

Adaptive Fault Estimation Design For T-S Fuzzy Systems With Interval Time Varying Delay

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Abstract—This paper is concerned with fault estimation design for continuous-time Takagi-Sugeno (T-S) fuzzy systems with time delay by using adaptive fault diagnostic observer. Through constructing an appropriate type of Lyapunov function, criteria is established to reduce the conservatism of the design procedure. Based on appropriate type of Lyapunov function, new sufficient delay-dependent conditions are given to guarantee the stability of error dynamics in terms of Linear Matrix Inequalities (LMI). Finally, the effectiveness of the proposed approach is illustrated through a simulation example.

Keyword: Linear Matrix Inequalities, Takagi-Sugeno fuzzy systems, fault tolerant control.

I. INTRODUCTION

In many applications the problem of fault detection and isolation FDI is a crucial issue that has been theoretically and experimentally investigated with different types of approaches. Adaptive Fault Diagnostic Observer (AFDO) approach has the advantage to estimate the state vector and actuator faults simultaneously. As consequence, the use of this approach has attracted an increasing interest in the last decade ([2], [3], [4]). Recent years have witnessed rapidly growing in the study of T-S fuzzy model since it is well known that this model is a powerful tool in approximating a wide class of nonlinear systems ([6], [7], [8], [9]). Still, time delay is one of the instability sources of dynamical processes. For this reason, T-S fuzzy model has been extended to deal with nonlinear systems with time delay. Different delay-independent methodologies have been proposed for analysis and synthesis for T-S fuzzy systems with time delay ([10], [11], [12]). It is generally recognized that delay-dependent results are usually less conservative than delay-independent ones. That's why, delay-dependent techniques have been reported in ([13], [14], [15]).

Considering the AFDO design, the problem of fault estimation for linear system are treated in [3]. This idea is extended to deal with T-S fuzzy models with constant time delay in [4]. The obtained result is delay-independent. By adopting the free weighting matrix technique, delay-dependent results are obtained for linear systems with bounded time delay [5]. However, the price under additional variables is the growth of computational burden in the meantime. In this paper, we investigate the delay-dependent AFDO design for T-S fuzzy systems with interval time varying delay without using any free

weighting matrices. The advantages of the developed criteria are: (1) the reduction of conservatism; (2) the reduction of computational complexity and (3) the removing of condition $\dot{\tau}(t) < 1$ which makes it applicable to both slow and fast time varying delay.

Notations $W + W^T$ is denoted as $W + (*)$ for simplicity.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a nonlinear system with time delay which could be approximated by a time delay T-S fuzzy model.

The i^{th} rule of the system is expressed in the following IF-THEN rules.

Plant Rule $i(i = 1, 2, \dots, r)$: If θ_1 is μ_{i1} and $\dots$ and θ_p is μ_{ip} THEN

$$\begin{aligned}\dot{x}(t) &= A_i x(t) + A_{\tau i} x(t - \tau(t)) + B_i u(t) \\ &\quad + E_i f(t) \\ y(t) &= C x(t) \\ x(t) &= \psi(t), \quad t \in [-\tau, 0]\end{aligned}\quad (1)$$

where $\theta_j(x(t))$ and $\mu_{ij}(i = 1, \dots, r, j = 1, \dots, p)$ are respectively the premise variable and the fuzzy sets; $\psi(t)$ is the initial condition; $x(t) \in \mathbb{R}^{n_x}$ is the state; $u(t) \in \mathbb{R}^{n_u}$ is the control input; $f(t) \in \mathbb{R}^{n_f}$ represents the actuator fault vector. It is assumed that the derivative of $f(t)$ with respect to time is norm bounded, i.e. $\|\dot{f}(t)\| \leq f_1$ and $0 \leq f_1 < \infty$. r is the number of IF-THEN rules.

The AFDO is constructed as

Observer Rule $i(i = 1, 2, \dots, r)$: If θ_1 is μ_{i1} and $\dots$ and θ_p is μ_{ip} THEN

$$\begin{aligned}\dot{\hat{x}}(t) &= A_i \hat{x}(t) + A_{\tau i} \hat{x}(t - \tau(t)) + B_i u(t) \\ &\quad + E_i \hat{f}(t) - L_i (\hat{y}(t) - y(t)) \\ \hat{y}(t) &= C \hat{x}(t)\end{aligned}\quad (2)$$

Denote $e_x(t) = \hat{x}(t) - x(t)$, $e_y(t) = \hat{y}(t) - y(t)$, $e_f(t) = \hat{f}(t) - f(t)$

then the error dynamic is given by

$$\begin{aligned}\dot{e}_x(t) &= (A(t) - L(t)C)e_x(t) \\ &\quad + A_{\tau}(t)e_x(t - \tau(t)) + E(t)e_f(t) \\ e_y(t) &= C e_x(t)\end{aligned}$$

where

$$A(t) = \sum_{i=1}^r h_i A_i; A_\tau(t) = \sum_{i=1}^r h_i A_{\tau i}; E(t) = \sum_{i=1}^r h_i E_i$$

The following result has been given in [4] which treats the case of constant delay ($\tau(t) = \tau$).

Theorem 1: [4] If there exist symmetric positive definite matrices $P > 0$, $Q > 0$, $M > 0$, Y_i and F_i such that the following LMI hold:

$$\Psi_{ij} + \Psi_{ji} < 0, \quad i, j = 1, 2, \dots, r, i \leq j \quad (3)$$

$$E_i^T P = F_i C, \quad i = 1, 2, \dots, r \quad (4)$$

then the fuzzy Fast Adaptive Fault Estimation (FAFE) algorithm

$$\dot{\hat{f}}(t) = -\Gamma \sum_{i=1}^r h_i F_i (\dot{e}_y(t) + e_y(t)) \quad (5)$$

can realize $e_x(t)$ and $e_f(t)$ uniformly ultimately bounded, where

$$\Psi_{ij} = \begin{bmatrix} \Psi_{ij}^{11} & P A_{\tau i} & -A_i^T P E_j + C^T Y_i^T E_j \\ * & -Q & -A_{\tau i}^T P E_j \\ * & * & -E_i^T P E_j - E_j^T P E_i + M \end{bmatrix}$$

in which

$$\Psi_{ij}^{11} = P A_i + A_i^T P - Y_i C - C^T Y_i^T + Q$$

In this case, the gains L_i are given by

$$L_i = Y_i P^{-1} \quad (6)$$

the symmetric positive definite matrix Γ is the learning rate.

In this paper, we will develop another design method which improves that in theorem (1).

III. IMPROVED RESULT

In this section, we first present useful lemma and then present the main result.

Lemma 1: Given matrices x and y .

For a symmetric positive definite matrix P , the following inequality holds

$$2x^T y \leq x^T P x + y^T P^{-1} y$$

Lemma 2 Consider a negative definite matrix $\Pi < 0$.

Given a symmetric matrix X of appropriate dimension such that $X^T \Pi X < 0$, then, $\exists \lambda \in \mathbb{R}^+$ such that

$$X^T \Pi X \leq -2\lambda X - \lambda^2 \Pi^{-1}$$

Lemma 3 Given matrices $M, E, F(t)$ with compatible dimensions and $F(t)$ satisfying $F(t)^T F(t) \leq I$.

Then, the following inequalities hold for any $\epsilon > 0$

$$M F(t) E + E^T F(t)^T M^T \leq \epsilon M M^T + \epsilon^{-1} E^T E$$

The following theorem gives the new LMI-based conditions in order to reduce the conservatism in theorem (1).

Furthermore, we consider interval time-varying delay rather than constant delay.

the interval time varying delay satisfies

$$0 < \tau_m \leq \tau(t) \leq \tau_M, \dot{\tau}(t) \leq \beta \quad (7)$$

Denote

$$\tau_1 = \frac{\tau_m + \tau_M}{2}; \tau_2 = \frac{\tau_M - \tau_m}{2}$$

Theorem 2: For given positif scalar λ , if there exist matrices $P > 0$, $Q_l > 0 (l = 1, 2, 3, 4, 5)$, $R_m > 0 (m = 1, 2, 3, 4)$ such that (4) and the following LMI hold

$$\Xi_{ij} + \Xi_{ji} < 0, \quad i, j = 1, 2, \dots, r, i \leq j \quad (8)$$

where

$$\Xi_{ij} = \begin{bmatrix} \xi_{ij}^{11} & R_4 & P A_{\tau i} & 0 \\ * & \xi_{22} & R_3 & Q_2 + R_1 \\ * & * & \xi_{33} & 0 \\ * & * & * & \xi_{44} \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ 0 & \xi_{ij}^{16} & \xi_{ij}^{17} \\ 0 & 0 & 0 \\ R_3 & -A_{\tau i}^T P E_j & A_{\tau i}^T P \\ -Q_2 + R_2 & 0 & 0 \\ \xi_{55} & 0 & 0 \\ * & \xi_{ij}^{66} & E_i^T P \\ * & * & \xi_{ij}^{77} \end{bmatrix}$$

in which

$$\begin{aligned} \xi_{ij}^{11} &= P A_i - Y_i C + A_i^T P - C^T Y_i^T + Q_4 + Q_5 - R_4; \\ \xi_{ij}^{16} &= -A_i^T P E_j + C^T Y_i^T E_j; \\ \xi_{ij}^{17} &= A_i^T P - C^T Y_i^T \\ \xi^{22} &= Q_1 - Q_5 - R_1 - R_3 - R_4; \\ \xi^{33} &= -(1 - \beta) Q_4 - 2R_3; \\ \xi^{44} &= Q_3 - Q_1 - R_1 - R_2; \\ \xi^{55} &= -Q_3 - R_2 - R_3; \\ \xi_{ij}^{66} &= -E_i^T P E_j - E_j^T P E_i + M; \\ \xi_{ij}^{77} &= -2\lambda P \\ &\quad + \lambda^2 ((\tau_2)^2 R_1 + (\tau_2)^2 R_2 + (2\tau_2)^2 R_3 + (\tau_m)^2 R_4) \end{aligned}$$

then the fuzzy Fast Adaptive Fault Estimation (FAFE) algorithm (5) can realize $e_x(t)$ and $e_f(t)$ uniformly ultimately bounded,

In this case, the L_i are given by (6).

Proof:

Consider the following Lyapunov function

$$V(t) = \sum_{i=1}^6 V_i(t) \quad (9)$$

where

$$\begin{aligned}
V_1(t) &= e_x(t)^T P e_x(t) + \int_{t-\tau_1}^{t-\tau_m} \{e_x(s)^T Q_1 e_x(s) \\
&\quad + 2e_x(s)^T Q_2 e_x(s - \tau_2) \\
&\quad + e_x(s - \tau_2)^T Q_3 e_x(s - \tau_2)\} ds \\
&\quad + \int_{t-\tau(t)}^t e_x(s)^T Q_4 e_x(s) ds \\
&\quad + \int_{t-\tau_m}^t e_x(s)^T Q_5 e_x(s) ds \\
V_2(t) &= \tau_2 \int_{-\tau_1}^{-\tau_m} \int_{t+\sigma}^t \dot{e}_x(s)^T R_1 \dot{e}_x(s) ds d\sigma \\
V_3(t) &= \tau_2 \int_{-\tau_M}^{-\tau_1} \int_{t+\sigma}^t \dot{e}_x(s)^T R_2 \dot{e}_x(s) ds d\sigma \\
V_4(t) &= 2\tau_2 \int_{-\tau_M}^{-\tau_m} \int_{t+\sigma}^t \dot{e}_x(s)^T R_3 \dot{e}_x(s) ds d\sigma \\
V_5(t) &= \tau_m \int_{-\tau_m}^0 \int_{t+\sigma}^t \dot{e}_x(s)^T R_4 \dot{e}_x(s) ds d\sigma \\
V_6(t) &= e_f(t)^T \Gamma^{-1} e_f(t)
\end{aligned}$$

Taking the derivation of $V(t)$, we have

$$\dot{V}_1(t) = 2e_x(t)^T P \dot{e}_x(t) + \Delta$$

where

$$\begin{aligned}
\Delta &= e_x(t - \tau_m)^T Q_1 e_x(t - \tau_m) + 2e_x(t - \tau_m)^T Q_2 e_x(t - \tau_1) \\
&\quad + e_x(t - \tau_1)^T Q_3 e_x(t - \tau_1) - e_x(t - \tau_1)^T Q_1 e_x(t - \tau_1) \\
&\quad - 2e_x(t - \tau_1)^T Q_2 e_x(t - \tau_M) - e_x(t - \tau_M)^T Q_3 e_x(t - \tau_M) \\
&\quad + e_x(t)^T Q_4 e_x(t) - (1 - \dot{\tau}(t)) e_x(t - \tau(t))^T Q_4 e_x(t - \tau(t)) \\
&\quad + e_x(t)^T Q_5 e_x(t) - e_x(t - \tau_m)^T Q_5 e_x(t - \tau_m)
\end{aligned}$$

$$\begin{aligned}
\dot{V}_2(t) &= \tau_2 \{ \dot{e}_x(t)^T R_1 \dot{e}_x(t) \\
&\quad - \int_{t-\tau_1}^{t-\tau_m} \dot{e}_x(s)^T R_1 \dot{e}_x(s) ds \}
\end{aligned} \tag{10}$$

Similarly, we obtain

$$\begin{aligned}
\dot{V}_3(t) &= \tau_2 \{ \tau_2 \dot{e}_x(t)^T R_2 \dot{e}_x(t) \\
&\quad - \int_{t-\tau_M}^{t-\tau_1} \dot{e}_x(s)^T R_2 \dot{e}_x(s) ds \} \\
\dot{V}_4(t) &= 2\tau_2 \{ 2\tau_2 \dot{e}_x(t)^T R_3 \dot{e}_x(t) \\
&\quad - \int_{t-\tau_M}^{t-\tau_m} \dot{e}_x(s)^T R_3 \dot{e}_x(s) ds \} \\
\dot{V}_5(t) &= \tau_m \{ \tau_m \dot{e}_x(t)^T R_4 \dot{e}_x(t) \\
&\quad - \int_{t-\tau_m}^t \dot{e}_x(s)^T R_4 \dot{e}_x(s) ds \}
\end{aligned}$$

By using the FAFE algorithm given in (5), we obtain

$$\begin{aligned}
\dot{V}_6(t) &= -2e_f(t)^T F(t) C (\dot{e}_x(t) + e_x(t)) \\
&\quad - 2e_f(t)^T \Gamma^{-1} \dot{f}(t)
\end{aligned}$$

By considering equality (4), we obtain:

$$\begin{aligned}
&\dot{V}_1(x(t)) + \dot{V}_6(x(t)) \\
&= 2e_x(t)^T P [(A(t) - L(t)C)e_x(t) + A_\tau(t)e_x(t - \tau(t)) \\
&\quad + E(t)e_f(t)] \\
&\quad + \Delta - 2e_f(t)^T F(t) C (\dot{e}_x(t) + e_x(t)) - 2e_f(t)^T \Gamma^{-1} \dot{f}(t) \\
&= 2e_x(t)^T P [(A(t) - L(t)C)e_x(t) + A_\tau(t)e_x(t - \tau(t))] \\
&\quad + \Delta - 2e_f(t)^T F(t) C \dot{e}_x(t) - 2e_f(t)^T \Gamma^{-1} \dot{f}(t) \\
&\quad + \underbrace{2e_x(t)^T P E(t) e_f(t) - 2e_f(t)^T F(t) C e_x(t)}_0
\end{aligned}$$

From lemma (1), we can obtain that

$$\begin{aligned}
-2e_f(t)^T \Gamma^{-1} \dot{f}(t) &\leq \\
e_f(t)^T M e_f(t) + \dot{f}(t)^T \Gamma^{-1} M^{-1} \Gamma^{-1} \dot{f}(t) &\leq \\
e_f(t)^T M e_f(t) + \delta &
\end{aligned} \tag{11}$$

where

$$\delta = f_1^2 \lambda_{max}(\Gamma^{-1} M^{-1} \Gamma^{-1})$$

Applying Jensen's integral inequality [1] to deal with the cross product items, we have

$$\begin{aligned}
&-\tau_2 \int_{t-\tau_1}^{t-\tau_m} \dot{e}_x(s)^T R_1 \dot{e}_x(s) ds \\
&\leq \begin{bmatrix} e_x(t - \tau_m) \\ e_x(t - \tau_1) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ * & -R_1 \end{bmatrix} \begin{bmatrix} e_x(t - \tau_m) \\ e_x(t - \tau_1) \end{bmatrix} \\
&-\tau_2 \int_{t-\tau_M}^{t-\tau_1} \dot{e}_x(s)^T R_2 \dot{e}_x(s) ds \\
&\leq \begin{bmatrix} e_x(t - \tau_1) \\ e_x(t - \tau_M) \end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 \\ * & -R_2 \end{bmatrix} \begin{bmatrix} e_x(t - \tau_1) \\ e_x(t - \tau_M) \end{bmatrix} \\
&-2\tau_2 \int_{t-\tau_M}^{t-\tau_m} \dot{e}_x(s)^T R_3 \dot{e}_x(s) ds \\
&\leq -(\tau(t) - \tau_m) \int_{t-\tau(t)}^{t-\tau_m} \dot{e}_x(s)^T R_3 \dot{e}_x(s) ds \\
&\quad - (\tau_M - \tau(t)) \int_{t-\tau_M}^{t-\tau(t)} \dot{e}_x(s)^T R_3 \dot{e}_x(s) ds \\
&\leq \begin{bmatrix} e_x(t - \tau_m) \\ e_x(t - \tau(t)) \\ e_x(t - \tau_M) \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 & 0 \\ * & -2R_3 & R_3 \\ * & * & -R_3 \end{bmatrix} \\
&\quad \begin{bmatrix} e_x(t - \tau_m) \\ e_x(t - \tau(t)) \\ e_x(t - \tau_M) \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
& -\tau_m \int_{t-\tau_m}^t \dot{e}_x(s)^T R_4 \dot{e}_x(s) ds \\
& \leq \begin{bmatrix} e_x(t) \\ e_x(t-\tau_m) \end{bmatrix}^T \begin{bmatrix} -R_4 & R_4 \\ * & -R_4 \end{bmatrix} \begin{bmatrix} e_x(t) \\ e_x(t-\tau_m) \end{bmatrix}
\end{aligned}$$

Let $\eta(t) = [e_x(t)^T, e_x(t-\tau_m)^T, e_x(t-\tau(t))^T, e_x(t-\tau_1)^T, e_x(t-\tau_M)^T, e_f(t)^T]^T$.

$$\begin{aligned}
\dot{V}(t) & \leq \eta(t)^T \Omega(t) \eta(t) \\
& + \dot{e}_x(t)^T ((\tau_2)^2 R_1 + (\tau_2)^2 R_2 + (2\tau_2)^2 R_3 + (\tau_m)^2 R_4) \dot{e}_x(t) + \delta
\end{aligned} \tag{12}$$

where

$$\Omega(t) = \begin{bmatrix} \omega_{11}(t) & R_4 & P A_\tau(t) & 0 & 0 & \omega_{16}(t) \\ * & \xi_{22} & R_3 & Q_2 + R_1 & 0 & 0 \\ * & * & \xi_{33} & 0 & R_3 & A_\tau(t)^T P E(t) \\ * & * & * & \xi_{44} & -Q_2 + R_2 & 0 \\ * & * & * & * & \xi_{55} & 0 \\ * & * & * & * & * & \omega_{66}(t) \end{bmatrix}$$

in which

$$\begin{aligned}
\omega_{11}(t) & = P(A(t) - L(t)C) + (A(t) - L(t)C)^T P \\
& + Q_4 + Q_5 - R_4; \\
\omega_{16}(t) & = -(A(t) - L(t)C)^T P E(t); \\
\omega_{66}(t) & = -2E(t)^T P E(t) + M
\end{aligned}$$

By the Schur complement, we obtain

$$\Phi(t) = \begin{bmatrix} \Omega(t) & \phi_{12}(t) \\ * & -P((\tau_2)^2 R_1 + (\tau_2)^2 R_2 + (2\tau_2)^2 R_3 + (\tau_m)^2 R_4)^{-1} P \end{bmatrix}$$

where

$$\begin{aligned}
\phi_{12}(t)^T & = [P(A(t) - L(t)C) \quad 0 \quad P A_\tau(t) \\
& 0 \quad 0 \quad P E(t)]^T
\end{aligned} \tag{13}$$

Applying lemma 2, if $\Xi_{ij} + \Xi_{ji} < 0, i, j = 1, 2, \dots, r, i \leq j$, then there exist a scalar $\epsilon > 0$ such that $\dot{V}(t) < -\epsilon \|\eta(t)\|^2 + \delta$. It follows that $\dot{V}(t) < 0$ for $\epsilon \|\eta(t)\|^2 > \delta$, which means that $\eta(t)$ converges to a small set $S = \{\eta(t) | \|\eta(t)\|^2 \leq \frac{\delta}{\epsilon}\}$ according to Lyapunov stability theory. Therefore, estimation errors of both the state and the fault are uniformly ultimately bounded.

We can extend the result in theorem 2 to the uncertain case: A_i and $A_{\tau i}$ are replaced by $A_i + \Delta A_i$ and $A_{\tau i} + \Delta A_{\tau i}$, respectively.

$$\Delta A_i = M_{Ai} F_i(t) E_{Ai}; \quad \Delta A_{\tau i} = M_{A\tau i} F_i(t) E_{A\tau i}$$

We obtain the following theorem

Theorem 3 For given positif scalar λ , if there exist matrices $P, Q_l (l = 1, 2, 3, 4, 5), R_m (m = 1, 2, 3, 4)$ and scalars ϵ_{Ai} and $\epsilon_{A\tau i}$ such that (4) and the following LMI hold

$$\Upsilon_{ij} + \Upsilon_{ji} < 0, \quad i, j = 1, 2, \dots, r, i \leq j \tag{14}$$

where

$$\begin{aligned}
\Upsilon_{ij} & = \begin{bmatrix} \Xi_{ij} & U_i & V_i \\ * & -\epsilon_{Ai} I & 0 \\ * & * & -\epsilon_{A\tau i} I \end{bmatrix} \\
& + \text{diag}(\epsilon_{Ai} E_{Ai}^T E_{Ai}, 0, \epsilon_{A\tau i} E_{A\tau i}^T E_{A\tau i}, 0_{1 \times 6})
\end{aligned}$$

in which

$$\begin{aligned}
U_i & = \begin{bmatrix} P M_{Ai} \\ 0_{4 \times 1} \\ -E_j^T P M_{Ai} \\ P M_{Ai} \end{bmatrix} \\
V_i & = \begin{bmatrix} P M_{A\tau i} \\ 0_{4 \times 1} \\ -E_j^T P M_{A\tau i} \\ P M_{A\tau i} \end{bmatrix}
\end{aligned}$$

then the fuzzy Fast Adaptive Fault Estimation (FAFE) algorithm (5) can realize $e_x(t)$ and $e_f(t)$ uniformly ultimately bounded,

In this case, the L_i are given by (6).

Proof:

In the case with uncertainty, we obtain theorem (2) in which Ξ_{ij} is replaced by $\bar{\Xi}_{ij}$ such that

$$\bar{\Xi}_{ij} = \Xi_{ij} + \Delta \Xi_{ij}$$

The uncertain part is given by

$$\Delta \Xi_{ij} = \begin{bmatrix} \Delta \Xi_{ij}^{11} & 0 & P \Delta A_{\tau i} & 0_{1 \times 2} & -\Delta A_i^T P E_j & \Delta A_i^T P \\ 0 & 0 & 0 & 0_{1 \times 2} & 0 & 0 \\ * & 0 & 0_{1 \times 2} & -\Delta A_{\tau i}^T P E_j & \Delta A_{\tau i}^T P \\ * & * & 0_{1 \times 2} & 0 & 0 \\ * & * & 0_{1 \times 2} & 0 & 0 \\ * & * & 0_{1 \times 2} & 0 & 0 \\ * & * & 0_{1 \times 2} & 0 & 0 \end{bmatrix}$$

in which

$$\Delta \Xi_{ij}^{11} = P \Delta A_i + \Delta A_i^T P$$

$$\begin{aligned}
\Delta \Xi_{ij} & = U_i F(t) [E_{Ai} \quad 0_{1 \times 6}] + (*) \\
& + V_i F(t) [0_{1 \times 2} \quad E_{A\tau i} \quad 0_{1 \times 4}] + (*)
\end{aligned} \tag{15}$$

By applying lemma 3 and schur complement, we obtain theorem 3.

Remark 1 It is easy to solve inequality (3) and (14) by using LMI Toolbox. For equations (4), we can make a transform into the following optimization problem:

Minimize $\rho > 0$ subject to

$$\begin{bmatrix} \rho I & E_i^T P - F_i C \\ * & \rho I \end{bmatrix} > 0, i = 1, \dots, r \quad (16)$$

Remark 2 Compared with theorem (1), theorem 2 contains some new improvements as follows:

(i) The result of theorem (2) is less conservative than that of theorem (1). This is because the conditions (3) is recovered by setting $Q_1 = Q_2 = Q_3 = Q_5 = R_1 = R_2 = R_3 = R_4 = 0$, $\tau(t) = \tau$, $\beta = 0$ in conditions (14).

(ii) Theorem 1 consider an interval time varying delay which is more general than constant time delay.

Furthermore, the uncertain case has been considered in theorem (3).

Remark 3 Based on the on-line fault estimate, an observer-based fast active fault tolerant controller can be designed to compensate for the effects of faults. This design is presented in ([4]) by using the basic concept of generalized inverse.

IV. SIMULATION RESULT

Consider the following T-S fuzzy model

$$\dot{x}(t) = \sum_{i=1}^2 h_i [A_i x(t) + A_{\tau i} x(t - \tau(t)) + B_i u(t)] \quad (17)$$

The membership functions for rules 1 and 2 are:

$$\begin{aligned} h_1(x_2(t)) &= 1 - \frac{x_2(t)^2}{2.25} \\ h_2(x_2(t)) &= 1 - h_1(x_2(t)) \end{aligned} \quad (18)$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.1125 & -0.0200 \\ 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -0.1125 & -1.5270 \\ 1 & 0 \end{bmatrix} \\ A_{\tau 1} &= \begin{bmatrix} -0.0125 & -0.0050 \\ 0 & 0 \end{bmatrix}, A_{\tau 2} = \begin{bmatrix} -0.0125 & -0.2300 \\ 0 & 0 \end{bmatrix} \\ B_1 &= B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix} \end{aligned}$$

Assume that time varying fault $f(t)$ is created as

$$\begin{aligned} f(t) &= 0, & 0 \leq t < 3 \\ &= 0.7 + 0.15 \sin(t), & 3 \leq t \leq 10 \end{aligned} \quad (19)$$

By using theorem 2 for $\tau(t) = 0.2 + 0.1 \sin(t)$ ($\tau_m = 0.1$; $\tau_M = 0.3$, $\beta = 0.1$) with the choice of $\lambda = 5$, we can obtain the following feasible solution:

$$\begin{aligned} P &= 10^3 \times \begin{bmatrix} 1.5409 & -1.5409 \\ -1.5409 & 3.3868 \end{bmatrix} \\ F &= 10^3 \times 1.8459 \\ L_1 &= \begin{bmatrix} 1.7056 \\ 2.1581 \end{bmatrix}, L_2 = \begin{bmatrix} 0.1081 \\ 2.1555 \end{bmatrix} \end{aligned}$$

The simulation result is shown in figure 1.

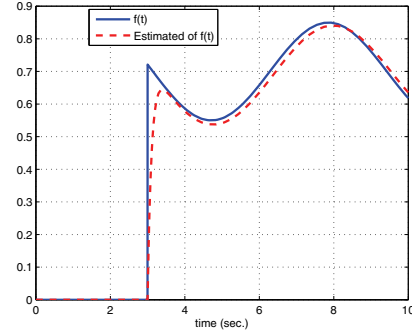


Fig. 1. Fault $f(t)$ and its estimated

V. CONCLUSION

The problem of delay dependent AFDO design to a class of T-S fuzzy systems with interval time varying delay has been investigated. Based on improved Lyapunov function, a delay dependent conditions for the existence of the fault estimator are given in terms of linear matrix inequalities. This result overcomes the drawbacks of delay independent results. An illustrative example has been presented to demonstrate the potential of the method.

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