

Higher Order Sliding Fuzzy Type-2 Interval Control for SISO Uncertain Nonlinear Systems

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Abstract—A higher order sliding fuzzy type-2 controller scheme for nonlinear uncertain perturbed systems is proposed in the paper. To overcome the constraint on the knowledge of the system model, local models related to some operating points were used to synthesize a nominal fuzzy type-2 global model. The controller uses integral sliding mode concept and contains two parts. The first one leads to achieve finite time stabilization of the higher order input output dynamics without uncertainties. The second part has the object to reject bounded uncertainties throughout the entire response of the system. Two adaptive fuzzy type-2 systems have been introduced to generate the two Super Twisting signals to avoid both the chattering and the constraint on the knowledge of upper bounds of both disturbances and uncertainties. These fuzzy type-2 systems are adjusted on-line by adaptation laws deduced from the stability analysis in Lyapunov sens. The advantages of the method are that its implementation is easy, the convergence time is chosen in advance and the robustness is ensured. A robot arm actuated by a DC motor is used as illustrative example. The obtained results show the good tracking performances and the applicability of the method.

Index Terms—Higher order sliding mode control, fuzzy type-2 sets, T-S fuzzy systems, integral sliding mode control.

I. INTRODUCTION

The sliding mode control (SMC) is a powerful method to control high-order nonlinear dynamic systems operating under uncertainty conditions [1],[2],[3]. The technique consists of two stages. First, a sliding surface, to which the controlled system trajectories must belong, is designed according to some performance criterion. Then, a discontinuous control is designed to force the system state to reach the sliding surface such that a sliding mode occurs on this manifold. When sliding mode is realized, the system exhibits robustness properties with respect to parameter perturbations and external matched disturbances [2]. In spite of claimed robustness properties, high frequency oscillations of the state trajectories around the sliding manifold known as chattering phenomenon [2],[3] are the major obstacles for the implementation of SMC in a wide range of applications. A number of methods have been proposed to reliably prevent chattering: among them, the boundary layer solution [4], [2], [5],[6], tangent hyperbolic [7] and observer-based solution [3]. These methods can eliminate chattering effect but a finite steady state error will exist [8] and requires a compromise between the level of tracking performance and a start-up control [9]. In [8], [9],[10] the modified switching signal (with saturation) is substituted by an adaptive fuzzy system. However, the convergence of the corresponding adaptive algorithm depends directly on the choice of the initial values of the adaptive fuzzy system. The state of the art of

classical sliding mode control with soft computing has been examined in [11].

The "higher order SMC" approach has been recently proposed and developed as a generalization of the classical first order sliding mode theory [12],[13],[14],[15]. The current papers result is based on this latter approach and its main idea can be described as follows. Let $s(x, t)$ ($x \in \mathbb{R}^n$ is the state variable, $t \in \mathbb{R}^+$ the time variable) be the sliding variable and $r \in \mathbb{N}$ the sliding order. The control forces to zero in finite time s and its $(r - 1)$ first higher time derivatives by acting discontinuously on the r th time derivative of s . Keeping the main advantages of standard SMC, the chattering effect is eliminated and higher order precision is provided. In the case of "real" SMC [12], if τ is the sampling time, the error is $o(\tau)$ in the case of standard SMC [16] whereas it is $o(\tau^r)$ in the r th order SMC [12].

In the case of second order SMC ($r = 2$), many works have given solutions. Several second order sliding mode algorithms are proposed in [14], [13] and [12]. Arbitrary-order sliding controllers for single-input single-output systems (SISO) with finite time convergence have been recently proposed in [15],[17],[18],[19]. In [20], the so-called terminal sliding modes control proposes a solution for arbitrary order finite time convergence sliding mode. A major drawback of this approach is that all trajectories have to start from a prescribed sector of state space in order to have finite control, which is restrictive in particular for practical applications. Many results concerning the high order SMC have been proposed for single Input single output in [13], [21] and [22]. However, the common drawback of this approach is the calculation of sliding gains used in the control law. Indeed, they are chosen to satisfy the finite time convergence condition, which required the well-knowledge of the system parameters.

The aim of this paper is to present an arbitrary-order sliding mode controller for uncertain SISO nonlinear systems. The main objective of this new approach is to propose a controller for which the implementation is easy, the convergence time is finite and well-known in advance and the computation of the gains control of the discontinuous part will be simplified. The design uses the integral sliding mode concept [23]. Actually, the problem of the higher order SMC of SISO minimum-phase uncertain systems is formulated in inputoutput terms only (as in [18]) through the differentiation of the sliding variable s , and is equivalent to the finite time stabilization of an r th order inputoutput linear dynamics with bounded uncertainties.

The control strategy presented in the sequel, whose basic idea has been introduced in [19], contains two parts: a part is discontinuous, forces the establishment of a sliding mode on the integral sliding manifold, and ensures the robustness with respect to bounded uncertainties, throughout the entire response of the system. The other part, which is obtained through optimal feedback control over finite time interval with fixed final states [24], is used to stabilize to zero in finite time the r th order inputoutput dynamics without uncertainties. In [22], this part is based on a close-loop control and has two dynamics, at $t \in [0, t_f]$ will be associated a forcing term δ and after $t > t_f$ will be removed. The computation of the control gains will be simplified by introducing two adaptive fuzzy type-2 systems in the second SMC. Their updates are performed using two adaptation laws derived from the study of stability in the sense of Lyapunov. Since the considered system is nonlinear and uncertain, we propose to use Takagi-Sugeno fuzzy system to construct a nominal model of the system. For this, we define some local models around some operating points, which will be used as conclusion of this Takagi-Sugeno system.

The paper is organized as follows. Section II gives an overview of the theory of the type-2 fuzzy systems. The problem formulation is presented in section III. The design procedure of higher SMC based on an integral sliding mode and its stability is discussed in section IV. The proposed sliding mode fuzzy type-2 interval controller is given in section V. Simulation example to demonstrate the performance of the proposed method is provided in section VI. Section VII gives the conclusions of this paper.

II. TYPE-2 FUZZY LOGIC SYSTEMS

The main property that distinguishes fuzzy logic is its capacity of representing and modeling imprecision and uncertainty by giving each point in fuzzy set a value belonging to the interval $[0, 1]$. However, the imprecision in such classical fuzzy system, which is called some times Type-1 fuzzy logic system (T1FLS) is not fully exploited and can deliver a non-satisfactory performance. Practically, three ways of uncertainty can be observed in the fuzzy logic system (FLS): (1) the used words in antecedents and consequents of rules can mean different things to different people; (2) consequents obtained by polling a group of experts will often be different for the same rule and (3) both of training data and measurements used to activate the FLS are noisy [25],[26].

In the last few years, a markable attention has been given to another type of fuzzy system called Type-2 fuzzy logic system (T2FLS). In T2FLS, the uncertainty is represented using a function, which is itself a Type-1 fuzzy number and that is why T2FLS is referred to that fuzzy system. The membership functions of T2 fuzzy sets are three dimensional and include a foot print of uncertainty (FOU) with a new third dimension of Type-2 fuzzy sets. FOU provides additional degrees of freedom that make it possible to directly model and handle uncertainties and consequently, T2FLS has the potential to outperform T1FLS [27],[28] in such cases. Such

advantage is quite important if we know that modeling of linguistic information and decision making represents the main application of FLS.

Figure 1 depicts the structure of a T2FLS [29],[30]; it is quite similar to a T1FLS, the only difference being that the antecedent and/or consequent sets in a T2FLS are type-2, so that each rule output set is a type-2. There are five principal parts in a T2FLS: fuzzifier, rule base, inference engine, type-reducer and defuzzifier.

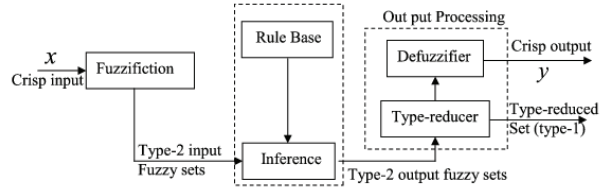


Fig. 1. The structure of T2FLS with its two out puts.

The type-reducer performs a type-reduction operation which is an extended version of T1 defuzzification. Type reduction yields a T1 set from the T2 rule output sets. The resulting T1 set is called type-reduced set. The type-reduced set can then be defuzzified to obtain a crisp output. We call the combination of type reduction and defuzzification output processing. The type reduced set of a T2FLS shows the possible variation in the crisp output of the FLS due to uncertain natures of the antecedents and/or consequents. The fuzzifier maps a crisp input vector with n inputs $x = (x_1, \dots, x_n)^T \in X_1 \times X_2 \times \dots \times X_n = X$ into input fuzzy sets which are T2 fuzzy input sets. However, we will use singleton fuzzification as it is fast to compute and thus suitable for real time applications. The T2 fuzzy rule base consists of a collection of IF-THEN rules in the following form [31]:

$R^i : IF \ x_1 \text{ is } \tilde{F}_1^i \text{ and } \dots \text{ and } x_n \text{ is } \tilde{F}_n^i \text{ THEN } y \text{ is } \tilde{G}^i$
Where $\tilde{F}_j^i$ s are antecedent T2 sets ($j = 1, 2, \dots, n$), $y \in Y$ is the output, $\tilde{G}^i$ s are consequent type-2 sets and $i = 1, 2, \dots, M$, where M is the total number of T2 fuzzy IF-THEN rules in the rule base.

The rule represents a T2 fuzzy relation between the input space $X_1 \times X_2 \times \dots \times X_n$ and the output space Y of the FLS. The fuzzy inference engine is decision making logic, which employs fuzzy rules from the fuzzy rule base to determine the mapping from X to Y . A T2FLS is a mapping $f : \mathbb{R}^n \rightarrow \mathbb{R}$. After fuzzification, fuzzy inference, type reduction and defuzzification, we obtain a crisp output. For an interval T2FLS, this crisp output is the center of type reduced set.

In interval T2FLS with meet under minimum or product t-norm, the firing interval $W^i = [\underline{w}^i, \bar{w}^i]$ of the i^{th} rule is an interval T1 set, which is determined by its left-most and right-most points $\underline{w}^i$ and $\bar{w}^i$ such that:

$$\begin{cases} \underline{w}^i = \left[\mu_{\tilde{F}_1^i}(\underline{x}_1) \times \dots \times \mu_{\tilde{F}_n^i}(\underline{x}_n) \right] \\ \bar{w}^i = \left[\bar{\mu}_{\tilde{F}_1^i}(\underline{x}_1) \times \dots \times \bar{\mu}_{\tilde{F}_n^i}(\underline{x}_n) \right] \end{cases} \quad (1)$$

Type reduction was proposed by Karnik and Mendel [32]. It is in extension of T1 defuzzification method and it is called type-reduction because this operation takes us from the T2 output sets of the FLS to a T1 set. This set may then be defuzzified to obtain a single crisp number. There exist many kinds of type-reduction [29],[33],[34]. In this paper, we will use the center of sets type reduction, which can be expressed in the case of interval T2 fuzzy sets as:

$$Y_{tr} = \int_{y^1 \in [y_l^1, y_r^1]} \cdots \int_{y^M \in [y_l^M, y_r^M]} \int_{w^1 \in [\underline{w}^1, \bar{w}^1]} \frac{\sum_i w^i y^i}{\sum_i w^i} \quad (2)$$

$$Y_{tr} = [y_l, y_r]$$

where Y_{tr} is the interval set determined by two end points y_l and y_r , $y^i \in Y^i = [y_l^i, y_r^i]$, Y^i the centroid of the T2 interval consequent set $\tilde{G}^i$ and $w^i \in W^i = [\underline{w}^i, \bar{w}^i]$ the firing interval.

III. PROBLEM FORMULATION

Consider the n th-order uncertain nonlinear system of the form:

$$\begin{cases} \dot{x} = f(x) + g(x) \cdot u(t) + d \\ y = x \end{cases} \quad (3)$$

where $f(x)$ and $g(x)$ are two nonlinear continuous and uncertain bounded functions, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the input and output of the system, respectively, d is an external unknown bounded disturbance. $x = [x, \dot{x}, \dots, x^{n-1}]^T \in \mathbb{R}^n$ is the measurable state vector and $s(x, t)$ is the sliding variable. We assume that the two functions $f(x)$ and $g(x)$ can be written respectively as the sum of a nominal function and an unknown bounded uncertainty:

$$\begin{cases} f(x) = f_0(x) + \Delta f(x); |\Delta f(x)| < \Delta_f \\ g(x) = g_0(x) + \Delta g(x); |\Delta g(x)| < \Delta_g \end{cases} \quad (4)$$

where Δ_f and Δ_g are two positive constants. Replacing (4) into (3), we obtain:

$$\begin{cases} \dot{x} = f_0(x) + g_0(x) \cdot u(t) + D \\ y = x \end{cases} \quad (5)$$

where

$$D = \Delta f(x) + \Delta g(x)u + d \quad (6)$$

The nominal model of the system can be obtained by identification or by approximation using a fuzzy system [35]. We consider the second case, because it allows us to exploit linguistic information from the expert human. The approximation can be made either using a Takagi-Sugeno classical fuzzy system or an adaptive one. This later can certainly get good results but requires a long computing time where we consider more than two inputs. Moreover, the linearisation techniques permit to transform the dynamics of the system into a local models around certain operating points. Thus, to calculate the nominal model, Takagi and Sugeno proposed in [36] a T1

fuzzy dynamic model formed by rules; where the conclusion part is a local model in the form :

$$x^{(n)} = A_i x + B_i u \quad (7)$$

A_i and B_i are two matrix with appropriate dimension ($i = 1, 2, \dots, r$). Then the corresponding IF-THEN fuzzy rule can be written as:

$$\text{IF } x \text{ is } H_1^i \text{ and } \dot{x} \text{ is } H_2^i \text{ and } \dots \text{ and } x^{(n-1)} \text{ is } H_n^i \text{ THEN } x^{(n)} = A_i x + B_i u \quad (8)$$

where H_j^i , ($j = 1, 2, \dots, n$) is the j^{th} fuzzy set of the i^{th} rule. However, as known in literature, the knowledge used to construct such rules are uncertain. This uncertainty leads to obtain rules whose premises or consequences uncertain, which can create uncertainty in the membership function. Since a T2 fuzzy system can take in account this uncertainties, we propose to extend the dynamic model given by (8), by introducing the concept of T2 fuzzy logic. Then, the nominal T2 fuzzy model related to the i^{th} rule will be given as follows [37]:

IF x is $\tilde{H}_1^i$ and $\dot{x}$ is $\tilde{H}_2^i$ and..... and $x^{(n-1)}$ is $\tilde{H}_n^i$ THEN

$$x^{(n)} = A_i x + B_i u \quad (9)$$

Where $\tilde{H}_j^i$, ($j = 1, 2, \dots, n$) is the j^{th} interval T2 fuzzy set of the i^{th} rule. For a given pair (x, u) , the fuzzy model resulting of the system appears as a weighted average of the local models used. If we use the product as an interference engine, the method of center set for the reduction type and center of gravity for defuzzification, the output fuzzy system will be giving by:

$$x^{(n)} = \frac{\sum_{i=1}^r w^i [A_i x + B_i u]}{\sum_{i=1}^r w^i} = \frac{\sum_{i=1}^r w^i [A_i x]}{\sum_{i=1}^r w^i} + \frac{\sum_{i=1}^r w^i [B_i u]}{\sum_{i=1}^r w^i} \quad (10)$$

If we denote by $f_0(x) = \left[\frac{\sum_{i=1}^r w^i A_i x}{\sum_{i=1}^r w^i} \right]$ and

$g_0(x) = \left[\frac{\sum_{i=1}^r w^i B_i}{\sum_{i=1}^r w^i} \right]$, the fuzzy nominal model will be giving by:

$$x^{(n)} = f_0(x) + g_0(x)u \quad (11)$$

where the variable w^i is the firing interval, which is determined by its most left and right points as given by (1). The reduction type algorithm given by karnik and Mendel [34] will be detailed in the third section .

The control objective is to fulfill the constraint of the sliding variable $s(x, t) = 0$ in finite time and to keep it exactly by discontinuous feedback control, and to ensure that the system out-put y follows a reference trajectory y_d . The set: $s^r = \{x | s(x, t) = \dot{s}(x, t) = \dots = s^{r-1}(x, t) = 0\}$ called r th order sliding set, is non-empty and is locally an integral set in the Filippov sense [38], the motion on s^r is called r th order sliding mode with respect to the sliding variable s . The r th

order SMC approach allows the finite time stabilization to zero of the sliding variable s and its $r - 1$ first time derivatives by defining a suitable discontinuous control function. s^r in function of the systems variables and the control variable u is given by [17]:

$$s^r = \phi(.) + \gamma(.)u \quad (12)$$

r means also the number of times to derive the sliding surface to appear explicitly the control variable.

In [22], [12], authors assume that: the functions $\phi(.)$ and $\gamma(.)$ are bounded uncertain functions, and, without loss of generality, let also the sign of the control gain $\gamma(.)$ be constant and strictly positive. Thus, there exist $K_m \in \mathbb{R}^{+*}$, $K_M \in \mathbb{R}^{+*}$, $C_0 \in \mathbb{R}^+$ such that $0 < K_m < \gamma < K_M$, $|\phi| \leq C_0$.

These assumption implies that this results are considered local. In the section V, we propose a method to avoid these restriction which makes the results global.

The r th order SMC with respect to the sliding variable s is equivalent to the finite time stabilization of [22]:

$$\begin{cases} \dot{z}_i = z_{i+1} \\ \dot{z}_r = \phi(.) + \gamma(.)u \end{cases} \quad (13)$$

with $1 \leq i \leq r - 1$ and $z = [z_1 \ z_2 \ \dots \ z_r]^T := [s \dot{s} \dots s^{(r-1)}]^T$.

IV. CONTROL DESIGN

The control law is composed of two parts. The first one, named ideal control [23], is continuous and stabilizes in finite time (13) at the origin when there are no uncertainties. In fact, this control part is used in order to generate trajectories that the system is forced to track. The second part, named integral sliding mode control, provides the complete compensation of uncertainty for $t \geq 0$ and ensures that control objectives are reached.

A. Continuous Control Part Design

Consider system (13), according to some assumptions it can be trivially rewritten as [22]:

$$\begin{cases} \dot{z}_i = z_{i+1} \\ \dot{z}_r = \underbrace{\phi(.) + [\gamma(.) - 1]u}_{\text{control part}} + u \end{cases} \quad (14)$$

for $1 \leq i \leq r - 1$, it yields

$$\begin{cases} \dot{z}_i = z_{i+1} \\ \dot{z}_r = \beta(.) + u \end{cases} \quad (15)$$

Let us define $u = u_0 + u_{dis}$, with u_0 being the ideal control, and u_{dis} being the integral sliding mode control. Consider now the particular case $\beta(.) = 0$. Then, as no control part u_{dis} is necessary in order to compensate the uncertainties, the control law u reads as $u = u_0$. One gets

$$\dot{z} = Az + Bu_0 \quad (16)$$

where A and B are defined by

$$A = \begin{bmatrix} 0 & 1 & \dots & 0 & \dots \\ \vdots & \ddots & \ddots & \ddots & \\ 0 & \ddots & \ddots & \dots & 1 \\ 0 & \ddots & \ddots & \ddots & 0 \end{bmatrix}_{r \times r}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r \times 1}.$$

The control objective is to drive the state of (16) to $z = 0$ at the fixed final time $t = t_f < +\infty$, with $z(0)$ being a bounded initial state vector. This objective can be done through the use of optimal control laws. The control law u_0 ensures the minimization of the following criterion:

$$J = \frac{1}{2} \int_0^{t_f} z^T Q z + u_0^2 dt \quad (17)$$

with $t_f < +\infty$, $|z(0)| < +\infty$ and Q a symmetric positive definite matrix under the fixed final state constraint $z(t_f) = 0$.

Theorem 1: [24] Consider linear system (16) with (A, B) reachable. A control law u_0 minimizing the criteria (17) and driving system (16) to $z(t) = 0$ at $t = t_f$ for an initial bounded condition $z(0)$ is given by (with $0 \leq t \leq t_f < +\infty$)

$$u_0 = -B^T M z(t) + B^T \delta(t) \quad (18)$$

with $\delta(t)$ and M are defined by

$$\begin{cases} \dot{\delta} = -(A^T - MBB^T)\delta \\ 0 = MA + A^T M - MBB^T M + Q \end{cases} \quad (19)$$

Initial condition $\delta(0)$ of $\delta(t)$ is selected in order to satisfy the terminal condition $z(t_f) = 0$.

The control law u_0 is defined in order to drive in finite time the system (16) to $z(t) = 0$ at $t = t_f$, then, the state variables of (16) converge exactly to the origin at $t = t_f$. The control law u_0 will maintain the required equilibrium state $z = 0$ even after the terminal time, i.e. $t > t_f$, provided that the forcing term $\delta(t)$ is removed at the terminal time, i.e. $u_0 = -B^T M z$ for $t > t_f$ [24]. Then, in order to reach in a finite time t_f the origin $z = 0$, and to maintain the system (16) at this point for $t > t_f$, a solution is [24]:

$$u_0 = \begin{cases} -B^T M z(t) + B^T \delta(t) & \text{for } 0 \leq t \leq t_f \\ -B^T M z(t) & \text{for } t > t_f \end{cases} \quad (20)$$

B. Integral Sliding Manifold

The basic idea consists in determining a sliding manifold such that the state trajectories start on this manifold at the initial time $t = 0$, which induces a sliding mode without reaching phase [2]. One has (for $1 \leq i \leq r - 1$)

$$\begin{cases} \dot{z}_i = z_{i+1} \\ \dot{z}_r = \beta(.) + u_0 + u_{dis} \end{cases} \quad (21)$$

u_{dis} is a discontinuous function designed in order to exactly reject the perturbation $\beta(.)$ throughout the entire response of the system. In order to reach this objective, the integral sliding mode control is used [23],[3]. Let $\sigma \in \mathbb{R}$ the sliding manifold defined as

$$\sigma = \sigma_0 + \zeta \quad (22)$$

where σ_0 may be designed as the linear combination of the state variables of (21). The term $\zeta \in \mathbb{R}$ induces the integral term and will be determined below. In order to determine the motion equations on the sliding manifold, the equivalent control method is used [23]. The time derivative of σ reads as

$$\dot{\sigma} = \frac{\partial \sigma_0}{\partial z} (Az + Bu_0) + \frac{\partial \sigma_0}{\partial z_r} (u_{dis} + \beta(.)) + \dot{\zeta} \quad (23)$$

A sufficient condition ensuring $\dot{\sigma} = 0$ for $t \geq 0$ is that the equivalent control u_{eq} of u_{dis} reads as

$$u_{eq} = -\beta(\cdot) \quad (24)$$

and $\dot{\zeta} = -\frac{\partial \sigma_0}{\partial z}(Az + Bu_0)$, $\zeta(0) = -\sigma_0(z(0))$, where $\zeta(0)$ is determined based on the requirement $\sigma(0) = 0$.

In this present work, to simplify the implementation of the method we consider the particle case

$$\sigma_0 = z_r \longrightarrow \dot{\sigma} = u + \beta(\cdot) + \dot{\zeta} \quad (25)$$

In this case, condition (24) holds if

$$\dot{\zeta} = -u_0, \zeta(0) = -z_r(0) \quad (26)$$

C. Design of the Discontinuous Control u_{dis}

The control law u_{dis} is designed in order to ensure that the sliding motion on the sliding manifold is guaranteed for $t \geq 0$ in spite of uncertainties and disturbances. Due to its relative simplicity of implementation and their robustness, Super-Twisting algorithm is among the 2-sliding algorithms which are widely used, and reads [12]:

$$u_{dis} = -u_1 - u_2 \quad (27)$$

where $\dot{u}_1 = -\lambda_1 \text{sign}(\sigma) \longrightarrow u_1 = \int_0^t \lambda_1 \text{sign}(\sigma) dt$ and $u_2 = \lambda_2 |\sigma|^{\frac{1}{2}} \text{sign}(\sigma)$. λ_1 and λ_2 are positive constants tuned such that the η -attractiveness condition is satisfied:

$$\sigma \dot{\sigma} \leq -\eta |\sigma|, \eta > 0 \quad (28)$$

The aim of our study is to determine a control law by employing 2-order sliding modes, so that the system output y follows a reference trajectory y_r and the tracking error $e = y - y_r$ converges to zero in the presence of uncertainties and disturbances. For this, we consider the following sliding surface [5]:

$$\begin{aligned} \sigma_0 &= \left(\frac{\partial}{\partial t} + \lambda \right)^{(n-1)} e \\ &= \sum_{k=0}^{n-1} \frac{(n-1)!}{k!(n-k-1)!} \left(\frac{\partial}{\partial t} \right)^{(n-k-1)} \lambda^k e \end{aligned} \quad (29)$$

$\lambda > 0$, the derivative of σ_0 can be expressed as:

$$\dot{\sigma}_0 = \delta_{\sigma_0} - y_r^{(n)} + f_0(\underline{x}) + g_0(\underline{x})u + D \quad (30)$$

which has the same form as given by equation (12), with $r = 1$, $\phi(\cdot) = \delta_{\sigma_0} - y_r^{(n)} + f_0(\underline{x}) + D$, $\gamma(\cdot) = g_0(\underline{x})$ and $\delta_{\sigma_0} = \sum_{k=1}^n \frac{(n-1)!}{k!(n-k-1)!} \left(\frac{\partial}{\partial t} \right)^{(n-k-1)} \lambda^k \dot{e}$. Since the system is relative degree 1, the second derivative of σ_0 is given by:

$$\ddot{\sigma}_0 = \varphi + \varrho \dot{u} \quad (31)$$

with $\varphi = \dot{\delta}_{\sigma_0} - y_r^{(n+1)} + \dot{f}_0(\underline{x}) + \dot{g}_0(\underline{x})u$ and $\varrho = g_0(\underline{x})$. In order to satisfy the constraint (28), Levant considers the system (31) without D and its derivative and gave the following sufficient conditions [15]:

$$\begin{cases} \Delta > 0, |\varphi| \leq \Delta, 0 < \Gamma_m \leq \varrho \leq \Gamma_M \\ \lambda_1 > \frac{\Delta}{\Gamma_m}, \lambda_2 \geq \frac{4\Delta}{\Gamma_m^2} \frac{\Gamma_M(\lambda_1 + \Delta)}{\Gamma_m(\lambda_1 - \Delta)} \end{cases} \quad (32)$$

In steady state, we will have $\sigma = 0$, choosing these sufficient conditions allow us easily to satisfy the condition (28) at startup. However, for optimal choice of λ_1 and λ_2 in the approaching phase, the knowledge of the D and $\dot{D}$'s upper bounds are required. Nevertheless even if the problem of determining these upper bounds is resolved, the presence of the function $\text{sign}(s(\sigma))$ causes a chattering phenomenon [37],[39],[40]. Even if its order is less important than in the classical SMC, it's still residual. One way to eliminate the chattering phenomenon consists to define a transition band near the sliding surface. But in order to keep the same tracking performances, the introduction of this band induced an important initial solicitation. Thus, there is a tradeoff between the level of performances tracking (small time response and control without chattering) and a starting suitable control. The objective of the proposed approach in the next section, is to use the same control structure with smooth control signal without reducing the tracking performances.

V. SLIDING MODE TYPE-2 FUZZY CONTROLLER

In this section, we will replace the two switching signals u_1 and u_2 respectively, by two control signals from two adaptive T2 fuzzy systems in order to smooth the control signal. The adaptation laws of adjustable parameters of the fuzzy systems are deduced from the stability study of the closed-loop process in the sense of Lyapunov. This adaptation in addition to smooth the control signal will enable us to better anticipate disturbances by providing us in each instant the optimal value (λ_1^* and λ_2^*) of λ_1 and λ_2 , which are hard to compute [41].

A. Proposed Type-2 Fuzzy Logic System

Both adaptive systems used to approximate the control gains, have the same structure as given by (2). The antecedent T2 Membership functions $\mu_{F_i^j}$ will be fixed as given in figure 3. The consequent sets which are the adjustable parameters will be fuzzy centroids T1. The output fuzzy systems are given by:

$$\begin{aligned} Y_{tr} &= [y_l, y_r] \\ &= \int_{\theta^1} \dots \int_{\theta^M} \int_{w^1} \dots \int_{w^M} 1 / \frac{\sum_i w^i \theta^i}{\sum_i w^i} \end{aligned} \quad (33)$$

In the case of T2 fuzzy systems, the universal approximation property is retained with an additional phase of reduction type. Since each set on the right-hand side of (33) is an interval T1 set, hence Y_{tr} is an interval T1 set. So, to find the type-reduced set Y_{tr} , we just need to compute the two end points of these interval. The maximum value of y is y_r and its minimum value is $y_l, \forall y \in Y_{tr}$, y can be represented as:

$$y = \frac{\sum_i w^i \theta^i}{\sum_i w^i} \quad (34)$$

The y_r points are associated solely with y_r^i , in same way y_l is associated solely with y_l^i . In the center of sets, Karnik and Mendel [32] have shown that the two end points of Y_{tr} , y_r and y_l depend only on a mixture $\underline{w}^i$ or $\overline{w}^i$ of values, since $w^i \in W^i = [\underline{w}^i, \overline{w}^i]$. In this case, each y_r and y_l can be represented as a vector of fuzzy basis functions (FBFs) expansion, that is:

$$y_l = \frac{\sum_{i=1}^M w_l^i \theta_l^i}{\sum_{i=1}^M w_l^i} = \sum_{i=1}^M \theta_l^i \xi_l^i \quad (35)$$

where w_l^i denotes the firing strength membership grade (either $\overline{w}^i$ or $\underline{w}^i$) of the i^{th} rule contributing to the left-most point y_l and $\xi_l^i = \frac{w_l^i}{\sum_{i=1}^M w_l^i}$ are the components of the first FBF vector ξ_l of y . Similarly:

$$y_r = \frac{\sum_{i=1}^M w_r^i \theta_r^i}{\sum_{i=1}^M w_r^i} = \sum_{i=1}^M \theta_r^i \xi_r^i \quad (36)$$

where w_r^i denotes the firing strength membership grade (either $\overline{w}^i$ or $\underline{w}^i$) contributing to the right-most point y_r and $\xi_r^i = \frac{w_r^i}{\sum_{i=1}^M w_r^i}$ are the components of the first FBF vector ξ_r of y .

Note that whereas a T1FLS is characterized by a single FBF expansion [42], an interval T2FLS is characterized by two FBF expansions. This difference is the consequence of the reduction type operation.

To obtain a crisp outputs from the T2FLS y , we must defuzzify the type-reduced set Y_{tr} . Since these type reduced set is interval set, therefore, the defuzzified output of y will be the average of y_l and y_r , that is:

$$y = \frac{y_l + y_r}{2} \quad (37)$$

Replacing (35) and (36) into (37), we obtain:

$$y = \frac{\theta_l^T \xi_l + \theta_r^T \xi_r}{2} = \underline{\theta}^T \left[\frac{\xi_l + \xi_r}{2} \right] = \underline{\theta}^T \underline{\xi} \quad (38)$$

Where $\underline{\xi} = \frac{\xi_l + \xi_r}{2}$ is the average FBF of y . In order to compute y_l and y_r Karnik and Mendel have developed a simple iterative algorithm. This algorithm is given as follows:

- 1 We assume that the parameters θ^i are arranged in ascending order, that is, $\theta^1 < \theta^2 < \dots < \theta^M$.
- 2 Compute y_r in (37) by initially setting $w_r^i = \frac{\overline{w}^i + \underline{w}^i}{2}$, for $i = 1, \dots, M$ and let $\dot{y}_r \equiv y_r$.
- 3 Find k ($1 \leq k \leq M-1$) such that $y_r^k \leq \dot{y}_r \leq y_r^{k+1}$.
- 4 Compute $y_r = \frac{\sum_{i=1}^M w^i \theta^i}{\sum_{i=1}^M w^i}$ with $w_r^i = \begin{cases} \overline{w}^i, & i \leq k \\ \underline{w}^i, & i > k \end{cases}$, and let $\dot{y}_r \equiv y_r$.
- 5 If $\dot{y}_r \neq y_r$, then go to setup 6. If $\dot{y}_r = y_r$, stop and put $\dot{y}_r \equiv y_r$.

6 Set $\dot{y}_r = \dot{y}_r$ and return to setup 3.

The procedure is similar to compute y_l , substituting y_r by y_l in setup 3. The calculation is done with

$$w_l^i = \begin{cases} \overline{w}^i, & i \leq k \\ \underline{w}^i, & i > k \end{cases}$$

Following the same reasoning the nominal model of the system given by equation (11) is calculated, and u_1 , u_2 are estimated.

B. Proposed Control Synthesis

In this subsection, we study the stability and the robustness of the closed loop system using two adaptive fuzzy systems to replace the functions $u_1 = \int_0^t \lambda_1 \text{sign}(\sigma) dt$ and $u_2 = \lambda_2 |\sigma|^{\frac{1}{2}} \text{sign}(\sigma)$. The terms of the high frequency component control will be given by: $\hat{u}_1(\sigma) = \underline{\theta}_1^T \underline{\xi} t_c$ and $\hat{u}_2(\sigma) = |\sigma|^{\frac{1}{2}} \underline{\theta}_2^T \underline{\xi}$. $\hat{u}_1(\sigma)$ and $\hat{u}_2(\sigma)$ are two fuzzy systems defined by (38), whose only input is the value of the sliding variable σ , and the outputs give the optimal values satisfying $\hat{u}_1^*(\sigma) = \lambda_1^* t_c$ and $\hat{u}_2^*(\sigma) = \lambda_2^* |\sigma|^{\frac{1}{2}}$ [37], t_c is the convergence time. In [37],[39] it is only estimated, in the present paper with the concept of integral sliding mode it's value is exactly determined in advance and chosen to be $t_c = t_f$.

So, the proposed control law is given by:

$$u = u_0 + u_{eq} - \hat{u}_1(\sigma) - \hat{u}_2(\sigma) \quad (39)$$

We consider the Lyapunov equation:

$$V = \frac{\sigma^2}{2} + \frac{1}{2\gamma_1} \tilde{\theta}_1^T \tilde{\theta}_1 + \frac{1}{2\gamma_2} \tilde{\theta}_2^T \tilde{\theta}_2 \quad (40)$$

with: $\tilde{\theta} = \theta - \theta^*$, θ^* is the optimal value of θ , γ_1 and γ_2 are two positive training constants.

The time derivative of V is :

$$\dot{V} = \dot{\sigma} + \frac{1}{\gamma_1} \tilde{\theta}_1^T \dot{\tilde{\theta}}_1 + \frac{1}{\gamma_2} \tilde{\theta}_2^T \dot{\tilde{\theta}}_2 \quad (41)$$

By replacing the proposed control into the derivative of the sliding variable, we obtain

$$\begin{aligned} \dot{\sigma} &= -\hat{u}_1(\sigma) - \hat{u}_2(\sigma) + D \\ &= -\hat{u}_1(\sigma) - \hat{u}_1^*(\sigma) + \hat{u}_1^*(\sigma) - \hat{u}_2(\sigma) - \hat{u}_2^*(\sigma) + \\ &\quad \hat{u}_2^*(\sigma) + D \end{aligned} \quad (42)$$

where $\hat{u}_2^*(\sigma) = |\sigma|^{\frac{1}{2}} \underline{\theta}_2^{*T} \underline{\xi}(\sigma)$ and $\hat{u}_1^*(\sigma) = \underline{\theta}_1^{*T} \underline{\xi}(\sigma) t_c$, then the derivative on the sliding surface can be given by:

$$\begin{aligned} \dot{\sigma} &= -(\underline{\theta}_1 - \underline{\theta}_1^*)^T \underline{\xi}(\sigma) t_c - (\underline{\theta}_2 - \underline{\theta}_2^*)^T |\sigma|^{\frac{1}{2}} \underline{\xi}(\sigma) \\ &\quad - \hat{u}_2^*(\sigma) - \hat{u}_1^*(\sigma) + D \end{aligned} \quad (43)$$

Substituting (43) into (41), we obtain:

$$\begin{aligned} \dot{V} &= -\sigma (\hat{u}_2^*(\sigma) + \hat{u}_1^*(\sigma)) + \frac{1}{\gamma_1} \tilde{\theta}_1^T (\dot{\tilde{\theta}}_1 - \gamma_1 \sigma \underline{\xi}(\sigma) t_c) \\ &\quad + \frac{1}{\gamma_2} \tilde{\theta}_2^T (\dot{\tilde{\theta}}_2 - \gamma_2 \sigma |\sigma|^{\frac{1}{2}} \underline{\xi}(\sigma)) + D \sigma \end{aligned} \quad (44)$$

If we choose the following adaptation laws:

$$\dot{\tilde{\theta}}_1 = \gamma_1 \sigma \underline{\xi}(\sigma) t_c \quad (45)$$

$$\dot{\tilde{\theta}}_2 = \gamma_2 \sigma |\sigma|^{\frac{1}{2}} \underline{\xi}(\sigma) \quad (46)$$

we obtain

$$\dot{V} = |\sigma|D - \left(\lambda_1^* t_c + \lambda_2^* |\sigma|^{\left(\frac{1}{2}\right)} \right) |\sigma| \leq 0 \quad (47)$$

Thus, the proposed control law guaranties both robustness and stability of the closed loop system. Moreover its simplicity allows an implementation for a real-time control.

VI. ILLUSTRATIVE EXAMPLE

In this section, the proposed control is used to control an arm robot with single joint, of mass m and languor l . This arm is actuated by a DC motor as shown in figure 2.

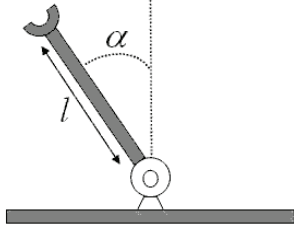


Fig. 2. Arm robot actuated by a DC motor

α means the angle rotation of the arm relative to the vertical. The dynamic of the robot is given by the 3rd order equation:

$$\alpha^{(3)} = f_0(\alpha, \dot{\alpha}) + g_0(\alpha, \dot{\alpha})u + D \quad (48)$$

$$\begin{cases} f_0(\alpha, \dot{\alpha}) = -\frac{R}{L}\ddot{\theta} - \left(\frac{K_b N^2 K_t}{m l^2 L} + \frac{g}{l} \cos \alpha \right) \dot{\alpha} - \frac{Rg}{lL} \sin \alpha \\ g_0(\alpha, \dot{\alpha}) = \frac{K_t N}{m l^2 L} \end{cases}$$

where $\alpha^{(3)}$, $\ddot{\alpha}$ and $\dot{\alpha}$ are the time derivatives of the angle α . g , L , R , N , K_b and K_t are respectively the gravity and the motor parameters whose signification is given in Table (1). $D = d + \Delta f + \Delta g$ means the sum of unknown external disturbances and model uncertainties.

TABLE I
MODEL PARAMETERS

Parameter	Notation	Value
Arm mass	m	2 Kg
Gravity	g	9.81 m/s
Arm languor	l	0.5 m
Resistance	R	1.5 Ω
Inductance	L	0.05 H
Constant EFM	K_b	0.2
Constant torque motor	K_t	0.3
Reduction ratio	N	60

As in [43],[37] the nominal fuzzy model is obtained using 2 rules describing the system dynamics around the operating points 0 and $\pm \frac{\pi}{2}$.

Rule1: IF $x_1(t)$ is close to 0

THEN

$$\begin{cases} \dot{x}(t) = A_1 x(t) + B_1 u(t) \\ y(t) = C x(t) \end{cases} \quad (49)$$

Rule2: IF $x_1(t)$ is close to $\pm \pi/2$

THEN

$$\begin{cases} \dot{x}(t) = A_2 x(t) + B_2 u(t) \\ y(t) = C x(t) \end{cases} \quad (50)$$

where:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{Rg}{lL} & -\frac{g}{l} - \frac{K_b N^2 K_t}{m l^2 L} & \frac{R}{L} \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{K_t N}{m l^2 L} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{Rg}{lL} & -\frac{K_b N^2 K_t}{m l^2 L} & \frac{R}{L} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{K_t N}{m l^2 L} \end{bmatrix}$$

$C = [1 \ 1 \ 0]^T$ and $x(t) = [\alpha \ \dot{\alpha} \ \ddot{\alpha}]^T$. The control objective is to control the state $x_1(t)$ of the system to track the reference trajectory $y_r(t) = \sin(2t)$. Suppose that structural uncertainty, which represent the variation in the mass in this example, is in the form $dm = 0.02m(\sin(2t) + \sin(3t))$ while the external perturbation is assumed to be $d = \sin(3t)$. According to the sliding surface given by Slotine [5] and since the system is of order ($n = 3$), the sliding surface is given by $s = \ddot{e} + 2\lambda\dot{e} + \lambda^2 e$. e is the tracking error position ($e = x_1 - y_r$), $\dot{e}$ is the tracking error velocity ($\dot{e} = x_2 - \dot{y}_r$). To construct the nominal model as given by (10), we define three T2 fuzzy interval sets depending on the variable $x_1(t)$. Similarly to generate the two adaptive fuzzy systems allows us to approximate u_1 and u_2 , we consider five interval fuzzy T2 sets: Negative, Small Negative, Zero, Small Positive and Positive, according to the variable s . In sliding regime, the transition between the approaching phase and the sliding phase occurs when the value of this one is zero. Therefore, in [9], it is showed that more the membership function corresponding to the fuzzy set zero is small, better are the tracking performances. The membership functions involved in building the model are given by the following equations:

$$\mu_{\tilde{H}_1^i} = 1 / (1 + \exp(1.8(x_1(t) + (0.61 + c_1)) / (\frac{\pi}{16})))$$

$$\mu_{\tilde{H}_2^i} = \exp\left(-\left(x_1(t) / \left(\frac{\pi}{4.5 + c_2}\right)\right)^2\right)$$

$$\mu_{\tilde{H}_3^i} = 1 / (1 + \exp(-1.8(x_1(t) + (0.61 + c_1)) / (\frac{\pi}{16})))$$

where c_1 and c_2 designate an area of uncertainty, in the case of our application $c_1 \in [0, 0.2]$ and $c_2 \in [0, 1]$. Similarly the antecedent sets of the two adaptive systems are obtained by adding an area of uncertainty. Figure 3 (a and b) shows the shape fuzzy sets of the nominal model and the adaptive systems.

The time derivative of s reads as $\dot{s} = \phi(\cdot) + \gamma(\cdot)u$ with $\phi(\cdot) = -\dot{y}_r^{(3)} + f_0(x) + 2\lambda\ddot{e} + \lambda^2 e$ and $\gamma(\cdot) = g_0(x)$. Let $z = s$, $A = [0]$ and $B = [1]$. Then as shown in section IV, the 2-order SMC is equivalent to the finite time stabilization of

$$\dot{z} = Az + Bu + [\phi(\cdot) + (\gamma(\cdot) - 1)u] \quad (51)$$

Let $t_f = 0.5s$. The state is initialized at $x(0) = [1 \ 0 \ 0]$ (the others constant are: $\lambda = 20$, $\gamma_1 = 80$ and $\gamma_2 = 40$), which implies that $z(0) = -320$. As mentioned in [24], the initial condition of δ are computed in order that the sliding variable

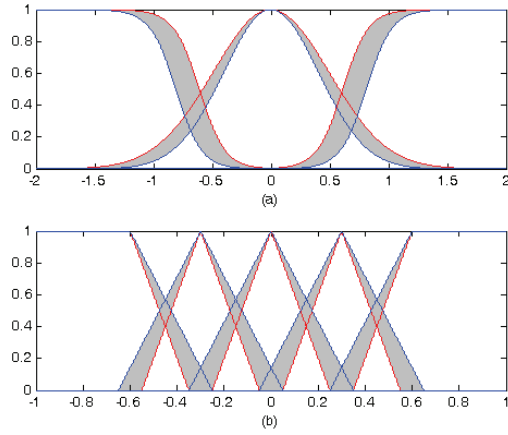


Fig. 3. (a) Interval type-2 antecedent membership functions of $x_1(t)$ defined for the interval nominal model; (b) Interval type-2 antecedent membership functions of s defined for the interval adaptive fuzzy systems.

s equal 0 at exactly $t = t_f = 0.5s$.

The first step consists in solving Riccati equation (second line of (19) for a matrix Q which is symmetric positive definite. The solution of this equation, M , is a symmetric positive definite matrix. Q is stated as $Q = [10]$, then one gets $M = 3.1623$. The second step consists in determining $\delta(0)$ such that $z(t_f) = 0$, knowing that only initial bounded condition $z(0)$ is known. From first line of (19), one gets

$$\delta(t) = e^{A_m t} \delta(0) \quad (52)$$

with $A_m = -[A^T - MBB^T]$. Then, by supposing that $t \leq t_f$, and from (16) and (20), one gets

$$\dot{z} = -A_m^T z + BB^T e^{A_m t} \delta(0) \quad (53)$$

By multiplying both sides of previous equation by $e^{A_m^T t}$, one gets

$$e^{A_m^T t} \dot{z} + e^{A_m^T t} A_m^T z = e^{A_m^T t} BB^T e^{A_m t} \delta(0)$$

$$\Rightarrow [[e^{A_m^T t}]^T z(t)] = e^{A_m^T t} BB^T e^{A_m t} \delta(0)$$

By integrating this last equation from $t = 0$ to $t = t_f$, with $z(t_f) = 0$, one gets

$$z(0) = \left[- \int_0^{t_f} e^{A_m^T t} BB^T e^{A_m t} dt \right] \delta(0) \quad (54)$$

$H = [\int_0^{t_f} e^{A_m^T t} BB^T e^{A_m t} dt]$ is the partial reachability gramian. Then, the initial condition $\delta(0)$ ensuring that $z(t_f) = 0$ can be derived from $\delta(0) = -H^{-1}z(0)$. H is evaluated using an algorithm from [44], its Matlab code being available in [45] as grampar function. We get $\delta(0) = 89.4549$. Figure 4 and figure 5(a) display the convergence of s , $\dot{s}$ and the quadratic error e to zero in exactly $0.5s$. Figure 5(b) displays the control signal which is smooth and free from chattering and abrupt variations. Figure 6 shows a good tracking of position and velocity trajectories. The state $(x_1(t), x_2(t))$ converge quickly to their references $(y_r, \dot{y}_r)$.

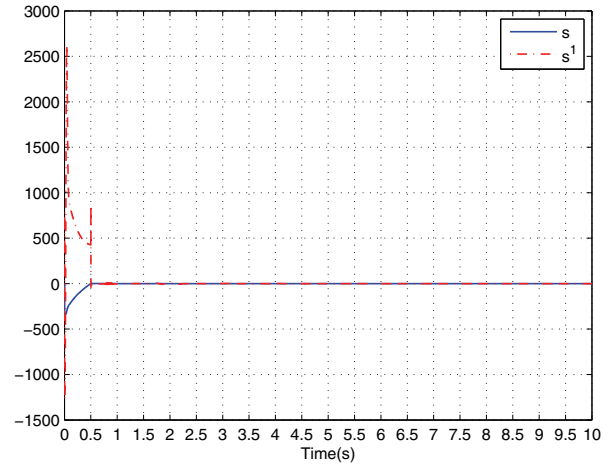


Fig. 4. s and $\dot{s}$ versus time(s)

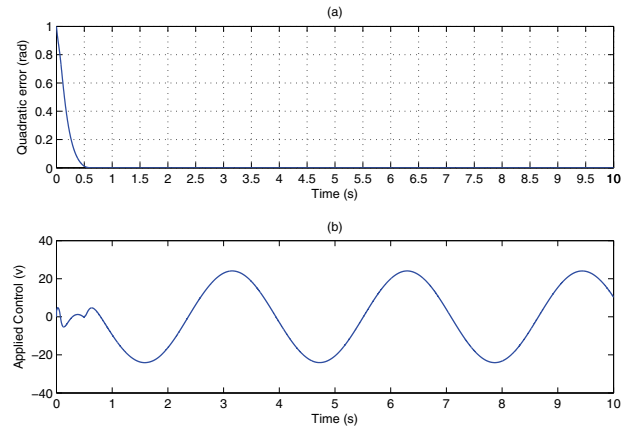


Fig. 5. (a) Trajectory of the quadratic tracking error position; (b) Control input u (v)

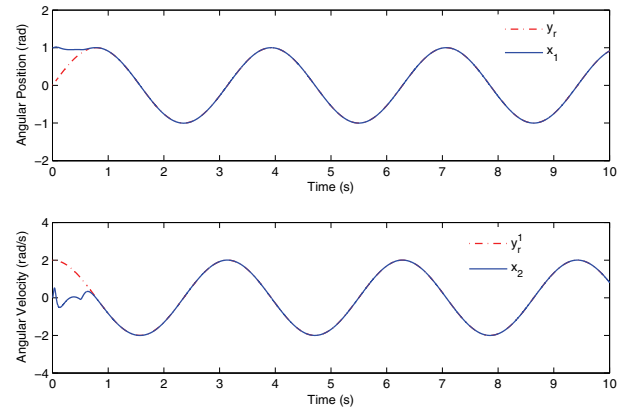


Fig. 6. (a) The state $x_1(t)$ (solid line) and its reference y_r (dashed line); (b) The state $x_2(t)$ (solid line) and its reference $\dot{y}_r$ (dashed line)

VII. CONCLUSION

In this paper, we have presented a type-2 fuzzy sliding modes controller for a perturbed uncertain nonlinear system. The control is generated from a nominal fuzzy type-2 model, who exploits the linear local models of the system. These local models are obtained by linearisation around some operating points. Two adaptive interval type-2 fuzzy systems are introduced to calculate the two switching control terms in order to overcome some restrictive constraints and to eliminate efficiency the chattering phenomenon. The stability and robustness of the closed loop system are proved analytically. The update of adjustable parameters of the two type-2 fuzzy systems are ensured by the adaptation laws derived using Lyapunov theory. The design uses the integral sliding mode concept. The controller is able to steer to zero in finite time the sliding surface and its derivatives; the convergence time can be fixed in advance. A simulation example has been presented to illustrate the effectiveness and the robustness of the proposed approach. Currently we are working on the generalization of this approach to multi-input multi-output system with real-time implementation.

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