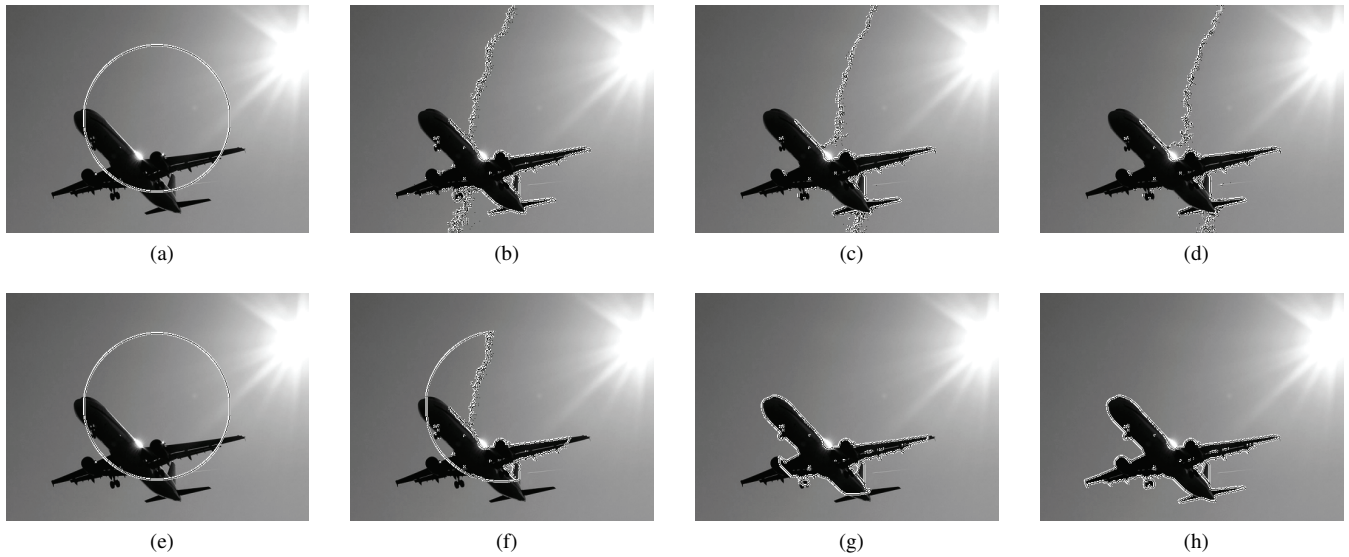


Figure 4: Comparison between the results of the segmentation procedures applying in an image of containing an airplane (including some intermediary steps): (a) to (d) using the FEB-AC model, and (e) to (h) using the MF-AC model.

near the boundaries are able to have their membership updated. In this case there is a localized energy decrease.

Figure 5 shows the segmentation results for FEB-AC and MF-AC models, using a flower image. This image has a distinct and well defined foreground and background and both models have similar results, except in the area near the center of the flower. The FEB-AC model correctly find the borders between the flower and the background but the interior, which is darker then the petals, is excluded from the object. The MF-AC model in addition to correctly detect the borders, was also able to include the interior of the flower. This behavior results from the localized membership update.

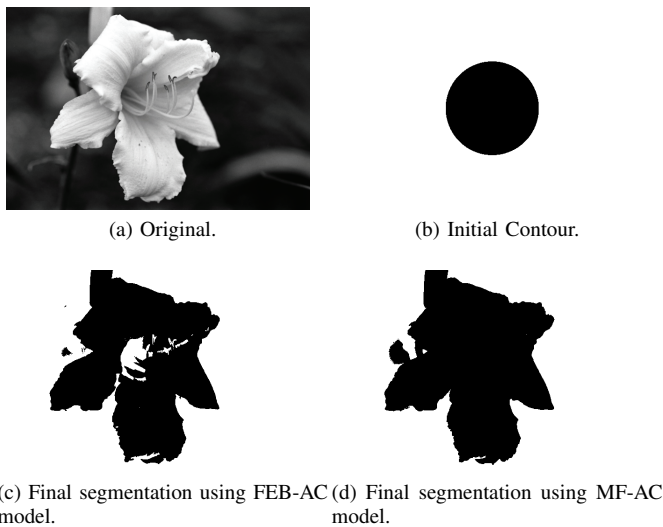


Figure 5: A flower image and the segmentation results using the FEB-AC and MF-AC models.

Figure 6 shows a comparison between the segmentation results between FEB-AC and MF-AC models. Note that the background in the space shuttle image presents high variation. As the FEB-AC model allows all pixels to have their membership updated at once, this variation in the background interfere in the computation of c_1 and c_2 constants and in the membership u itself. Due to this fact, the model was unable to correctly segment the contour of the desired object. In this image, the MF-AC model was able to extract the desired object because of the restriction on membership update.

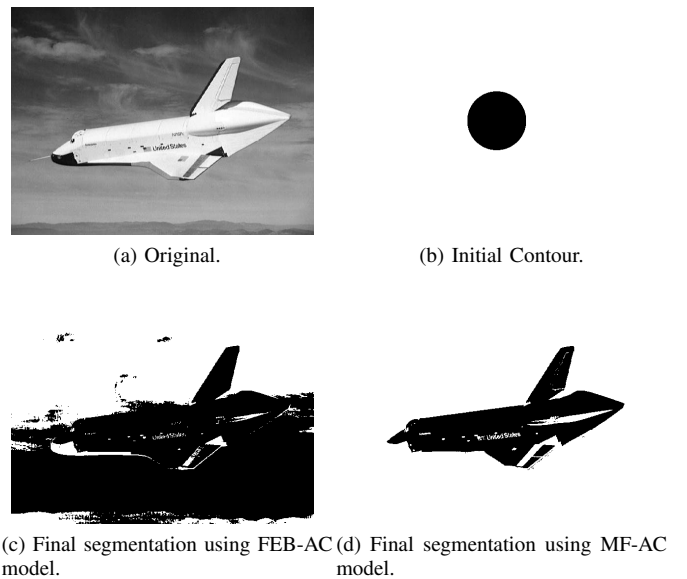


Figure 6: A Space shuttle image and the segmentation results using the FEB-AC and MF-AC models.

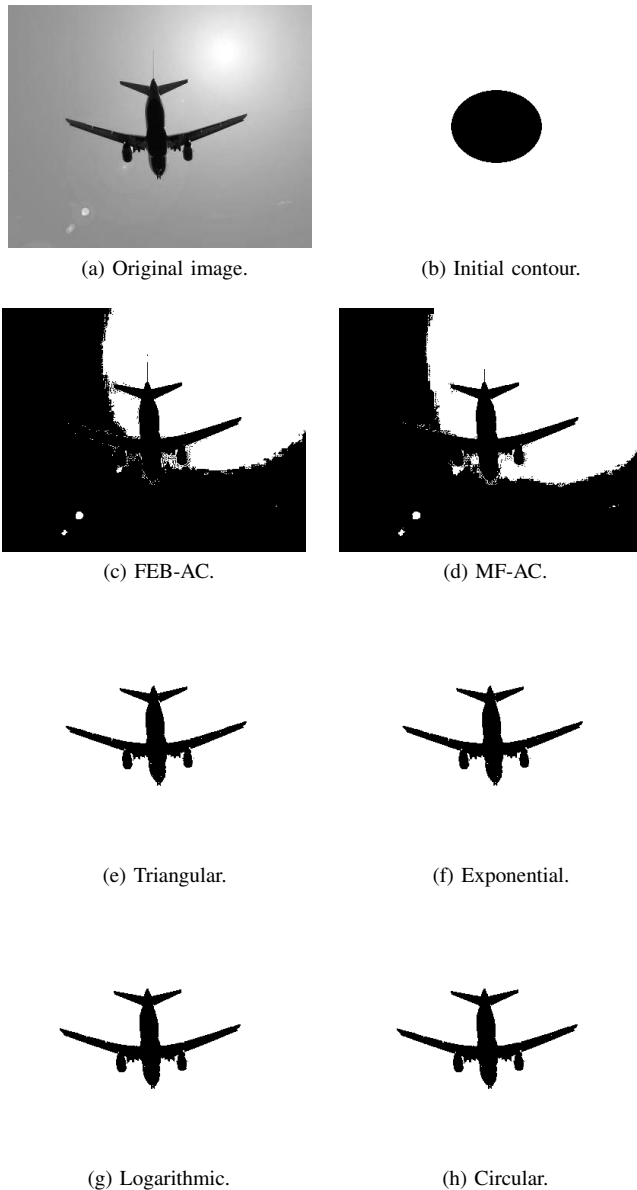


Figure 7: Segmentation results using: (c) FEB-AC; (d) MF-AC; (e, f, g, and h) MFT2-AC with different type-2 functions. The comparison shows that type-2 models are able to detect the airplane under high background variation.

In Figure 7 experimental results using another airplane image are presented. In this case the MFT2-AC model was used. This image presents a strong spot of light that contributes to a high illumination variation in the background of the scene, causing the FEB-AC and MF-AC models to loose the contour of the central object. This effect does not happen with any of the versions of the proposed MFT2-AC model. The introduced type-2 mapping functions contribute to the algorithm to find the correct contour of the object.

The use of triangular, exponential, and logarithmic type-2 functions spreads the membership values between inner and

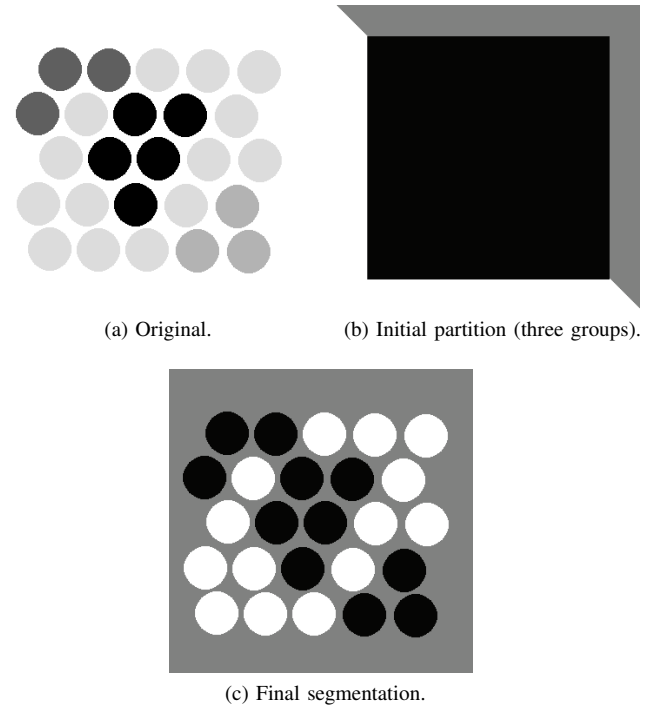


Figure 8: An artificial grayscale image, segmented using FM-AC model.

outer regions. The circular type-2 function, however, narrows membership values to a small range near the value of the contour ($u = 0.5$). Besides that, in this image, the circular function was also able to segment the airplane.

The use of fuzzy multiple centers model (FM-AC) is showed in Figure 8. In this experiment, an artificial image composed by dots of different grayscale intensities was used. The image, originally with five different intensities was segmented in three groups using pixel intensity similarity to join the groups.

These experiments give an overview of the capabilities of proposed models: by applying an evolution scheme in the base model, the algorithm is capable to handle the segmentation more efficiently. Type-2 fuzzy sets can deal with some scenes that the MF-AC is not able to correctly segment. The change in the membership function may, for instance, give more importance to a higher membership values and decreases even more the memberships that are already small. This adjustment, depending on the image, contribute to the separation process between the object and the background, by adding a new level of uncertainty. The multiple centers model generalizes the base model so that its is able to identify an arbitrary number of groups in a given image. It is useful when one needs to separate multiple objects.

V. CONCLUSIONS

In this paper, we proposed a time step evolution to the Fuzzy Energy-Based Active Contour model. In order to achieve this behavior, morphological operators are used to limit the fuzzy membership updates at each iteration. We also extended

this new model to use type-2 fuzzy sets, assigning to each membership value a secondary membership, that adds a new level of uncertainty to the membership. We also generalized the original algorithm to allow the division of the image in an arbitrary number of groups. The experiments showed that each proposed modification improves the power of the model to segment correctly certain types of images.

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