

Design of a Reliable Hub-and-spoke Network Using an Interactive Fuzzy Goal Programming

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Abstract. A Hub Location Problem (HLP) deals with finding the locations of hub facilities and assignment of demand nodes to established facilities. Hubs play a central role in many networks such as telecommunication networks and their unavailability may lead to network breakdown or poor service levels. An objective in design of a hub-and-spoke network is maximization of reliability to transfer flows. This paper puts forward design of a reliable single-allocation hub-and-spoke network using an interactive fuzzy goal programming. To model and solve the problem, a fuzzy goal programming approach was developed for design of network in an interactive manner between decision maker and the model. To validate the model and the proposed solution approach, a test problem is presented and comparison of results is made using this problem.

Keywords. *Reliable hub location problem, Interactive decision making, Fuzzy goal programming, Network design, Bi-objective programming.*

I. INTRODUCTION

Decisions are usually categorized as strategic, tactical and operational. A strategic or long-term decision does not take place on a regular basis and needs major capital investments. An example of a long-term decision is determination of facility location ([1]). Facility Location (FL) has been an important concern for around 100 years. Applications of FL could be found in both public and private sectors, since more and more managers realize the importance of decisions made concerning location of facilities.

Hub location problem (HLP) has been an attractive, yet tricky variant of facility location. It arises in many different contexts such as telecommunications, cross-docking, airline industry, computer networks, etc. For example, nowadays many LTL trucking companies designed their networks based on the hub-and-spoke structure. This has led to better efficiency and improved service levels. In a typical HLP, the following questions are to be answered: a) Where to locate hub facilities b) How to allocate demand nodes to hubs, so the objective of the whole network is optimized. Since these decisions are interconnected, their separation may easily

lead to sub-optimal results. Therefore, these questions are to be considered in a single model.

HLPs are often categorized as single-allocation and multiple allocation types. While in a single-allocation HLP, each spoke is allocated to one and only one hub node, in the multiple-allocation case, a spoke may be allocated to more than one hub. In this paper, a single-allocation HLP will be studied. Another interesting issue to be considered in design of hub-and-spoke networks is the discount factor associated with arcs which is a value between zero and one to represent the economies of scale. Generally, a flow in a hub-and-spoke network is in accordance with one of the seven types shown in fig. 1. In each of these types, the reliabilities of hubs and arcs play a pivotal role in a successful, on-time delivery of flow. Since hubs and arcs in a network are liable to disruption, design of a reliable network is of great value to decision makers. Kim and O'Kelly [2] proposed the reliable hub location problem and presented two models. As far as we know, this subject has not been considered much to date.

Perez et al. [3] claimed that real applications of facility location can be full of linguistic vagueness that can be appropriately modeled using fuzzy variables. In a HLP, many parameters could not be determined precisely such as demands, distances or reliabilities. In addition, it is sometimes difficult to describe parameters such as reliability as random variables. In these situations, fuzzy logic has been proven to be useful. However, reviewing the literature of HLP, we came to notice that the fuzzy version of HLP has gained little attention.

The rest of this paper is organized as follows: In the preceding section, a brief literature review of HLP and its recent publications is presented. Then, the mathematical formulation of this paper will be proposed. Section 4 is devoted to a brief introduction of fuzzy goal programming. The proposed interactive fuzzy goal programming (IFGP) will be set out in section 5. Afterwards, computational experiments are given in section 6. Finally, conclusions and areas for future research will be given in section 7.

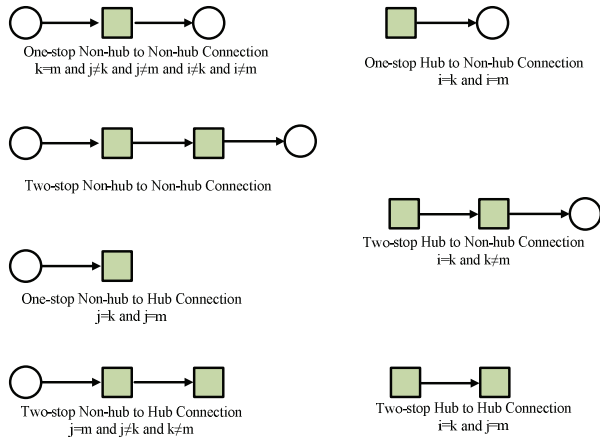


Figure 1. Different types of flows in a hub-and-spoke network

II. LITERATURE REVIEW

Hub Location Problem (HLP) has a long and wealthy background. From the beginning of the 21st century, an increase in the number of publications regarding HLP has been witnessed. The emergence of HLP dates back to late 60s when Goldman [4] defined the network hub location problem for the first time. Later, O'Kelly [5] presented the first seminal paper of HLP by proposing a mathematical formulation for a hub location problem. He studied the American airline passenger network and proposed the single allocation p -hub median problem which has gained the most attention so far among various hub location problems. He also proposed a well-known dataset (CAB) for hub location problems involving the passenger flows between 25 US cities. Since then, contributions to HLP have been presented in a variety of ways. It should be noted that a great deal of publications has been about the HLP models in which cost or time have been considered.

The most recent review of HLP is Alumur and Kara [6] which is an elegant and comprehensive review of network hub location problems. They classified their review into four distinct sections as p -hub median problem, the hub location problem with fixed costs, the p -hub center problem and hub covering problems. Since Alumur and Kara [6] is a comprehensive and valuable review, in this paper, only some recent publications will be presented. One may refer to [6] or other reviews and references therein to get more knowledge about the HLP literature before 2008.

Recently, some heuristics and metaheuristics have been proposed to solve various HLPs. Using tabu search to solve the single-allocation hub covering HLP in incomplete networks (Calik et al. [7]), using ant colony optimization for the single-allocation p -hub center problem (Meyer et al. [8]), VNS for the single-allocation p -hub median problem (Illic et al. [9]), using Lagrangian relaxation for the single-allocation HLP (Contreras et al. [10]) are some instances, to name a few.

Using fuzzy logic in HLP has been recently considered by Chou [11]. He studied the container transshipment hub locations in southeastern Asia. He proposed a fuzzy multiple criteria decision making model to select the container transshipment hub port. Davari et al. [12] studied the single-allocation p -hub maximum reliability model of Kim and O'Kelly [2] in a fuzzy sense. They considered reliability of

each arc to be a fuzzy variable and presented an effective simulation-embedded simulated annealing to solve this problem.

Additionally, the Partitioning-Hub-Location-Routing Problem (PHLRP) by Catanzaro et al. [13], HLP in a competitive environment by Gelareh et al. [14] and Eiselt and Marianov [15], the time-definite p -hub median problem by Campbell [16], HLP assuming hub congestion by De Camargo et al. [17], considering the size of hubs as a decision variable by Correia et al. [18], stochastic p -hub center problem by Sim et al. [19], hierarchical HLP (Yaman [20]) are some recent extensions to the classical HLP.

Obviously, there has not been much attempt in literature to model fuzzy reliable hub location problems. Considering the publications to date, this paper contributes to the literature of HLP in two distinct aspects. First, a new method is proposed to calculate the reliability of a path in a hub-and-spoke network which seems to represent the real-world applications better. In addition, using fuzzy logic in hub location problems has not gained enough consideration so far and is addressed in this paper. Moreover, this paper will present using IFGP in solving HLP considering fuzzy aspiration levels. To the best of our knowledge, this has not been considered in the literature of HLP before.

III. MODEL FORMULATION

The model in this paper is an extension to Kim and O'Kelly [2]. Before presenting the problem formulation, let's define indices, parameters and variables.

Parameters

W_{ij}	The flow between nodes i and j
R_{ij}^{km}	The reliability of the flow between nodes i and j which is routed via hubs k and m
P	Number of hubs to be located

Variables

X_{ij}^{km}	Equals 1 if the flow between i and j is routed via hubs k and m
Z_k	Equals 1 if a hub is located at hub k

Model:

$$\text{Max} \sum_i \sum_j \sum_k \sum_m W_{ij} R_{ij}^{km} X_{ij}^{km} \quad (1)$$

$$\text{Max } \omega \quad (2)$$

$$\omega \leq W_{ij} R_{ij}^{km} X_{ij}^{km} + (1 - X_{ij}^{km})M \quad \forall (i, j, k, m); i \neq j \quad (3)$$

$$\sum_k Z_{kk} = P \quad (4)$$

$$\sum_k Z_{ik} = 1 \quad \forall i \quad (5)$$

$$Z_{ik} \leq Z_{kk} \quad \forall i, k (i \neq k) \quad (6)$$

$$\sum_m X_{ij}^{km} = Z_{ik} \quad \forall j > i; k \quad (7)$$

$$\sum_k X_{ij}^{km} = Z_{jm} \quad \forall j > i; m \quad (8)$$

$$Z_{ik} \in \{0, 1\} \quad (9)$$

$$0 \leq X_{ij}^{km} \leq 1 \quad (10)$$

Equation (1) is the objective function, where the weighted reliability of the network is to be maximized. The fitness of a

solution is found by adding the weighted reliabilities of each OD pair together. Equations (2) and (3) represent the second objective presenting the center objective which maximizes the minimum reliability in the network. Constraint (4) holds that P hubs are to be opened. Constraint (5) states that each node must be assigned to one and only one hub. Constraint (6) states that an assignment could be made to only hub nodes. Constraints (7) and (8) force that a flow from i to j is routed via hub pair (k, m) only when hubs are established in these two nodes. Constraint (9) guarantees that the values of Z are binary variables. Finally, (10) represents the values that the X variable could take.

Kim and O'Kelly [2] presented a procedure to compute the reliability of a path based on single reliabilities of its arcs. However, it can be argued that in some cases, it is unable to represent the reliability properly.

It is known that in a hub-and-spoke network, to reach a destination, the flow must pass hubs and arcs. Therefore, the reliability of each hub is of high importance, along with the reliability of arcs. This paper adds the reliabilities of hubs to the procedure of reliability calculation by presenting a procedure to sequential multiplication of single reliabilities of hubs and arcs. Moreover, it is assumed that same as the arcs, there should be a discount factor associated with hubs which will be shown as β in the sequel. It should be noted that this value could be completely different from the arcs discount factor. Fig. 2 shows two common types of flows in a hub-and-spoke network. The reliabilities of these two types of flows could be found as:

$$(I) R_{ij}^{km} = r_{ik} * (r_{kk})^{1-\beta} * (r_{km})^{1-\alpha} * (r_{mm})^{1-\beta} * r_{mj} \quad (11)$$

$$(II) R_{ij}^{km} = r_{ik} * (r_{kk})^{1-\beta} * r_{kj} \quad (12)$$

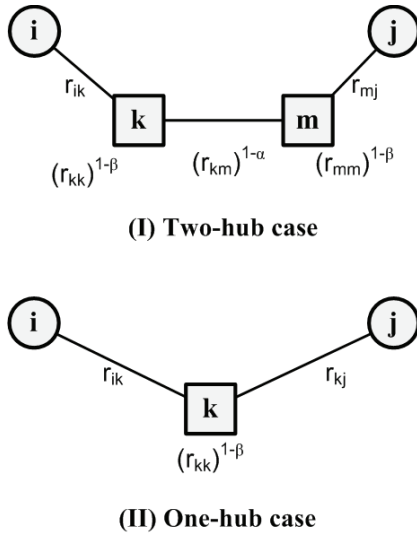


Figure 2. The two common types of flows in hub-and-spoke networks

IV. FUZZY GOAL PROGRAMMING

Goal Programming (GP), first introduced by Charnes and Cooper [21], has been studied for many years as an efficient tool for multi-objective decision making (MODM) problems. Using a GP, decision maker(s) define aspiration levels and seeks an optimal solution where the undesirable deviations are minimized. However, determining precise aspiration levels for

each objective in real world problems is a difficult task for decision makers (Yaghoobi et al. [22]).

Fuzzy Goal Programming (FGP) is a variant of the conventional GP, dealing with the imprecision came across by the DM, while expressing one's aspiration levels. The use of Fuzzy set theory (FST), proposed by Zadeh [23] to assist DMs to deal with this imprecise situation and uncertainties in GP was first proposed by Narasimhan [24] and then improved by Hannan [25].

Definition 2 (Zimmermann [26]). Assume that we are given a fuzzy goal $\tilde{G}$ and a fuzzy constraint $\tilde{C}$ in a space of alternatives X . Then $\tilde{G}$ and $\tilde{C}$ combine to form a decision $\tilde{D}$ which is a fuzzy set resulting from intersection of $\tilde{G}$ and $\tilde{C}$. Bellman and Zadeh [27] proposed the min-operator to obtain a decision by intersection of the fuzzy sets of all objective functions.

DMs often need to know the crisp optimal solution, so it seems appropriate to suggest the highest degree of membership in the fuzzy set decision. Therefore, by using "maximizing solution", we have:

$$\text{Max}_{x \geq 0} \text{Min}_i \{ \mu_i(x) \} = \text{Max}_{x \geq 0} \mu_{\tilde{D}}(x) \quad (13)$$

Introducing a new variable, λ , this conventional linear programming problem can be formulated as follows:

$$\text{Max } \lambda \quad (14)$$

$$\mu_i(x) \geq \lambda \quad i = 1, \dots, K \quad (15)$$

$$Ax \leq b \quad (16)$$

$$x \geq 0 \quad (17)$$

$$\lambda \in [0,1] \quad (18)$$

V. THE PROPOSED INTERACTIVE FUZZY FOAL PROGRAMMING (IFGP)

Many real-world problems are associated with a degree of uncertainty about goals and constraints. Recently, many academicians and practitioners have realized that uncertainty is an inevitable element in decision making. In addition, usually there is not enough historical data, or it is extremely difficult to gather reliable data, or even there is no underlying statistical model to build the model. Therefore, traditional probabilistic approaches are futile in many cases. In these sorts of problems, fuzzy logic has been proven to be a great tool. This paper will present how IFGP could be used to solve RHLF. Like any other MODM approach, IFGP is separated in two distinct stages. First, efficient solutions are found and then, a search is followed to find a compromise solution.

In the standard IFGP, one assumes that the DM can specify preference function with respect to the combination of the individual objective functions in advance, either as weights (utilities) or as distance functions.

Werners [28] presented a "fuzzy-and" operator which could be used in IFGP and is a strongly monotonically increasing function. It is presented as follows:

$$\mu_D(X) = \text{Max} \left\{ \gamma \text{Min}_i \{ \mu_i(x) \} + (1-\gamma) \left(\frac{1}{K} \right) \sum_i^K \mu_i(x) \right\} \quad (19)$$

where, $\mu_i(x)$ is the corresponding membership functions of K fuzzy objectives and constraints. DMs can interact with the

model to obtain preferred compromise solution by using γ as the coefficient of compensation defined within the interval $[0, 1]$. Later, Selim and Ozkarahan [29] modified the model by adding w_i as the relative weights for the membership functions of the objectives in (20):

$$\mu_D(X) = \text{Max} \left\{ \gamma \text{Min}_i \{ \mu_i(x) \} + (1 - \gamma) \sum_i^K w_i \mu_i(x) \right\} \quad (20)$$

Results of using the Selim and Ozkarahan's [29] operator shows that the modified operator may provide different and even more preferable results compared to the Werners's "fuzzy and" approach. We will examine the usefulness of this operator on RHLP and analyze results.

The IFGP-based solution approach for solving RHLP is described as follows: First, individual best solutions are found using the single objective optimization and the CPLEX solver. Then, the decision maker is asked to determine if she/he is satisfied with the solution. If the results are not satisfactory, then using the payoff table (which contains the optimal values of objectives), membership functions are determined for objectives. Then, changing the values of γ gradually from 1 to 0, different solutions are obtained and presented to the decision maker. This process is followed until finding a preferred solution by the decision maker.

VI. COMPUTATIONAL EXPERIMENTS

In order to generate test problems, it was assumed that the reliability of a link is a function of its length. Therefore, the CAB dataset was modified to address the need to generate reliability values. To do so, the search was restricted to nodes 11 through 20. The set of cities is comprised of eastern, central, and western USA and include the following nodes.

- Kansas City
- Los Angeles
- Memphis
- Miami
- Minneapolis
- New Orleans
- New York
- Philadelphia
- Phoenix
- Pittsburg

Thus, it can be concluded that the dataset is sparse enough. Then, reliability of each link was found as a function of its length. In other words, reliability value of each arc was considered to be a value between 0.5 and 0.9 based on its length. Then, the hub reliabilities (which are diagonal values in the reliability matrix) were generated uniformly between 0.9 and 1. Moreover, arc and hub discount factors were considered to be 0.95 and 0.9 respectively.

Fig. 3 shows the normalized fitness values of solutions for the test problem. Obviously, there is a tradeoff between the two objectives of the problem. Moreover, fig. 4 shows the behavior of both objectives and the aggregate function versus γ . It has been shown that although objectives change considerably by changes in γ , after a threshold ($\gamma = 0.3$), the aggregated value remains almost steady.

To show the dissimilarities among the three models, fig. 5 depicts the solutions found using various values of γ . It clearly

shows how the outputs differ in the locations of hubs and assignment of spokes to hubs.

Fig. 5 soundly confirms that there is an apparent discrepancy between solutions which can be attributed to changes in the value of γ . A rigorous analysis of results shows that New York is the only city which is present in all the solutions. Moreover, other cities which have been included in these solutions are Miami, Memphis, Phoenix, and Los Angeles. Since we assumed the reliabilities of arcs as a function of their length, it could be verified that the hubs are sparse in order to be able to cover the flows throughout the network.

VII. CONCLUSION AND FUTURE RESEARCH

At this point, to bring the study to a close, a conclusion is given. This paper presented a new method to compute the reliability of paths in a hub-and-spoke network. Moreover, it showed how IFGP could be used to solve RHLP. Results showed that IFGP could be a worthwhile tool to find solutions of RHLP in an interactive way with the decision maker. Another advantage of using IFGP is the fact that solutions could be found even if there is not a unanimous agreement between decision makers.

RHLP is still immature and needs more attention among academicians. An appealing future research could be considering the same problem in a stochastic environment. One may go through the performance of IFGP on other HLP variants such as incomplete hub networks or even hierarchical hub location problem. In addition, other variants of IFGP could be developed and their performance is to be assessed. Another interesting avenue for future research is building a model in which both cost and reliability are considered. Such an assumption makes the problem more realistic and in worth consideration in future. Last, but not least, is the possibility of considering multiple-allocation in the HLP.

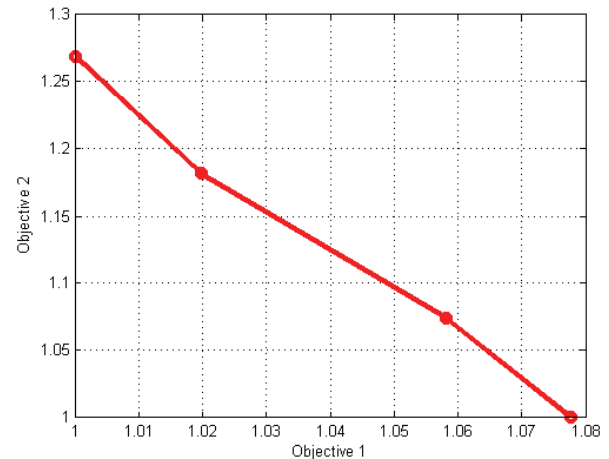


Figure 3. The normalized fitness values

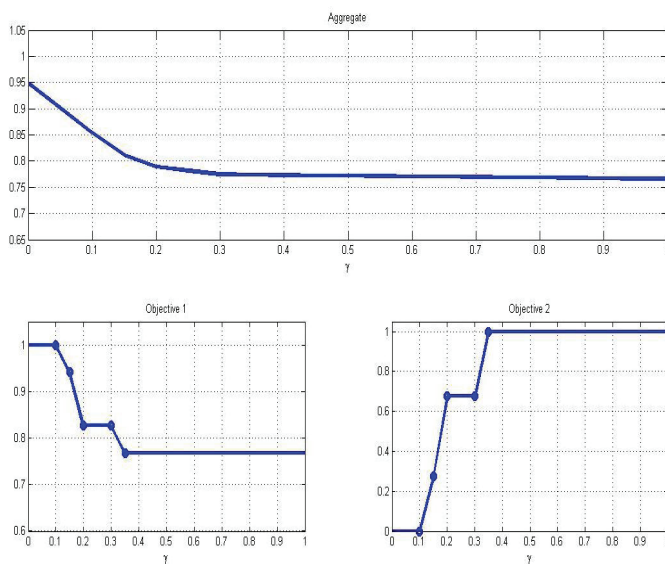


Fig 4. Results based on the values for γ

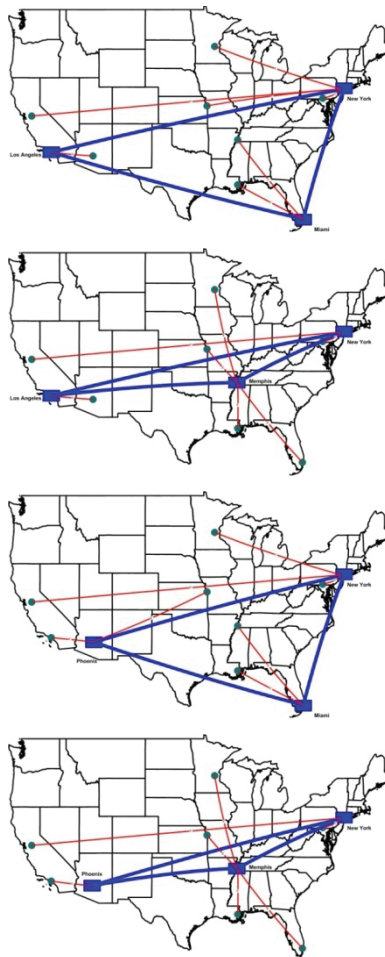


Fig 5. Comparing the solutions found

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