

A Fuzzy Stochastic Programming Approach to Solve the Capacitated Lot Size Problem Under Uncertainty

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Abstract— This paper develops a fuzzy stochastic multi-objective linear programming (FSMOLP) model for a multi-level, multi-item capacitated lot sizing problem (CLSP) in a multi assembly shop. The proposed model attempts to minimize the total cost consisting of total production variation cost, inventory cost, backlog cost and total setup cost while maximizing the resource utilization simultaneously. To cope with the uncertainty associated with the most of the input data, e.g., the demand and process-related parameters they are treated as fuzzy stochastic parameters with identical stochastic membership function during the planning horizon. To show the usefulness of the proposed solution method, a numerical example is first solved. Then, the usefulness of the proposed model is validated over a set of randomly generated test problems.

Keywords; *Multi-level, multi-item capacitated lot sizing problem, Fuzzy Stochastic Multi-Objective Linear Programming, Multi assembly shop*

I. INTRODUCTION

Shops, in which several assembly products are mounted simultaneously, are called Multi-Assembly Shops (MSA). In these shops, the raw material after being converted to sub-components by initial operations enters the main assembly line with different lot sizes. In such environments, many products with various sub-assemblies are assembled in different levels and then changed into final products (see [1]-[3]). MSA production planning models involves multiple items, restrictive capacities and setup times and/or cost. In this multi-level, multi-item production system, assumes a serial item structure with one or more final products that requires a multiple component. Each item is produced on a resource.

The multi-item multi-period CLSP with dynamic demand is a well known optimization problem that finds a wide variety of applications in real-world problems. Due to its vast industrial applicability, researchers have devoted special attention to it (see [4]-[5]). The problem arises in production environments where the changeover of a resource from one product type to another causes setup time and/or setup costs. Numerous models to support dynamic lot sizing for multi-level production systems have been presented over the last decades. Reviews of the literature on dynamic lot sizing are given by [6]-[8]. Big-bucket model formulations multi-level CLSP determines production quantities and periods only, without consideration of the actual production sequence of the orders within a time period [9]. The problem is complicated by

the existence of setup costs as well as inventory holding costs [10].

The deterministic multi-item dynamic capacitated lot-sizing problem has been studied for a long time (For recent overviews see [11]-[13]). However, only a limited number of researchers have considered dynamic capacitated lot sizing under imprecise parameters. Industrial planning practice usually applies a forecasting procedure that provides a deterministic time series of expected future parameters. Uncertainty is taken into consideration by developing stochastic or fuzzy programming methods.

However, only a limited number of researchers have considered dynamic CLSP under uncertain demand. A literature overview is presented in Sox et al. [14]. Sox and Muckstadt [15] solve a variant of the stochastic dynamic CLSP, where item and period specific backorder costs as well as extendible production capacities are considered. Brandimarte [16] considers the stochastic CLSP where the uncertainty of the demand is represented through a scenario tree. In this case, the period demands are modeled as discrete random variables. Tempelmeier [17] consider the stochastic version of the dynamic multi-item CLSP. The problem is to determine lot sizes to satisfy demands for multiple products over a finite discrete time horizon.

Another imprecision is that, in actual practice, the capacity is frequently vague and difficult to predict. Moreover only a limited amount of (production, space, or budget) capacity might be available for a type of product, and this capacity might vary over time due to its own or other products seasonality. The fuzzy set theory, which can represent this vague future capacity easily, is an ideal approach. Pai [18] extended the work of Florian and Klein [19] by using fuzzy capacity. Various investigators have used fuzzy set theory to investigate the lot-sizing problem. Lee and Yao [20] used the fuzzy concept to deal with lot sizing problem while demand and production quantity are uncertain. Chang et al. [21] modified the economic reorder point problem by introducing a fuzzy backorder quantity. The results offered a better sense of using economic fuzzy quantities. Chen and Wang [22] proposed a functional principle to deal with the lot sizing problem in a fuzzy environment. Yao and Lee [23] used a triangular fuzzy number to represent the fuzzy order quantity. Pappis and Karacapilidis [24] derived an appropriate number of production runs and lot sizes for multi-products with fuzzy

demands. Several methods have been developed in the literature to deal with the possibilistic models involving the imprecise coefficients in both objective functions and constraints. Notably, the main part of proposed method is based on Jimenez et al. [25], Jimenez, [27] and Torabi and Hassini [26] because of their several advantages as follow:

1. Jimenez, [27] introduced general ranking method which can be applied to different kinds of membership functions such as triangular, trapezoidal and nonlinear ones in both symmetric and a symmetric forms.
2. The Jimenez et al. [25] method is based on the strong mathematical concepts such as expected interval and expected value of fuzzy numbers.
3. By increasing the size of problem, the complexity of proposed model should not be increased exponentially.
4. TH method [26] is very promising fuzzy multi-objective approach which can produce efficient solutions based on the decision maker's preferences along with offering appropriate flexibility to provide different solutions to help the decision maker in selecting the final preferred compromise solution.

However, the most popular approach to convert the fuzzy linear program (FLP) into the conventional deterministic linear program (LP) is the method of ordering fuzzy numbers such as area compensation method [28], the expected mid-point of fuzzy numbers [29], the grade of possibility and necessity [30] and the signed distance method [31].

Similarly, the fuzzy stochastic linear program has also been handled by defuzzifying and derandomizing fuzzy stochastic variables in two manner as sequential or simultaneously. In the sequential approaches, the defuzzifying process is performed first. Then the derandomizing process is done later (see [32]-[33]).

In this paper, we develop a fuzzy stochastic formulation for CLSP in which an optimal production plan is provided for multi products with imprecise dynamic demands over a given planning horizon. The production system is capacitated and setup times/costs occur when changing from one product to another while the model's parameters are vague. The objective is to find an optimal production plan that minimizes production variation, setup, and backlog and inventory cost while maximizes the resource utilization. Compared with deterministic models, fuzzy stochastic CLSP models are even more important in practice. In many real-world problems, customer demands, inventory bounds, and costs are all subject to uncertainty, and to make things worse, they could be dependent on each other in consecutive periods. The study of lot-sizing began with Wagner and Whitin [33], and there is now a sizeable literature in this area extending the basic model to consider capacity constraints, multiple items, multiple levels, etc. However, most previous work on lot-sizing has been directed towards the deterministic case. The practical problem is that in general much, if not all, of the future demands, capacity, resource consumption and costs (i.e. setup, production variation, holding and backorder cost) have to be forecast. However, the existence of forecast errors radically

affects the behavior of the lot sizing procedures [34]. In contrast to the deterministic CLSP, we assume that every parameters such as cost multiple, demand, available resource and consumption resource are fuzzy stochastic variable. The period demands are non stationary (to permit dynamic effects such as seasonal variations, promotions, or general mixtures of known customer orders with random portions of period demands). Because they depend on many factors. Thus, it is difficult to determine exactly the values of these parameters. Moreover, the factors, which are fluctuating due to uncertain environment, could make these parameters vary. These circumstances often happen in long term planning, development strategies and engineering design, in which the described conditions (objectives, constraints, and coefficients) cannot be determined precisely and certainly.

In addition, we developed a new fuzzy stochastic multi-objective linear programming (FSMOLP) approach to solve the proposed model. We first convert the imprecise constraints into the crisp ones by using the expected value of corresponding imprecise parameters as well as the imprecise nature of the objective function by using upper and lower bound of the fuzzy stochastic objective functions coefficients. Then, after applying appropriate strategies for converting this possibilistic model into an auxiliary crisp multi-objective linear model (MOLP), we apply an interactive fuzzy approach to solve this MOLP and finding a preferred compromise solution. The main contributions of this paper could be highlight as follows:

1. Proposing a new bi-objective multi item, multi level CLSP model.
2. Developing a new CLSP model based on the structure of multi assembly shops.
3. Considering the imprecision and ambiguousness of CLSP parameters and modeling a fuzzy stochastic CLSP.
4. Proposing a new algorithm to find the efficient solutions of the proposed fuzzy stochastic CLSP.

II. PROPOSED MODEL

We classified the parameters of the proposed CLSP into two categories as supplier-related and demand-related parameters. Demand-related parameters include demand, backorder cost which are related (or set) to the costumers, while supplier-related parameters are determined by the supplier. Production variation, holding and setup cost, the amount of needed resource to produce one item and the available resource at each level in each period are fallen in the second category. CLSP is a multi period, big-time bucket, production planning problem that lot sizes of the product should be determined for long planning horizon. Thus, it is difficult to determine exactly the values of supplier-related and demand –related parameters. Moreover, the factors, which are fluctuating due to uncertain environment, could make these parameters vary. These circumstances often are different for each category that could introduce as follows:

1. Long planning horizon

2. Risk
3. Impact of the environmental factors
4. Change shops and workers situation
5. Vagueness and imprecise parameters

According to the above mentioned, many researchers focus on the stochastic or fuzzy techniques to deal with the vagueness and imprecision nature of the real case problems. Although both of stochastic and fuzzy approach separately are worthwhile techniques. According to the nature of problem parameters, fuzzy stochastic parameters are more desirable than stochastic or fuzzy parameters, because the parameters of the model determined based on the incomplete historical data (or ill-known data). In addition considering the input parameter as fuzzy stochastic variable could help the decision makers to close into the real problem. Consequently, due to the unavailability or incompleteness of data in real world circumstances, especially in big-time bucket production planning problem, every parameters embedded in such CLSP model have an imprecise and vague nature. So, in order to model the lack of knowledge about these parameters we use appropriate fuzzy stochastic membership function. The following assumption has been considered to formulate proposed CLSP model.

A. Assumptions

1. Each item needs just an identical resource.
2. The amounts of available resources in shop floor are imprecise and consider as fuzzy stochastic variable.
3. At the end of each period available resource will be retrieved.
4. Several products are produced, each of which is assembled from two or more item from upper level.
5. The considered CLSP problem is a multi period big-time bucket problem.
6. When all tasks of a level are finished, the tasks of the lower level could be started.
7. The demand of the product is non deterministic parameter with identical fuzzy stochastic membership function.
8. Customers demand should be satisfied completely at the end of planning horizon.
9. The backorder and inventory holding is allowed, while both of them are equal to zero at the begging and ending of the planning horizon.
10. The production system has been designed as multi-level that in each level various item should be assembled.
11. In terms of number of products is a multi-item production planning that there are several products should be organized during planning horizon.
12. The capacity constraint is considered on available resource.
13. Setup time is considered as constraint in model and setup cost appeared as cost in objective function.

The following notations are used in the formulation of the proposed CLSP model:

B. Notations

Indices:

i : End product, $i=1, \dots, P$

v : Item, $v=1, \dots, V$

l : level, $l=1, \dots, L$

j : Machine, $j=1, \dots, J$

h : period(Time bucket), $h=1, \dots, H$

Parameters:

$\tilde{C}_{ivj}^{Sh}$: Setup cost of item v of level l , from end product i on machine j in period h

$\tilde{C}_i^{Hh}$: Holding cost of end product i in period h

$\tilde{C}_i^{Bh}$: Back order cost of product i in period h

$\tilde{d}_i^h$: Demand of end product i at the end of period h

RT_{ivj} : The rate of production item v at level l of end product i on machine j

t_{ivj}^h : The processing time of item v at level l of end product i on machine j at period h

b_{ilv} : The number of item v at level l needed for producing product i .

ST_{ivj} : Setup time of item v at level l of end product i on machine j

$\tilde{a}_{iv}$: The amount of resource needed to produce one unit of item v of level l from end product i

$\tilde{R}_j$: The accessible amount of resource j

λ_{ivj} : 1, If item v of level l , from end product i is need machine j , 0 otherwise

Decision variable:

I_i^h : Inventory level of end product i at the end of period h

B_i^h : Back order level of end product i at the end of period h

Q_i^h : The lot size of end product i at period h

C. Fuzzy Stochastic CLSP formulation

The objective of the proposed CLSP model is to determine the lot-sizes of the products whereas satisfying the cost and utility of the multi assembly shop. There are two objectives associated with the model: one is to minimize the costs related to the assembling operations, resource setups, backorder and inventory holding; the other is to maximize the utilization of the available resource. The two objectives are described below.

The first objective function is:

$$\begin{aligned} \text{Min} \quad & \sum_{h=1}^H \sum_{i=1}^P \sum_{l=1}^L \sum_{v=1}^V \tilde{C}_{ivj}^{Sh} . Y_i^h + \sum_{h=1}^H \sum_{i=1}^P \tilde{C}_i^{Hh} . I_i^h + \\ & \sum_{h=1}^H \sum_{i=1}^P \tilde{C}_i^{Bh} . B_i^h + \sum_{h=1}^H \sum_{i=1}^P \sum_{l=1}^L \sum_{v=1}^V \tilde{C}_{ivj}^{Ph} . Q_i^h . b_{ilv} \end{aligned} \quad (1)$$

The four terms indicate, respectively, resource setup cost, inventory holding cost, backorder cost and production variation cost. In the first term, $\tilde{C}_{ivj}^{Sh}$ is the setup cost of

resource j and Y_i^h is an auxiliary 0-1 integer variable which assumes a value of one if product i is produced in period, and zero otherwise. In the second and third term, $\tilde{C}_i^{ph}$ and $\tilde{C}_i^{bh}$ are the holding and backorder cost for a unit of product i that holding or non delivery at the end of period. In the fourth term, $\tilde{C}_{ilv}^{ph}$ is per unit cost of performing operation item (ilv). This includes both the assembling and consumed resource cost.

The second objective function is formulated as follows:

$$\text{Min } \sum_{h=1}^H \sum_{l=1}^L \sum_{j=1}^J (\tilde{R}_j - R_j^h) \quad (2)$$

where,

$$R_j^h = \sum_{i=1}^P \sum_{v=1}^V b_{ilv} \cdot Q_i^h \cdot \tilde{a}_{ilv} \cdot \lambda_{ilvj} \quad \forall h, l, j$$

The $\tilde{R}_j$ is the total amount of resource j that is available during planning horizon. We split the multi assembly shop into a number of the level including various item from different products. When any level is started, all related item will be assembled until the next level begin. At the start of next level all resources will be retrieved and $\tilde{R}_j$ resource is available again. In the second objective, we try to close the amount of total available resource with consuming resource ($\tilde{R}_j^h$) for each resource at each level in each period. The following constraint sets are in effect:

The demand of the customers should be satisfied during planning horizon, while according to the assumption the backorder and holding are allowed. So the demand balance constraints are defined as three equations.

$$I_i^{h-1} - B_i^{h-1} + Q_i^h = I_i^h - B_i^h + \tilde{d}_i^h \quad \forall i, h \geq 2, 3, \dots, H \quad (3)$$

$$Q_i^1 - I_i^1 + B_i^1 - \tilde{d}_i^1 = 0 \quad \forall i \quad (4)$$

$$I_i^H = B_i^H = I_i^0 = B_i^0 = 0 \quad \forall i \quad (5)$$

The first and second equation balance the amount of the backorder, holding and lot sizes of all products from $h=2$ to end of the planning horizon and $h=1$ respectively. The third formulation shows that at the start and end of the planning horizon backorder and holding is equal to zero.

The capacity constraint, ensure that the overall consumption must remain lower than the available capacity. The available resource $\tilde{R}_j$ would be retrieve at the end of each level for each resource at each level in each period.

$$\sum_{i=1}^P \sum_{v=1}^V b_{ilv} \cdot Q_i^h \cdot \tilde{a}_{ilv} \cdot \lambda_{ilvj} \leq \tilde{R}_j \quad \forall h, l, j \quad (6)$$

Production equation, grantee that if any product is produced in each period ($Q_i^h > 0$) the decision variable Y_i^h is become one, and zero otherwise.

$$Q_i^h \leq M \cdot Y_i^h \quad \forall i, h \quad (7)$$

$$Y_i^h = \{0, 1\} \quad \forall i, h \quad (8)$$

$$Q_i^h, B_i^h, I_i^h \in \text{Integer} \quad \forall i, h \quad (9)$$

Variable constraint, show the type of each decision variable

$$Y_i^h = \{0, 1\} \quad \forall i, h \quad (10)$$

$$Q_i^h, B_i^h, I_i^h \in \text{Integer} \quad \forall i, h \quad (11)$$

III. RANKIN OF FUZZY STOCHASTIC VARIABLE

There are several types of definition of fuzzy stochastic variable. Here, the attention is restricted to the concepts of Luhandjula [35] for the definition of the fuzzy stochastic variable and its characteristics (see [36]) as shown in figure 1.

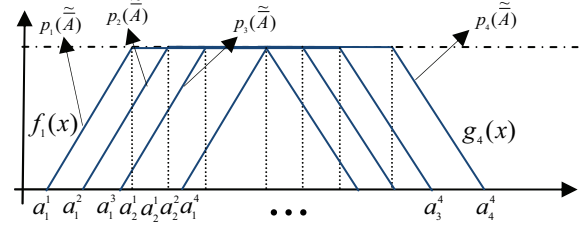


Figure 1. Fuzzy Stochastic number

When $\tilde{A}$ is an uncertain value parameter, the membership degree $\mu_{\tilde{A}}(x)$ can be viewed as the credibility degree of $\tilde{A}$ taking value x [37]. $\tilde{A}$ ($\tilde{A} = (a_1, a_2, a_3, a_4)$) is a trapezoidal fuzzy number with membership degree $\mu_{\tilde{A}}(x)$ as (12) if $f_a(x)$ and $g_a(x)$ are the linear function.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & (-\infty, a_1] \\ f(x) & [a_1, a_2] \\ 1 & [a_2, a_3] \\ g(x) & [a_3, a_4] \\ 0 & [a_4, +\infty) \end{cases} \quad (12)$$

According to the $\tilde{A}$ definition, the expected interval of $\tilde{A}$ was define by Heilpern, [38] as (13).

$$EI(\tilde{A}) = [E_1^{\tilde{A}}, E_2^{\tilde{A}}] \quad (13)$$

where

$$E_1^{\tilde{A}} = \int_0^1 f_{\tilde{A}}^{-1}(y) dy = \int_0^1 ((a_2 - a_1) \cdot y + a_1) dy = \frac{a_2 + a_1}{2}$$

$$E_2^{\tilde{A}} = \int_0^1 g_{\tilde{A}}^{-1}(y) dy = \int_0^1 ((a_3 - a_4) \cdot y + a_4) dy = \frac{a_3 + a_4}{2}$$

From (13), the expected interval of fuzzy stochastic variable $\tilde{A}$ is obtained by (14). Moreover the expected value is define as the average of the $E_1^{\tilde{A}}$ and $E_2^{\tilde{A}}$.

$$EI(\tilde{A}) = [E_1^{\tilde{A}}, E_2^{\tilde{A}}] = \left[\frac{\bar{a}_1 + \bar{a}_2}{2}, \frac{\bar{a}_3 + \bar{a}_4}{2} \right] \quad (14)$$

where

$$\bar{a}_i = E(a_i) = \sum_k p_k(\tilde{A}) \times a_i$$

While there is existed wide range of ranking approach to rank fuzzy numbers in the literature, notably some issued should be considered to choose efficient ranking method as follow:

1. Due to the complicated structure of the mathematical models, the applied method should not increase the size of the problem (neither variable nor constraints).
2. The ranking method should not be approximated approach while we prefer to find the best solution if possible.

Definition 1. Assuming $\tilde{A}_1$ and $\tilde{A}_2$ are two fuzzy stochastic variable. The membership degree $\mu_{\tilde{A}_1 \geq \tilde{A}_2}(x)$ can be viewed as the credibility degree of $\tilde{A}_1 \geq \tilde{A}_2$ taking value x and as follow:

$$\mu_{\tilde{A}_1 \geq \tilde{A}_2} = \begin{cases} 0 & E_2^{\tilde{A}_1} - E_1^{\tilde{A}_1} < 0 \\ \frac{E_2^{\tilde{A}_1} - E_1^{\tilde{A}_1}}{E_2^{\tilde{A}_1} - E_1^{\tilde{A}_1} - (E_1^{\tilde{A}_1} - E_2^{\tilde{A}_1})} & 0 \in [E_1^{\tilde{A}_1} - E_2^{\tilde{A}_1}, E_2^{\tilde{A}_1} - E_1^{\tilde{A}_1}] \\ 1 & E_1^{\tilde{A}_1} - E_2^{\tilde{A}_1} > 0 \end{cases} \quad (14)$$

where $[E_1^{\tilde{A}_1}, E_2^{\tilde{A}_1}]$ and $[E_1^{\tilde{A}_2}, E_2^{\tilde{A}_2}]$ are the expected interval of $\tilde{A}_1$ and $\tilde{A}_2$. When $\mu_{\tilde{A}_1 \geq \tilde{A}_2} \geq \chi$ we will say that $\tilde{A}_1$ is bigger than, or equal, to $\tilde{A}_2$ at least in a degree χ and we will represent it by $\tilde{A}_1 \geq_\chi \tilde{A}_2$.

Definition 2 [36]. Given a decision vector $x \in R^n$ we will say that it is feasible in degree χ if

$$\min_{m=1, \dots, M} \{\mu_m(\tilde{A}_1 x, \tilde{A}_2)\} = \chi \quad (15)$$

That is to say

$$\tilde{A}_1 x \geq_\chi \tilde{A}_2 \quad (16)$$

It is equivalent to (from (14)),

$$[\chi E_1^{\tilde{A}_1} + (1 - \chi) E_2^{\tilde{A}_1}] x \leq \chi E_2^{\tilde{A}_2} + (1 - \chi) E_1^{\tilde{A}_2} \quad (17)$$

IV. OBJECTIVE FUNCTION WITH FUZZY STOCHASTIC COEFFICIENT

In this section we try to convert the fuzzy stochastic coefficient of the objective functions into the crisp ones such that using maximum performance of the fuzzy stochastic number. Assumed that P is a fuzzy stochastic programming problem as follow:

$$P: \min Z_n = \tilde{c}_n^l x \quad \text{for } n = 1, \dots, N \\ \text{s.t.} \\ x \in X$$

$\tilde{c}_n$ is a fuzzy stochastic coefficient of the objective function (n^{th}) and X is feasible space of the problem.

Definition 3. For fuzzy stochastic number ($\tilde{c}_n$) we define the upper and lower bound of $\tilde{c}$ ($\tilde{c}_n^l$ and $\tilde{c}_n^u$) as follow:

Lower bound of $\tilde{c}_n$ ($\tilde{c}_n^l$): The maximum movement that $\tilde{c}_n$ could move to left hand side as follows:

$$\tilde{c}_n^l = \min a_n^k = (\min a_{n1}^k, \min a_{n2}^k, \min a_{n3}^k, \min a_{n4}^k) \quad (18)$$

Upper bound of $\tilde{c}_n$ ($\tilde{c}_n^u$): The maximum movement that $\tilde{c}_n$ could move to right hand side as follows:

$$\tilde{c}_n^u = \max a_n^k = (\max a_{n1}^k, \max a_{n2}^k, \max a_{n3}^k, \max a_{n4}^k) \quad (19)$$

According to $\tilde{c}_n^l$ and $\tilde{c}_n^u$ the membership function of each objective function would be obtained (20) and (21) for minimizing and maximizing respectively.

$$\mu_n^-(z) = \begin{cases} 0 & Z > z_u^{n(PIS)} \\ \frac{z_l^{n(PIS)} - Z}{z_u^{n(NIS)} - z_l^{n(PIS)}} & z_l^{n(PIS)} \leq Z \leq z_u^{n(NIS)} \\ 1 & Z \leq z_l^{n(PIS)} \end{cases} \quad (20)$$

$$\mu_n^+(z) = \begin{cases} 1 & Z > z_u^{n(PIS)} \\ \frac{Z - z_u^{n(PIS)}}{z_u^{n(PIS)} - z_l^{n(NIS)}} & z_l^{n(NIS)} \leq Z \leq z_u^{n(PIS)} \\ 0 & Z \leq z_l^{n(NIS)} \end{cases} \quad (21)$$

Often the researchers to find the positive ideal solution (PIS) and negative ideal solution (NIS) solve the MOLP problem 2N time (N number of the objective functions). To avoid iterative procedure and reduce running time, we define $z_l^{n(PIS)}$ and $z_u^{n(NIS)}$ and $z_u^{n(PIS)}$ and $z_l^{n(NIS)}$ for minimizing and maximizing objective functions respectively as definition 6 and 7.

Definition 4. Define $z_l^{n(PIS)}$ and $z_u^{n(NIS)}$ for minimization:

$$z_l^{n(PIS)} = \min z_l^n(x) \quad (\text{For each } h \text{ individually})$$

$$z_u^{n(NIS)} = \max_{\hat{x}} \{z_u^n(\hat{x}_n)\} \quad (\hat{x} \text{ is the solution of } z_u^n, \text{ for maximizing each } n \text{ individually})$$

Definition 5. Define $z_u^{n(PIS)}$ and $z_l^{n(NIS)}$ for maximization:

$$z_u^{n(PIS)} = \max z_u^n(x) \quad (\text{For each } h \text{ individually})$$

$$z_l^{n(NIS)} = \min_{\hat{x}} \{z_l^n(\hat{x}_n)\} \quad (\hat{x} \text{ is the solution of } z_l^n, \text{ for minimizing each } n \text{ individually})$$

where z_l^n and z_u^n is formulated as (22) and (23).

$$Z_n = \min EV(\tilde{c}_n^l).x \\ \text{s.t.} \\ x \in X \quad (22)$$

$$Z_n = \min EV(\tilde{c}_n^u).x \\ \text{s.t.} \\ x \in X \quad (23)$$

Torabi and Hassini, [26], proposed a novel interactive fuzzy approach to solve multi objective linear programming (MOLP) problem and finding a preferred compromise solution. We applied their aggregate function to solve P as follow:

$$\max \eta(z) = \zeta \eta_0 + (1 - \zeta) \sum_n \theta_n \mu_n(z) \\ \text{s.t.} \\ \eta_0 \leq \mu_n(z) \quad n = 1, \dots, N \\ z \in F(z) \\ \zeta \in [0, 1] \quad (24)$$

where $\mu_n(z)$ and $\eta_0 = \min_n \{\mu_n(z)\}$ denote the satisfaction degree and minimum satisfaction degree of objective functions, respectively. This formulation has a new achievement function defined as a convex combination of the lower bound for satisfaction degree of objectives (η_0), and the weighted sum of these achievement degrees ($\mu_n(z)$) to ensure yielding an adjustably balanced compromise solution.

Moreover, θ_n and ς indicate the relative importance of the n^{th} objective function and the coefficient of compensation, respectively. The θ_n parameters are determined by the decision maker based on her/his preferences such that $\sum_n \theta_n = 1$, $\theta_n > 0$. Also, ς controls the minimum satisfaction

level of objectives as well as the compromise degree among the objectives implicitly. That is, the proposed formulation is capable of yielding both unbalanced and balanced compromised solutions for a given problem instance based on the decision maker's preferences through adjusting the value of parameter ς [17].

By defining upper and lower bound of fuzzy stochastic coefficient (definition 3) we convert the imprecision objective function into crisp one for single or multi objective fuzzy stochastic programming problem.

V. THE PROPOSED FUZZY SOLUTION APPROACH

To solve the proposed MOLP models with fuzzy stochastic parameters several methods have been developed in the literature. By considering the imprecision situation as fuzzy stochastic parameters, both experimentally nature of the data and satisfaction level of decision makers are taken into account simultaneously. But it would increase the complexity of the solution method. Hop, [16] convert the fuzzy stochastic MOLP into the corresponding deterministic MOLP problem by using superiority and inferiority measures of the fuzzy stochastic variables. In another work, he presented a model to measure attainment value of fuzzy stochastic goals and then applied it to de-randomize and defuzzify the fuzzy stochastic goal programming problem and obtain a standard linear program [39]. Although in both papers he obtained acceptable results, two issues could be discussed: A) he considered fuzzy stochastic number as different fuzzy number with identical probability (discrete distribution). B) He changed the structure of the model by adding the imprecise constraint into objective function. However, in this paper to solve the proposed fuzzy stochastic MOLP model of CLSP, we proposed an interactive fuzzy stochastic solution approach by combining the [25], [26], [36]. The proposed hybrid approach is as follows:

Step 1: Determine the appropriate fuzzy stochastic membership function for imprecise parameters and forming the fuzzy stochastic CLSP model.

Step 2: Convert the uncertain constraints into the crisp ones by using the expected value of corresponding imprecise parameters as equation (17).

Step 3: Convert fuzzy stochastic coefficient of the objective functions into the fuzzy ones by using upper and lower bound of coefficient (equations (18) and (19)).

Step 4: Convert fuzzy objective functions into the crisp ones by using the expected value of the upper and lower bound of the coefficients and find the Z^u , Z^l .

Step 5: Determine the membership function of the objective functions by using (20)-(21) and definition 6 and 7.

Step 6: Convert the equivalent crisp MOLP model into a single one using TH aggregation functions [26], (24) which resulting the efficient solutions.

To run the proposed algorithm the decision maker should specify the value of the: feasibility degree of the constraints (χ); the deviation of the objective function coefficient (γ); coefficient of compensation (ς); relative importance of the fuzzy goals θ_n . Jimenez, [25] proposed a three steps procedure to maximize the satisfaction (feasibility) degree. The deviation of the objective function coefficient (γ) is set by the DM. Also, the coefficient of compensation and relative importance of the fuzzy goals are specified by decision makers generally. If the decision makers are not satisfied with the obtained results, another solution will be provided by changing these values.

VI. NUMERICAL ILLUSTRATION

To illustrate the efficiency of the proposed method, we consider Luhandjula's example. The results are compared with Luhandjula [35] and Hop [36] results. In this example a triangular fuzzy number $((m-0.5, m, m+0.5))$ with a discrete distribution $p_1 = 0.25$ and $p_2 = 0.75$ has been considered. The results show the capability of the proposed model in against of existed method [36] and [35] as reported in table 1.

$$\begin{aligned} \max \quad & \begin{pmatrix} 3 \\ 4 \end{pmatrix} x_1 + \begin{pmatrix} 2 \\ 3 \end{pmatrix} x_2 \\ \text{s.t.} \quad & \tilde{A}x \leq \tilde{b} \\ & x \geq 0, \end{aligned} \quad (\tilde{A}, \tilde{B}) = \begin{cases} (\tilde{A}_1, \tilde{B}_1) = \left\{ \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right\} \\ (\tilde{A}_2, \tilde{B}_2) = \left\{ \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \end{pmatrix} \right\} \end{cases}$$

According to the step 1 to 6 the model would be rewritten as follow:

$$\begin{aligned} \max \quad & \eta_0 \\ \text{s.t.} \quad & \eta_0 \leq \mu_n(z) \quad n=1 \\ & [0.75\chi + (1-\chi)1.25]x_1 + [2.25\chi + (1-\chi)2.75]x_2 \leq [4.25\chi + (1-\chi)4.75] \\ & [\chi + (1-\chi)1.5]x_1 + [1.5\chi + (1-\chi)2]x_2 \leq [3.75\chi + (1-\chi)4.25] \end{aligned}$$

where

$$\mu_1(z) = \begin{cases} 1 & z > 15 \\ \frac{z-11.25}{3.75} & 11.25 \leq z \leq 15 \\ 0 & z < 11.25 \end{cases}$$

Table 1. Obtained results

Method	Decision vector	Objective function
Proposed method	$x_1 = 3.6$ $x_2 = 0.1$	15
Hop [16]	$x_1 = 4$ $x_2 = 1$	14
Luhandjula [25]	$x_1 = 1.34$ $x_2 = 1$	6.02

From the obtained results, it is clear that the proposed method gives acceptable solution based on the Luhandjula [35] and Hop [36] results. In addition, the proposed methodology is also superior considering the amount of calculations and complexities embedded in other methodology, especially, the Hop's method because of a few number of constraints and the simplicity of conversion. Moreover, the proposed method compares fuzzy stochastic variables based on their relative relationships. This comparison will relax the constraints from the deterministic ones. Thus, the larger spreads of fuzzy numbers led to the more relaxation amount [36].

To illustrate the validity of the proposed model, a numerical experiment is solved and results are reported in table 2. The characteristics of the problem are as follows:

1. Membership function of each fuzzy stochastic parameter (FSP) is trapezoidal with discrete uniform distribution.
2. Setup and production variation costs are a FSP with trapezoidal fuzzy membership function and are generated randomly with uniform distribution. Similarly, Backorder and holding cost of product are $\tilde{C}_{div}^{bh} = 0.55 \times \tilde{C}_{div}^{ph}$ and $\tilde{C}_{div}^{bh} = 1.2 \times \tilde{C}_{div}^{ph}$ respectively.
3. b_{iv} were generated from a uniform distribution U [1,5].
4. $\tilde{a}_{iv}$ were generated randomly from uniform distribution U [3,5].
5. $\tilde{r}_j$ were generated randomly from uniform distribution U [4000,16000].

Based on the structure of the proposed CLSP model, problem dimensions has been obtained according to five parameters as period, product, level, item and resource and is equal to $(h \times i \times l \times v \times j) = (3 \times 3 \times 4 \times 3 \times 3)$.

The test problem is carried out with a branch-and-bound (B&B) method by using the LINGO 9.0 software. We have considered a problem with three products, four levels, three item which should be produced during three periods with three resources. According to the step 1 to 4 of proposed method the Fuzzy stochastic CLSP model is reformulated and the solution of Z_i^n , Z_i^n , $Z_i^{n(PIS)}$, $Z_u^{n(NIS)}$ has been calculated firstly. We determine the membership function of the each objective function and then convert the equivalent MOLP model into a single crisp ones based on the step 6 and obtaining results as $Z_1=6971$ and $Z_2=288$ with $\zeta = 0.1$, $\bar{\theta} = (0.7, 0.3)$ as reported in table 2.

VII. CONCLUSION

We develop a fuzzy stochastic multi-objective linear programming model for big bucket multiple periods lot-sizing problems in which an optimal production plan is provided for multiple products with dynamic demands over a given

planning horizon. The significance of accounting for uncertainty and risk in such networks spurs an interest to develop appropriate production planning tools to cope with uncertain and imprecise parameters in multi assembly shop lot sizing. To deal with imprecise and ambiguous nature of data, we introduce appropriate fuzzy stochastic membership functions for imprecise parameters.

Then an interactive fuzzy stochastic approach has been developed. Uncertain constraints as well as imprecise objective functions are converted into their crisp one by using the expected interval and value of the ill-known parameters, respectively. Finally, the equivalent crisp MOLP model is transferred into a single objective model using an appropriate aggregation function which ensures yielding efficient solutions. The proposed solution method is validated over a number of test problem and the corresponding results seem to be promising. This approach can also be used for solving other fuzzy stochastic MOLP models due to its computationally advantages.

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Table 2. Obtained results

Objective function			Period								
			h=1			h=2			h=3		
			Q	Products	I	Q	Products	I	Q	Products	I
Lower	Cost	Utility	23-24-24	0-0-0	14-14-14	0-0-0	0-0-0	6-6-7	0-0-0	0-0-0	0-0-0
Upper	6971	288	0-0-0	9-10-9	0-0-0	0-0-0	17-17-16	0-0-0	23-24-24	0-0-0	0-0-0
	8769	352									