

Intuitionistic Fuzzy Reasoning with Cognitive Maps

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Abstract—Fuzzy cognitive maps have proven an exceptional means to reasoning for decision support. In this paper we propose a novel approach to hesitancy-aware reasoning based on cognitive maps and intuitionistic fuzzy logic. Intuitionistic fuzzy sets are considered for the linguistic representation of both the concepts of a cognitive map and the relations defined between them. A comparative advantage of this approach over the state of the art is an intrinsic mechanism of modeling human hesitancy, as introduced in the construction of a cognitive model, and propagated through the reasoning process to a final decision. A numerical example demonstrates its effectiveness which can extend to a variety of real-world applications.

Keywords—fuzzy cognitive maps; intuitionistic fuzzy sets; reasoning; decision support

I. INTRODUCTION

Human decisions are commonly expressed with a degree of hesitation, originating from uncertain or imperfect facts and imprecise knowledge. The theory of intuitionistic fuzzy sets (IFSs) [1] offers a sound mathematical framework to modeling hesitation through a tradeoff between the membership and the non-membership of an element to such a set.

A pioneering approach to intuitionistic fuzzy reasoning for decision support have been proposed in [3], where different forms of fuzzy propositional expressions and their relations have been investigated. In [4] Sanchez's approach to fuzzy reasoning has been extended with IFSs and applied for medical diagnosis. As an alternative to that method, a distance-based method has been proposed in [5]. Recent advances in field include an intuitionistic fuzzy Petri net model and reasoning algorithm [6], and an extension of the fuzzy cognitive map (FCM) model for intuitionistic fuzzy reasoning [7].

FCMs are fuzzy directed graphs whose nodes represent concepts which can be causally interrelated through the graph edges and describe knowledge with fuzzy logic [8]-[9]. In [7] IFSs have been used to define fuzzy relations between the concepts including the hesitancy of the experts in defining such relations during the construction of an intuitionistic FCM (iFCM). Advantages of this approach include a) capability to model the expert's hesitation in the construction of the cognitive map b) coping with incomplete or even conflicting information, c) easy construction and parameterization.

The improved results obtained with the iFCM in comparison with the conventional FCM approach on a real

application for medical decision support [7] motivated us to investigate an enhanced iFCM model for decision support. This model, presented in this paper, co-evaluates human hesitancy not only in the definition of the relations between the concepts but also in the definition of the concept values. It can be considered as a generalization of the original iFCM enabling the expression of hesitancy both in the construction of cognitive models by experts and in their application by users.

The rest of this paper consists of three sections. Section II briefly describes the previous iFCM approach [7], to which we will refer to as iFCM-I, and presents the novel iFCM approach for intuitionistic fuzzy reasoning. In order to differentiate it from the previous one, we will call it iFCM-II. A numerical example that demonstrates the effectiveness of the latter one is presented in Section III, whereas the conclusions of this study are summarized in Section IV.

II. INTUITIONISTIC FUZZY COGNITIVE MAPS

A. Intuitionistic Fuzzy Sets

Given a universe of discourse E an IFS is defined as [1]

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in E \} \quad (1)$$

where $\mu_A : E \rightarrow [0,1]$ and $\gamma_A : E \rightarrow [0,1]$ define the degree of *membership* and *non-membership*, respectively, of the element $x \in E$ to the set $A \subset E$. For every element $x \in E$ it holds that $0 \leq \mu_A(x) \leq 1$, $0 \leq \gamma_A(x) \leq 1$ and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$. For $x \in E$, if $\gamma_A(x) = 1 - \mu_A(x)$, A represents a fuzzy set. The *hesitancy* of the element $x \in E$ to the set $A \subset E$ is defined as

$$\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x) \quad (2)$$

A variety of operations over IFSs have been defined [2]. These include the summation and the multiplication of two fuzzy sets, which are defined as

$$A \oplus B = \{ \langle x, \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x), \gamma_A(x) \cdot \gamma_B(x) \rangle \mid x \in E \} \quad (3)$$

$$A \otimes B = \{ \langle x, \mu_A(x) \cdot \mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x) \cdot \gamma_B(x) \rangle \mid x \in E \} \quad (4)$$

B. iFCM-I

Similarly to a conventional FCM [10], iFCM-I models a decision making task as a graph [7]. Its nodes represent N concepts that sufficiently describe this task. Each concept is

This work was realized in the framework of the project DebugIT which is co-funded by the EC FP7 grant agreement n° FP7-217139. The EC, DG INFOS, is not liable for any use that may be made of the information in this document. It reflects solely the views of the authors.

modeled as a *linguistic variable* C_i , $i=1,2,\dots,N$ that can take linguistic values represented by fuzzy sets $\tilde{L}_n^{C_i}$, $n=1,\dots,l$, on U_i , where U_i represents the respective universe of discourse.

The experts describe the *relations* between the concepts by if-then *rules* of the form:

“If a *change* X occurs in the value of concept C_j then a *change* Y is caused in the value of concept C_i .”

where X and Y are linguistic variables quantifying the respective *change* with a linguistic value represented by a fuzzy set $\tilde{R}_n$, on E , from $\tilde{R} = \{\tilde{R}_r\}$, $n=1,\dots,r$.

From each rule the *influence* of concept C_j on concept C_i is inferred as a linguistic value represented by a fuzzy set $(\tilde{I}_n)_{ji}$ on E , from $\tilde{I} = \{\tilde{I}_n\}$, $n=1,\dots,g$, according to [10]. These fuzzy sets express the degree to which a concept C_j influences another concept C_i , $i=1,2,\dots,N$, $j=1,2,\dots,N$.

Unlike a conventional FCM, iFCM-I can model the *hesitation* of the expert to express the rule and therefore the influence among two concepts, C_i and C_j , $i=1,\dots,N$, $j=1,\dots,N$. This is represented by a fuzzy set $(\tilde{H}_n)_{ij}$, on $[0,1]$, from $\tilde{H} = \{\tilde{H}_n\}$, $n=1,\dots,h$, which is subsequently used to construct a set Ω of IFSs from $\tilde{I} \times \tilde{H}$, defined as

$$\Omega_n = \{ \langle x, \mu_{\Omega_n}(x), \gamma_{\Omega_n}(x) \rangle | x \in E, n=1,\dots,\omega \leq m \cdot p \} \quad (5)$$

with a membership function $\mu_{\Omega_n}(x) = \mu_{\tilde{I}_m}(x)$, $m=1,\dots,g$, a non-membership function $\gamma_{\Omega_n}(x) = 1 - \mu_{\tilde{I}_m}(x) - \mu_{\tilde{H}_p}(x)$, $p=1,\dots,h$, and a hesitancy function $\pi_{\Omega_n}(x) = \mu_{\tilde{H}_p}(x)$ as Eq.(2), where $0 \leq \mu_{\Omega_n}(x) \leq 1$, $0 \leq \gamma_{\Omega_n}(x) \leq 1$, $0 \leq \pi_{\Omega_n}(x) \leq 1$.

The reasoning process with iFCM-I involves iterative calculation of the following equation until a steady state is reached [7]:

$$s_i^{k+1} = f \left((2s_i^k - 1) + \sum_{j=1, j \neq i}^N ((2s_j^k - 1) \cdot w_{ji}^\mu \cdot (1 - w_{ji}^\pi)) \right) \quad (6)$$

where $s_i^k \in [0,1]$, $i=1,2,\dots,N$ represent real node values at an iteration k , $w_{ji}^\mu \in [-1,1]$ and $w_{ji}^\pi \in [0,1]$ represent the influence weight and the hesitancy weight corresponding to the edge directed from node j to node i . Function f is of sigmoid form and aims to keep the node values within $[0, 1]$. The initial weights are estimated by defuzzification of the aggregated, linguistically expressed, influences between the concepts [10].

C. iFCM-II

The previous iFCM approach considers only the hesitancy in expression of the influence between the concepts. However, in human decision making tasks hesitancy may also be introduced in the determination of concept values, and it may also be propagated through the reasoning process to the final decision. In order to incorporate such a consideration into a concise decision making model, we propose a more general, novel, formulation of the iFCM.

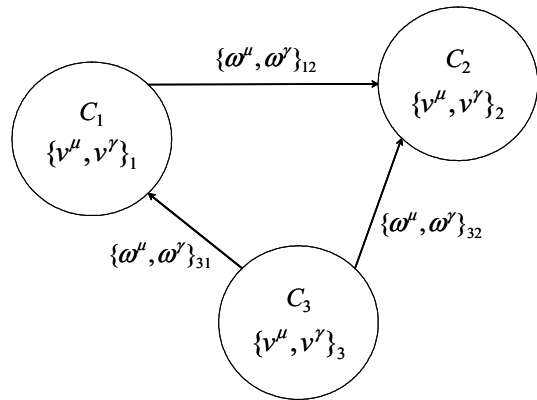


Figure 1. The proposed iFCM-II model.

The proposed approach considers that concepts C_i , $i=1,2,\dots,N$ are modeled as linguistic variables that can take linguistic values represented by IFSs of the form $L_n^{C_i} = \{ \langle x, v_i^\mu(x), v_i^\gamma(x) \rangle | x \in U_i \}$, $n=1,\dots,l$, on U_i , where U_i represents the respective universe of discourse. Therefore, each concept is characterized by a pair $\{v^\mu, v^\gamma\}_i$ representing the membership and the non-membership to the respective IFS. Similarly, considering the use of IFSs in the description of the influence of a concept C_i to another concept C_j , a pair of membership and non-membership values $\{\omega^\mu, \omega^\gamma\}_{ji}$ can be used for the representation of this influence. This enhanced iFCM model is illustrated graphically in Fig.1. It should be noted that the pair $\{\omega^\mu, \omega^\gamma\}_{ji}$ defines an IFS as in Eq.(1), whereas in the case of iFCM-I which in turn results in (w_{ji}^μ, w_{ji}^π) , represents a pair of weight values characterizing the influence between the concepts and not an IFS.

Having such an IFS-based representation of the concepts and the influences between the concepts, we reformulate the equation of the conventional FCM [10], which is

$$s_i^{k+1} = f \left(s_i^k + \sum_{j=1, j \neq i}^N s_j^k w_{ji} \right) \quad (7)$$

so that it can be applied on IFSs, for $i=1,2,\dots,N$

$$\{v^\mu, v^\gamma\}_i^{k+1} = F \left(\{v^\mu, v^\gamma\}_i^k \oplus \left(\bigoplus_{j=1, j \neq i}^N (\{v^\mu, v^\gamma\}_j^k \otimes \{\omega^\mu, \omega^\gamma\}_{ji}) \right) \right) \quad (8)$$

where the arithmetic summation and multiplication of Eq.(7) have been replaced by the respective operations on IFSs defined by Eqs.(3-4), and $F(A)$ is a function defined on IFSs as

$$F(A) = \{f_\mu(\mu_A), f_\gamma(\gamma_A)\} \quad (9)$$

where A is an IFS, and f_μ , and f_γ are sigmoid functions such that $0 \leq f_\mu(\mu_A) + f_\gamma(\gamma_A) \leq 1$. In this work we consider

$$f_\mu(x) = f_\gamma(x) = \tanh(x) \quad (10)$$

It should be noted that function F is considered mainly as a means to push the system to converge to a steady state by deforming the real membership and non-membership values and not for the normalization of the real membership and the non-membership values, which by definition range in a standard interval i.e. $[0, 1]$.

By using Eq.(4) on Eq.(8) for the calculation of the IFS multiplication, it is obtained:

$$\{v^\mu, v^\gamma\}_i^{k+1} = F \left\{ \{v^\mu, v^\gamma\}_i^k \oplus \left(\bigoplus_{j=1, j \neq i}^N \left(\{v_j^\mu \omega_{ji}^\mu, v_j^\gamma + \omega_{ji}^\gamma - v_j^\gamma \omega_{ji}^\gamma\} \right) \right) \right\} \quad (11)$$

and by using Eq.(3) on the right-hand term of Eq.(9) it becomes

$$\{v^\mu, v^\gamma\}_i^{k+1} = F \left\{ \{v^\mu, v^\gamma\}_i^k \oplus \left\{ m_{iN}, \prod_{j=1, j \neq i}^N (v_j^\gamma + \omega_{ji}^\gamma - v_j^\gamma \omega_{ji}^\gamma) \right\}^k \right\}$$

which again due to Eq.(3) results in

$$\{v^\mu, v^\gamma\}_i^{k+1} = F \left\{ \left\{ v_i^\mu + m_{iN} - v_i^\mu m_{iN}, v_i^\gamma \cdot \prod_{j=1, j \neq i}^N (v_j^\gamma + \omega_{ji}^\gamma - v_j^\gamma \omega_{ji}^\gamma) \right\}^k \right\}$$

or

$$\{v^\mu, v^\gamma\}_i^{k+1} = F \left\{ \left\{ v_i^\mu + (1 - v_i^\mu) \cdot m_{iN}, v_i^\gamma \cdot \prod_{j=1, j \neq i}^N (v_j^\gamma + \omega_{ji}^\gamma - v_j^\gamma \omega_{ji}^\gamma) \right\}^k \right\} \quad (12)$$

where

$$m_{Ni} = \bigoplus_{j=1, j \neq i}^N v_j^\mu \omega_{ji}^\mu, i=1,2,\dots,N. \quad (13)$$

In this equation $\oplus^\mu$ represents the summation operator applied on the memberships of the IFSs in Eq.(3), i.e.

$$\mu_A \oplus^\mu \mu_B = \mu_A + \mu_B - \mu_A \mu_B. \quad (14)$$

Equation (12) can be calculated recursively for all $j=1 \dots N$

$$m_{ij} = \begin{cases} v_i^\mu \omega_{ii}^\mu & j=1 \\ m_{(j-1)i} \oplus^\mu v_j^\mu \omega_{ji}^\mu, & j>1 \end{cases} \quad (15)$$

which due to Eq.(13) can also be expressed as

$$m_{ij} = \begin{cases} v_i^\mu \omega_{ii}^\mu & j=1 \\ m_{(j-1)i} + v_j^\mu \omega_{ji}^\mu - m_{(j-1)i} v_j^\mu \omega_{ji}^\mu, & j>1 \end{cases} \quad (16)$$

Obviously the complexity of Eq.(11) is higher than that of Eq.(6) and Eq.(7); however, the result obtained will be more

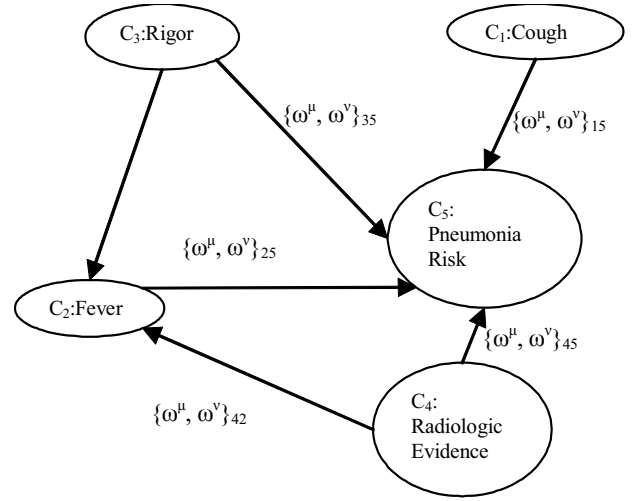


Figure 2. iFCM for pneumonia risk assessment.

informative since a propagated degree of hesitation will be quantifiable through the application of Eq.(2) on Eq.(11).

The hesitancy of the proposed model at steady state is calculated from Eq.(2) by using the real membership and non-membership values. These are obtained by application of the inverse F on the decision concepts i.e. if c is a decision concept the real membership and non-membership values at the iteration K where convergence is met will be $F^{-1}(\{v^\mu, v^\gamma\}_c^K)$, which is defined as $\{f_\mu^{-1}(v^\mu), f_\gamma^{-1}(v^\gamma)\}_c^K$.

III. NUMERICAL EXAMPLE

The medical decision making task of pneumonia risk assessment, described in [11], has been considered for our case study. This is an important medical decision making task because of the serious, life threatening, adverse consequences that could occur in the critically ill patients if the pneumonia infection is misdiagnosed [12].

In order to present a numerical example that would be easily reproducible by other researchers, despite the complex IFS operations involved, we constructed a reduced version of the cognitive model presented in [11]. This iFCM model consists of only five concepts instead of 34 (Fig. 2). We demonstrate its construction and simulation for three cases, using both the iFCM-I and the iFCM-II approaches.

A. Model construction

The pneumonia risk assessment model was based on the knowledge of three expert physicians. The opinions of these experts were pooled to define the number and types of parameters/factors associated with the particular decision making task, as well as the interconnections among these parameters. The parameters have been considered as concepts of the FCM and include the symptoms of cough (C_1), fever (C_2), rigor (C_3), the presence of radiological evidences of pneumonia in the patient's chest radiograph (C_4), and the risk

TABLE I. EXPERTS' OPINIONS ON INFLUENCES BETWEEN CONCEPTS

Relation ($C_i \rightarrow C_j$)	Experts' Opinions					
	I-1	H-1	I-2	H-2	I-3	H-3
$C_1 \rightarrow C_5$	low	v.low	low	v.low	low	low
$C_2 \rightarrow C_5$	med	v.low	med	v.low	med	v.low
$C_3 \rightarrow C_5$	low	low	med	low	med	low
$C_4 \rightarrow C_5$	high	low	med	v.low	high	v.low
$C_3 \rightarrow C_2$	med	low	low	v.low	low	v.low
$C_4 \rightarrow C_2$	high	low	v.high	med	high	low

TABLE II. RELATIONSHIPS AMONG IFCM CONCEPTS

Relation ($C_i \rightarrow C_j$)	Membership, Non-membership and Hesitancy iFCM-I / iFCM-II		
	$w_{ji}^\mu / \omega_{ji}^\mu$	$w_{ji}^\gamma / \omega_{ji}^\gamma$	$w_{ji}^\pi / \omega_{ji}^\pi$
$C_1 \rightarrow C_5$	0.250	0.630	0.120
$C_2 \rightarrow C_5$	0.500	0.430	0.070
$C_3 \rightarrow C_5$	0.345	0.500	0.150
$C_4 \rightarrow C_5$	0.584	0.300	0.100
$C_3 \rightarrow C_2$	0.300	0.600	0.100
$C_4 \rightarrow C_2$	0.700	0.070	0.220

TABLE III. DECISION RANGES FOR CONCEPT C_5

Decision Node Values	Ranges Proposed by Physicians			
	<0.50	0.50-0.70	0.70-0.85	>0.85
Infection Risk	Low	Moderate	High	Very high

of infection (C_5), which is the decision concept. These concepts are well documented in the literature [12]. They represent the main variables that play an important role in the final diagnostic decision about the risk of pneumonia. For this application, the concepts take two, three, four or five possible discrete or fuzzy values [7]. For example the symptom of cough takes three fuzzy values (no cough, moderate cough, serious cough state) according to the initial patient stage. The decision concept (C_5) represents the risk of pulmonary infection and takes four fuzzy values (low, moderate, high, very high).

According to the proposed methodology (Section II.C), the opinions of three experts were pooled together to assign new if-then rules that describe the influences from concepts C_i to concepts C_j , $i=1\dots 5$, $j=1\dots 5$. The inferences of the rules are linguistic weights described by IFSs $\{\omega^\mu, \omega^\gamma\}_{ji}$. The linguistic weights obtained by the experts for the construction of the iFCM model are apposed in Table I. For each influence ($I-E_i$) between two concepts, the expert E_i , $i=1,2,3$ expresses his/her hesitancy ($H-E_i$). The linguistic values of hesitancy vary from very low (v.low) to high. The respective membership, non-membership and hesitancy values derived from Table I are common for both the iFCM-I and iFCM-II models, and they

are summarized in Table II. The interpretation of the numeric risk for pneumonia infection obtained from the output concept C_5 is performed according to Table III.

In what follows, we provide three example patient cases for which we would like to assess the pneumonia risk using the constructed model for both iFCM-I and iFCM-II approaches.

B. First Case

In the first case, we consider a patient with suspected pneumonia having moderate cough ($C_1=0.6$), high fever ($C_2=0.7$), and no other information is available. Based on these data, the experts assessed the infection risk as moderate.

In iFCM-I, as noted in [7], the concepts with unknown values are set to 0.5, i.e. $C_3=C_4=0.5$. In iFCM-II the membership values of the respective concepts are set to zero, i.e. $C_3=C_4=0$. The non-membership values of these concepts are initially set to 0.5. However, the initial non-membership values do practically affect the final decision, because, as it can be noticed by the obtained results, they reach zero at convergence.

The results of the reasoning process obtained with iFCM-I (Eq. 6), at each iteration k , till convergence at a steady state are apposed in Table IV. The first row includes the initial concept values, and the last row, in boldface, indicates the concept values at steady state. The respective results with the use of the proposed iFCM-II approach are presented in Tables V and VI. We provide all concept values for validation of the results of other researchers that would like to implement our iFCM models. Table V shows that iFCM-II reaches a steady state at a numerical value that is close to the membership value obtained with iFCM-II, whereas the decisions derived

TABLE IV. FIRST CASE: REASONING USING IFCM-I

Iteration k	Concept Values Till Steady State				
	C_1	C_2	C_3	C_4	C_5
0	0.6000	0.7000	0.5000	0.5000	0.0000
1	0.6000	0.7000	0.5000	0.5000	0.5000
2	0.6000	0.7000	0.5000	0.5000	0.5342
3	0.6000	0.7000	0.5000	0.5000	0.5512
4	0.6000	0.7000	0.5000	0.5000	0.5596
5	0.6000	0.7000	0.5000	0.5000	0.5637
6	0.6000	0.7000	0.5000	0.5000	0.5657
7	0.6000	0.7000	0.5000	0.5000	0.5667
8	0.6000	0.7000	0.5000	0.5000	0.5672
9	0.6000	0.7000	0.5000	0.5000	0.5674
10	0.6000	0.7000	0.5000	0.5000	0.5676
11	0.6000	0.7000	0.5000	0.5000	0.5677
12	0.6000	0.7000	0.5000	0.5000	0.5677

TABLE V. FIRST CASE: REASONING USING iFCM-II

Iteration k	Concept Membership Values Till Steady State				
	C_1^μ	C_2^μ	C_3^μ	C_4^μ	C_5^μ
0	0.6000	0.7000	0.0000	0.0000	0.0000
1	0.5370	0.6044	0.0000	0.0000	0.4198
2	0.4908	0.5402	0.0000	0.0000	0.5713
3	0.4548	0.4931	0.0000	0.0000	0.6203
4	0.4258	0.4567	0.0000	0.0000	0.6330
5	0.4018	0.4274	0.0000	0.0000	0.6333
6	0.3815	0.4031	0.0000	0.0000	0.6295
7	0.3640	0.3826	0.0000	0.0000	0.6245
8	0.3488	0.3650	0.0000	0.0000	0.6194
9	0.3353	0.3496	0.0000	0.0000	0.6144
10	0.3233	0.3360	0.0000	0.0000	0.6097
11	0.3124	0.3239	0.0000	0.0000	0.6053
12	0.3027	0.3130	0.0000	0.0000	0.6012
13	0.2937	0.3032	0.0000	0.0000	0.5974
14	0.2856	0.2942	0.0000	0.0000	0.5938
15	0.2781	0.2860	0.0000	0.0000	0.5904
16	0.2711	0.2785	0.0000	0.0000	0.5842
17	0.2710	0.2755	0.0000	0.0000	0.5842

TABLE VI. FIRST CASE: REASONING USING iFCM-II

Iteration k	Concept Non-Membership Values Till Steady State				
	C_1^γ	C_2^γ	C_3^γ	C_4^γ	C_5^γ
0	0.30000	0.20000	0.50000	0.50000	0.50000
1	0.29131	0.08539	0.46212	0.46212	0.09794
2	0.28334	0.03348	0.43181	0.43181	0.01576
3	0.27600	0.01220	0.40683	0.40683	0.00224
4	0.26920	0.00417	0.38578	0.38578	0.00030
5	0.26288	0.00135	0.36772	0.36772	0.00004
6	0.25698	0.00042	0.35199	0.35199	0.00000
7	0.25147	0.00012	0.33814	0.33814	0.00000
8	0.24630	0.00003	0.32582	0.32582	0.00000
9	0.24144	0.00001	0.31476	0.31476	0.00000
10	0.23685	0.00000	0.30476	0.30476	0.00000
11	0.23252	0.00000	0.29566	0.29566	0.00000
12	0.22842	0.00000	0.28734	0.28734	0.00000
13	0.22453	0.00000	0.27968	0.27968	0.00000
14	0.22083	0.00000	0.27261	0.27261	0.00000
15	0.21731	0.00000	0.26605	0.26605	0.00000

Iteration k	Concept Non-Membership Values Till Steady State				
	C_1^γ	C_2^γ	C_3^γ	C_4^γ	C_5^γ
16	0.21395	0.00000	0.25995	0.25995	0.00000
17	0.21075	0.00000	0.25425	0.25425	0.00000
18	0.20768	0.00000	0.24891	0.24891	0.00000

by both iFCM approaches, according to Table III are in agreement with the aggregated fuzzy experts' opinion i.e. moderate. The non-membership values of all the concepts of iFCM-II are monotonically decreasing. The steepest decrease is observed at the output concept C_5 , whose non-membership value reaches zero after converging at a steady state. Furthermore a real hesitancy estimation can be obtained from the decision concept, which according to Section II.C where $\tanh^{-1}(0.5842)=0.6688$, is 0.3312 using Eq.(2).

C. Second Case

The second case refers to a patient with suspected pulmonary infection having moderate cough ($C_1=0.6$), low fever ($C_2=0.3$), and rigor ($C_3=1$). Based only on these symptoms, the experts decided that the infection risk is moderate.

The convergence of both iFCM-I and iFCM-II follows a similar pattern with the first case, so we provide a condensed graphic representation of the values till steady state in Figs. 3 and 4. The respective values of the concepts are provided in Tables VII-IX. In these tables the first rows include the initial concept values, and the last rows, appearing in boldface, indicate the concept values at steady state. Both iFCM approaches are in accordance with the experts' opinion since they both evaluate to moderate risk according to Table III. The hesitancy at the output node is where $\tanh^{-1}(0.6874) = 0.8430$, is 0.1581 using Eq. (2).

TABLE VII. SECOND CASE: REASONING USING iFCM-I

Iteration k	Concept Values Till Steady State				
	C_1	C_2	C_3	C_4	C_5
0	0.6000	0.3000	1.0000	0.5000	0.0000
13	0.6000	0.6290	1.0000	0.5000	0.6896

TABLE VIII. SECOND CASE: REASONING USING iFCM-II

Iteration k	Concept Membership Values Till Steady State				
	C_1^μ	C_2^μ	C_3^μ	C_4^μ	C_5^μ
0	0.6000	0.3000	1.0000	0.0000	0.0000
16	0.2781	0.5978	0.7616	0.0000	0.6874

TABLE IX. SECOND CASE: REASONING USING iFCM-II

Iteration k	Concept Non-Membership Values Till Steady State				
	C_1^γ	C_2^γ	C_3^γ	C_4^γ	C_5^γ
0	0.30000	0.20000	0.00000	0.50000	0.50000
16	0.21395	0.00000	0.00000	0.25995	0.00000

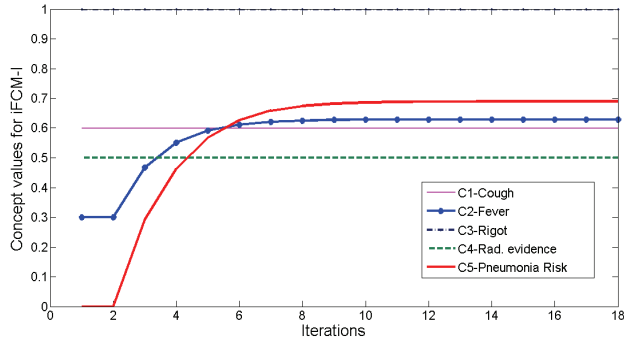


Figure 3. Convergence of iFCM-I for the second case.

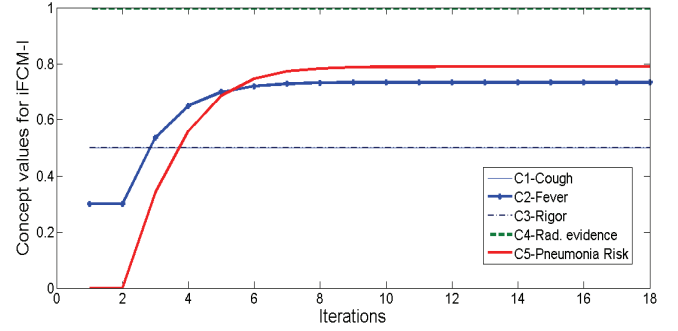


Figure 5. Convergence of iFCM-I for the third case.

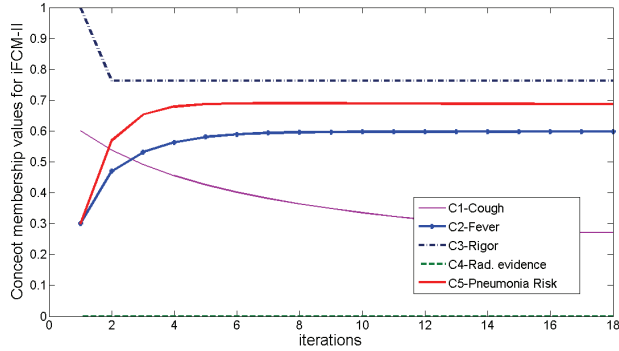


Figure 4. Convergence of iFCM-II for the second case.

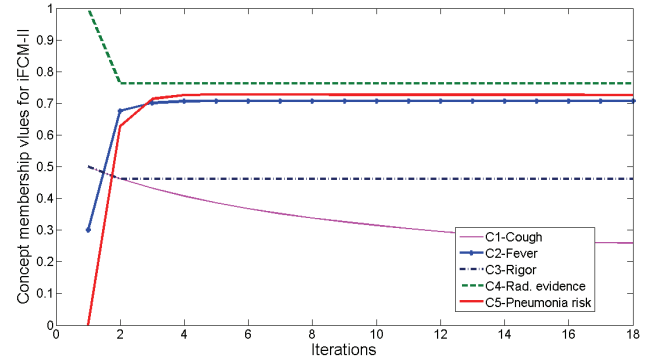


Figure 6. Convergence of iFCM-II for the third case.

D. Third Case

In this case, we consider a patient with low fever ($C_2=0.3$) and presence of radiological evidences of pneumonia ($C_4=1$).

TABLE X. THIRD CASE: REASONING USING iFCM-I

Iteration k	Concept Values Till Steady State				
	C_1	C_2	C_3	C_4	C_5
0	0.5000	0.3000	0.5000	1.0000	0.5000
12	0.5000	0.7337	0.5000	1.0000	0.7895

TABLE XI. THIRD CASE: REASONING USING iFCM-II

Iteration k	Concept Membership Values Till Steady State				
	C_1^μ	C_2^μ	C_3^μ	C_4^μ	C_5^μ
0	0.5000	0.3000	0.5000	1.0000	0.0000
15	0.2781	0.7076	0.4621	0.7616	0.7266

TABLE XII. THIRD CASE: REASONING USING iFCM-II

Iteration k	Concept Non-Membership Values Till Steady State				
	C_1^γ	C_2^γ	C_3^γ	C_4^γ	C_5^γ
0	0.3000	0.5000	0.1000	0.2000	0.5000
15	0.2173	0.0000	0.2660	0.2660	0.0000

There is no any information on other symptoms. According to the experts there is a high risk of this patient having pneumonia.

The convergence of both iFCM-I and iFCM-II is graphically represented in Figs. 5-6. The initial concept values and the concept values after converging at a steady state are apposed in Tables X-XII. According to Table III it can be noticed that both iFCM models result in a decision that is in agreement with the experts' opinion, i.e. high pneumonia risk. The hesitancy obtained at the decision node of iFCM-II is 0.0785 using Eq. (2), where $\tanh^{-1}(0.7266) = 0.9214$.

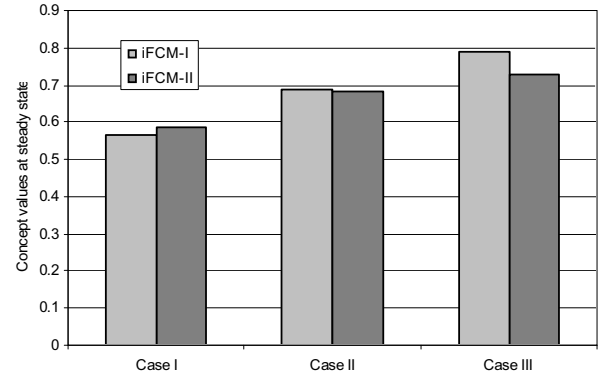


Figure 7. Comparison of the decision concept values at steady state with iFCM-I and iFCM-II for the three different patient cases presented in this study.

In the examined cases the results of both iFCM techniques are identical according to the calculated decision concepts (see Figure 7) and agree with the experts' opinion. But the advantageous point of iFCM-II technique is that it further assesses the hesitancy on concepts values and especially on decision nodes for which the physicians are interested.

IV. CONCLUSIONS

We presented a novel approach to intuitionistic fuzzy reasoning for decision support based on a novel iFCM model, which is referred to as iFCM-II. The proposed iFCM model is elegantly formulated using the mathematical formalism of IFS operations and exhibits several advantages over the previously proposed one. These include:

- It enables modelling of the uncertainty and imprecision introduced not only in the definition of the relations between the concepts of the cognitive map but also in the definition of the concept values.
- It is more general and could be used to closer approximate human decision making. It reduces to the previous iFCM model if the hesitancy in the determination of all concept values is zero.
- The output of the reasoning process may include a degree of hesitation expressed through IFS hesitancy.

The proposed iFCM-II model was applied for pneumonia risk assessment on three patient cases to demonstrate its functionality. The results showed that both iFCM-I and iFCM-II lead to decisions that are in accordance to the decisions of the experts. However, as illustrated in Fig. 7, the calculated decision concepts may exhibit differences.

Future research will focus on the application and the evolution of the proposed model towards an even improved

modelling of uncertainty and human indecision.

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