

Sliding Adaptive Fuzzy Control for a Class of Time-Delayed Chaotic Systems

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Abstract— In this study, sliding adaptive fuzzy control (SAFC) is proposed to chaos control for a class of time-delayed chaotic systems. An adaptation law based on Gradient Descent (GD) method is utilized to determine the parameters of fuzzy controller. The first derivation of Lyapunov function is considered as the cost function which must be minimized, so the stability of proposed method is guaranteed with Lyapunov stability theory. Moreover, the changes of parameters and delay time are investigated. Simulation results show that the proposed method is appropriate to chaos control and it is robust with respect to changes of parameters. Also it is illustrated that the performance of proposed method is better than the performance of sliding mode controller.

Keywords: *Chaos control, Sliding mode control, Adaptive fuzzy Control, Time-delayed chaotic systems*

I. INTRODUCTION

Chaos is the long and finite aperiodic behavior in deterministic systems which are extremely sensitive to initial conditions [1, 2]. This phenomenon has occurred in many fields such as physical, chemical, biological process and secure communications [3, 4]. Chaos is often undesirable or unwanted, so the chaos control is required to attain the system with the regular behavior. However, sometimes it is useful as in mixing and coding process [5, 6]. Since Ott, Grebogi and Yorke proposed the OGY method in 1990 [7], chaos control has been one of the interesting topics for researchers. Many various control methods have been studied to chaos control such as fuzzy [5, 6], sliding mode control (SMC) [8, 9], delayed feedback [10], and radial basis function (RBF) sliding mode controller [11].

Since Mackey and Glass observed the oscillation and chaos in physiological systems by first order differential delay equations in 1977 [12], time-delayed chaotic systems have attracted many interests. After that, many time-delayed chaotic systems were found with one state variable, such Ikeda model

simplified the model of a laser with optical feedback. Logistic model is the other one which describes the model of population increasing in an unchangeable environment. One of the important researches studied the behavior of these systems is presented in [13]. In 2004, Ikeda model and bifurcation phenomenon are restudied in [14]. Logistic model is studied in [15]. The bifurcation phenomenon in Logistic model is investigated with considering the different delay in [16].

Time-delayed chaotic systems introduce infinite dimension, so chaos control in these systems is more complicated than in the chaotic systems with finite dimension. Many studies have been done on chaos control in these systems, for example, the occasional proportional feedback [17], and the delayed feedback controller (DFC) [18]. Hybrid control including SMC and delayed feedback controller is designed to stabilize unstable periodic orbits (UPO's) and unstable fixed points (UFP's) [19]. An observer-based approach to control of time-delay chaotic systems via Takagi – Sugeno fuzzy model is studied in [20].

In this study, an adaptive fuzzy controller is applied to stabilize UPO's and UFP's. The input of this controller is a sliding surface. The basic idea of this method is minimizing the first derivation of Lyapunov function, so that the stability is achieved. Notice that, to stabilize UPO's, sliding adaptive fuzzy control (SAFC) based delayed feedback is proposed in this paper. Also, the effect of model parameters and time delay are investigated with considering different values of them. The SAFC is implemented on Mackey-Glass and Logistic models.

This paper is organized as follows: in section 2, the model of time-delayed chaotic systems and its properties are described. The structure of proposed controller constitutes section 3. Stability analysis is studied in section 4. Simulation results and conclusion are given in section 5 and 6, respectively.

II. MODEL OF SYSTEM

A class of time-delayed systems is considered as follows:

$$\dot{x}(t) = -ax(t) + f_b(x(t-\tau)) + u(t) \quad (1)$$

where, $x, u \in R$ are the state variable and control input, respectively. a, b, τ are the system parameters, $\tau > 0$. Time-delayed system in (1) can show chaotic behavior when the nonlinear function $f_b(\cdot)$ satisfies the following conditions [21, 22]:

- (i) $f_b(0) = 0$;
- (ii) $\forall x \quad xf_b(x) \geq 0$;
- (iii) $\exists x_{eq} > 0 \quad f_b(x_{eq}) = x_{eq}$;
- (iiii) $\exists r \in (0, x_{eq}) : \forall x \quad f_b(x) \leq f_b(r) = M, M > 0$.

Some time-delayed chaotic systems satisfied above conditions are Mackey-Glass, Ikeda and Logistic models. These models are given as

Mackey-Glass Model [12, 21]

$$f_b(z) = \frac{bz}{1 + |z|^C} \quad (C > 1) \quad (2)$$

Ikeda model [13, 21, 23]

$$f_b(z) = \begin{cases} b \sin(z) & |z| \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Logistic model [22]

$$f_b(z) = \begin{cases} bz(1-z) & 0 \leq z \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

In this paper, Mackey-Glass and Logistic models are studied.

III. DESIGN OF CONTROLLER

In this section, sliding adaptive fuzzy control is designed to chaos control in time-delayed chaotic systems. Gradient Descent (GD) is applied to train the parameters of fuzzy controller. In this method, minimizing the first derivation of Lyapunov function is considered, so that the stability is achieved. This derivation is applied as a cost function to determine unknown parameters to design of controller. Based

on the Lyapunov theorem, the sliding surface reaching condition is $s\dot{s} < 0$.

Sliding surface is defined as follows:

$$s(t) = e(t) = \begin{cases} x(t) - x_{eq} & ; \text{to stabilize UFP's} \\ x(t) - \alpha x(t-T) & ; \text{to stabilize UPO's} \end{cases} \quad (5)$$

where $0 < \alpha < 1$. If $\alpha = 1$, then a stable periodic solution is obtained and if $\alpha < 1$, the origin will be stabilized. Notice that to stabilize UPO's, it is assumed that $x^*(t)$ is considered as the desired UPO with $x^*(t+T) = \alpha x^*(t)$. The fuzzy system used in this study has product inference engine, singleton fuzzifier, center average defuzzifier, and Gaussian membership function. So, the fuzzy system is given as

$$u_1 = \frac{\sum_{l=1}^E y^l \left[\prod_{i=1}^n \exp\left(-\frac{(s-c_i^l)^2}{\sigma}\right) \right]}{\sum_{l=1}^E \left[\prod_{i=1}^n \exp\left(-\frac{(s-c_i^l)^2}{\sigma}\right) \right]} = \hat{Y}^T \phi \quad (6)$$

where $\hat{Y} = [y^1, \dots, y^n]^T$, E is fixed, y^l , c_i^l and σ are free parameters. $s \in R^n$ is the input of the fuzzy system, and $u_1 \in R$ is the output of the fuzzy system. y^l is the center of fuzzy set. Parameters c_i^l and σ are centers and radius of basis function, respectively. In this paper, c_i^l and σ are chosen arbitrarily. Since this fuzzy system is considered with one input, n is equal to 1. To design the controller, the parameter y^l must be determined based on the reaching condition, $s\dot{s} < 0$. Thus, the GD method is applied to determine it as follows:

$$\dot{y}^l = -\gamma \frac{\partial s(t)\dot{s}(t)}{\partial y^l(t)} \quad (7)$$

Equation (7) can be rewritten as

$$\dot{y}^l = -\gamma \frac{\partial s(t)\dot{s}(t)}{\partial y^l(t)} = -\gamma \frac{\partial s(t)\dot{s}(t)}{\partial u(t)} \frac{\partial u(t)}{\partial y^l(t)} \quad (8)$$

Let us to define

$$u = u_1 + u_2 \quad (9)$$

where $u_2 = -f_b(x(t - \tau))$, so the non-linear term is cancelled. Then

$$\begin{aligned} \dot{y}^l &= -\gamma \frac{\partial s(t) \dot{s}(t)}{\partial y^l(t)} = -\gamma \frac{\partial s(t) \dot{s}(t)}{\partial u(t)} \frac{\partial u(t)}{\partial y^l(t)} \\ &= -\gamma \frac{\partial s(t) \dot{s}(t)}{\partial u_1(t)} \frac{\partial u_1(t)}{\partial y^l(t)} \\ &= -\gamma s(t) \frac{\exp(-(\frac{s - c^l}{\sigma})^2)}{\sum_{l=1}^E \exp(-(\frac{s - c^l}{\sigma})^2)} = -\gamma s(t) \phi_l(s) \end{aligned} \quad (10)$$

where γ is the learning rate parameter. Finally, y^l can be on-line adjusted as it was shown in (10). The SAFC will be compared with SMC described as

$$u(t) = -k_1 \text{sat}\left(\frac{x(t) - x_{eq}}{k_2}\right) \quad (11)$$

where x_{eq} is the desired fixed point and k_1, k_2 are the controller parameters. This controller has been used in [19]. Since (1) shows chaotic behavior, many unstable periodic orbits exist in its attractor. The SAFC based delayed feedback can be useful to stabilize UPO's with desired period. The SAFC based delayed feedback is described as

$$u(t) = \alpha u(t - T) + u_1 + u_2 \quad (12)$$

Notice that $x^*(t + T) = \alpha x^*(t)$, and $x^*(t)$ is considered as the desired UPO.

IV. STABILITY ANALYSIS

Lyapunov stability analysis is applied to prove the stability of UFP's and UPO's. From the theory of sliding mode control, a perfect control law is obtained as

$$u_{eq} = \begin{cases} \dot{x}(t) + \alpha x(t) + \dot{s}(t) + \lambda s(t) & ; \text{for UFP's} \\ -\alpha u(t - T) + \dot{x}(t) & \\ + \alpha x(t) + \dot{s}(t) + \lambda s(t) & ; \text{for UPO's} \end{cases} \quad (13)$$

where s is presented in (5).

First, to analyze the stability, (9) is substituted in (1):

$$\dot{x}(t) = \begin{cases} -\alpha x(t) + u_1(t) & ; \text{for UFP's} \\ -\alpha x(t) + \alpha u(t - T) + u_1(t) & ; \text{for UPO's} \end{cases} \quad (14)$$

if u_1 is substituted with u_{eq} then $\dot{s}(t) + \lambda s(t) = 0$. Since λ is a positive constant parameter, the sliding surface variable s will converge to zero, gradually. So, e will converge to zero. From (13) and (14):

$$\dot{s}(t) = -\lambda s(t) + (u_{eq}(t) - u_1(t)) \quad (15)$$

Theorem ([24]). Suppose that the input universe of discourse U is a compact set in R^n . Then, for any given real continuous function $g(z)$ on U and arbitrary $\xi > 0$, there exists a fuzzy system $F(z)$ in the form of (10) such that

$$\sup_{z \in U} \|g(z) - F(z)\| \leq \xi$$

That is, the fuzzy systems with product inference engine, singleton fuzzifier, center average defuzzifier, and Gaussian membership functions are universal approximators.

Therefore, the control law u_1 can be obtained via the optimal weighting vector $\bar{Y}$ of the fuzzy system. The designed control is applied to estimate the u_{eq} . This approximation can be achieved with the error which is smaller than ξ .

$$\max |\bar{u}(x, \bar{Y}) - u_{eq}(x)| < \xi$$

where $\bar{u}(x, \bar{Y}) = \sum_{l=1}^E \bar{y}^l \phi_l = \bar{Y}^T \phi$ and $u_{eq}(x) = \bar{Y}^T \phi + \xi$.

Define $\tilde{Y} = \bar{Y} - \hat{Y}$ as the weighting error, and then (15) can be rewritten as

$$\dot{s}(t) = -\lambda s(t) + (\tilde{Y}^T \phi + \xi) \quad (16)$$

Notice that $\bar{Y}$ and $\hat{Y}$ are the optimal and estimated values of Y , respectively.

Define the Lyapunov function as follows:

$$V(s, \tilde{Y}) = \frac{1}{2} s^2 + \frac{1}{2\gamma} \tilde{Y}^T \tilde{Y} \quad (17)$$

The first derivation of (17) with respect to time is obtained as

$$\dot{V} = s\dot{s} + \frac{1}{\gamma} \tilde{Y}^T \dot{\tilde{Y}} \quad (18)$$

Substituting (10), (16) in to (18), then

$$\dot{V} = s(-\lambda s + \tilde{Y}^T \phi + \xi) + \frac{1}{\gamma} \tilde{Y}^T (-\gamma s \phi) = -\lambda s^2 + s\xi \quad (19)$$

Thus

$$\dot{V} \leq |s|(-\lambda|s| + \xi) \quad (20)$$

So, if $|s| \geq \xi/\lambda$, then $\dot{V} \leq 0$ which means that this Lyapunov function will decrease gradually. It can be concluded that the error of state variable will converge into a boundary layer ξ/λ around $e = s = 0$ in the phase plane. Hence, it can be resulted that the SAFC is stable and the error of state variable will converge into a small value.

V. SIMULATION RESULTS

A. Implementation with constant values of a, b, τ

In this section, the behavior of two considered models with $\tau = 10$, $a = 1$, $b = 2$ (for Mackey-Glass model), $b = 4$ (for Logistic model), $C = 10$ and $x(0) = 0.1$ is studied. For Mackey-Glass and Logistic models, UFP's other than the origin are $\pm 10\sqrt{\frac{b-a}{a}} = \pm 1$ and $\frac{b-a}{b} = 0.75$, respectively.

Fig. 1 shows the phase plane of the uncontrolled system.

The SAFC and the SAFC based delayed feedback are applied to stabilize the UFP's and UPO's, respectively. The parameters of SMC are $k_1 = 3, k_2 = 0.1$ and the reason of these choices has been described in [19]. System responses are shown in Fig. 2. As it is shown in Fig. 3 the control effort of SAFC is less than the control effort of SMC and the control signal of SAFC is smoother than the control signal of SMC. Notice that the speed of SAFC and SMC is controllable. The SAFC speed is improved with a proportional gain. Also, the speed of response with SMC becomes better if k_1 is chosen larger than 3 or k_2 is chosen smaller than 0.1. Notice that if k_1 is chosen larger than 3, the control signal will become larger. In this mode, system may be damaged, and if k_2 is chosen smaller than 0.1, the control effort will become more that it is not desired. Furthermore, it is assumed that $|s(t)| \leq k_2$ is $s(t)$'s domain of attraction, so very small value of k_2 is not desired. In summary, with comparing the response speeds and control efforts of SAFC and SMC, it is obtained that the performance of SAFC is better than the performance of SMC. To show the effect of small value of k_2 , its value is changed to 0.0001. Fig. 3 shows that in this case, the control effort becomes more, although the speed of response becomes better.

To stabilize UPO's, it is assumed that $\alpha = 1, T = 13$. The results obtained after using SAFC based delayed feedback at

$t = 100$ are shown in Fig. 5. As it is shown in this figure, the period of the response is exactly the same as it has been desired.

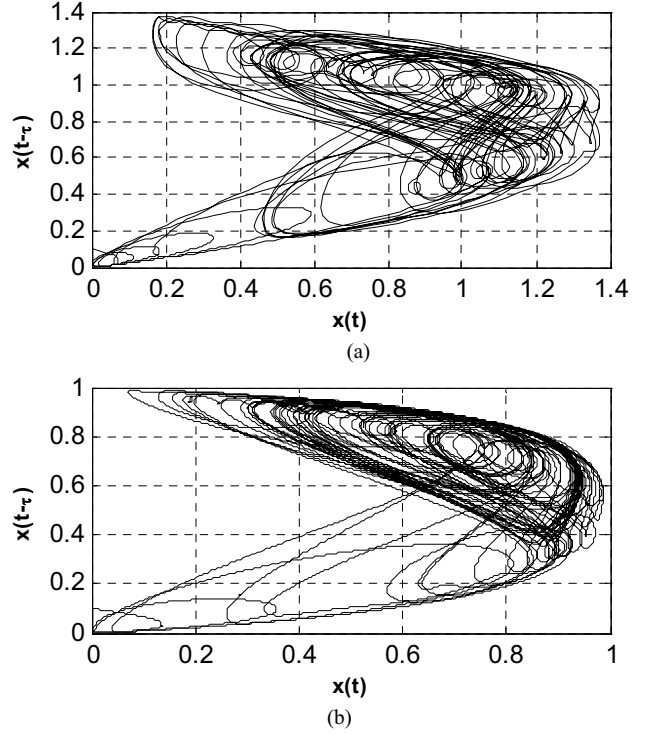


Figure 1. Phase plane of the uncontrolled systems, for (a) Mackey-Glass, (b) Logistic model.

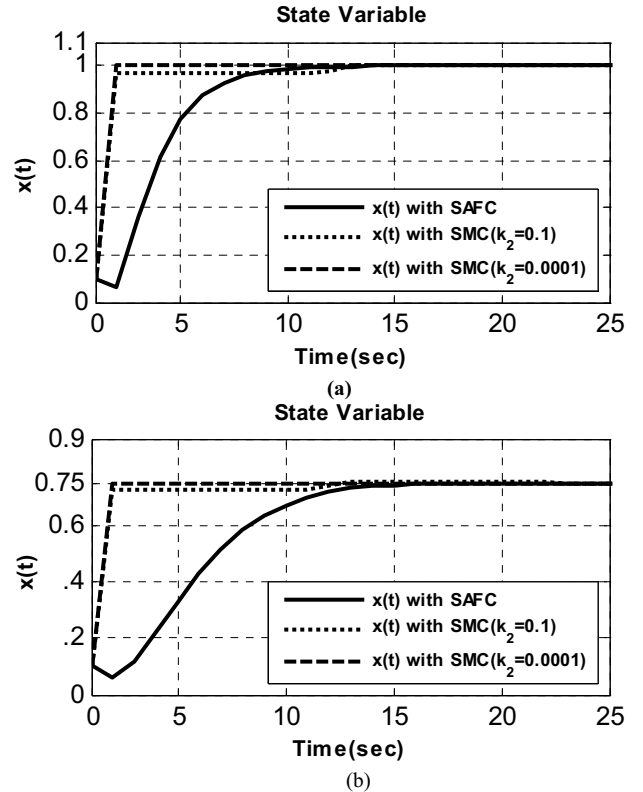


Figure 2. System response of (a) Mackey-Glass, (b) Logistic model to SAFC and SMC.

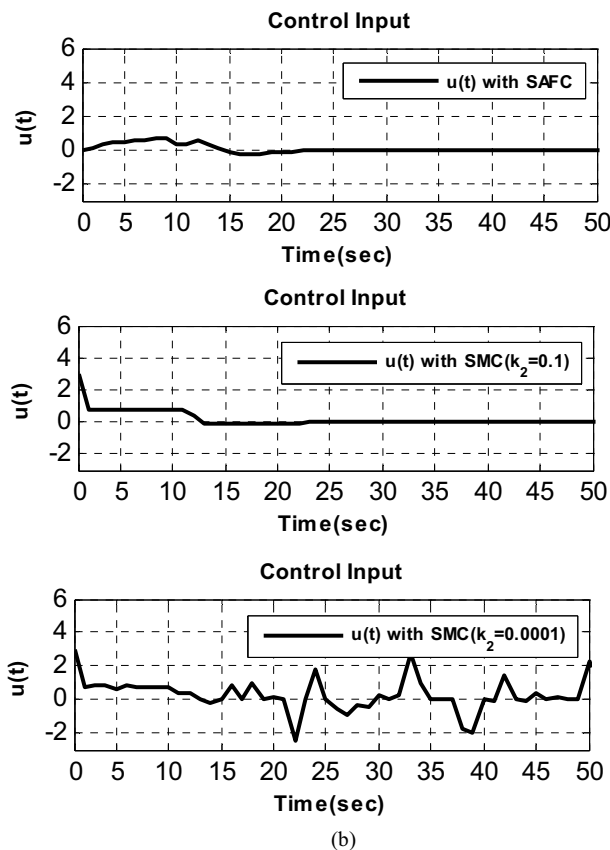
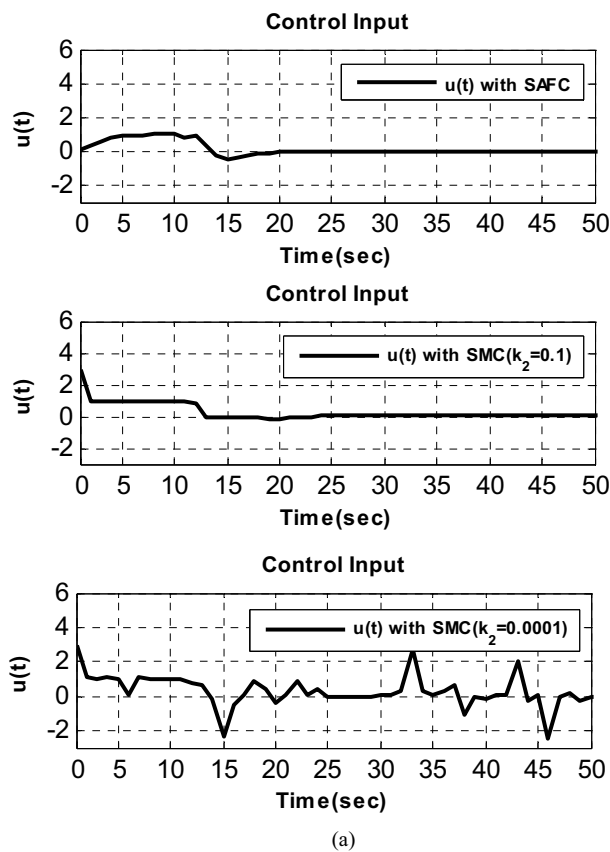


Figure 3. Control effort of (a) Mackey-Glass, (b) Logistic model in SAFC and SMC cases.

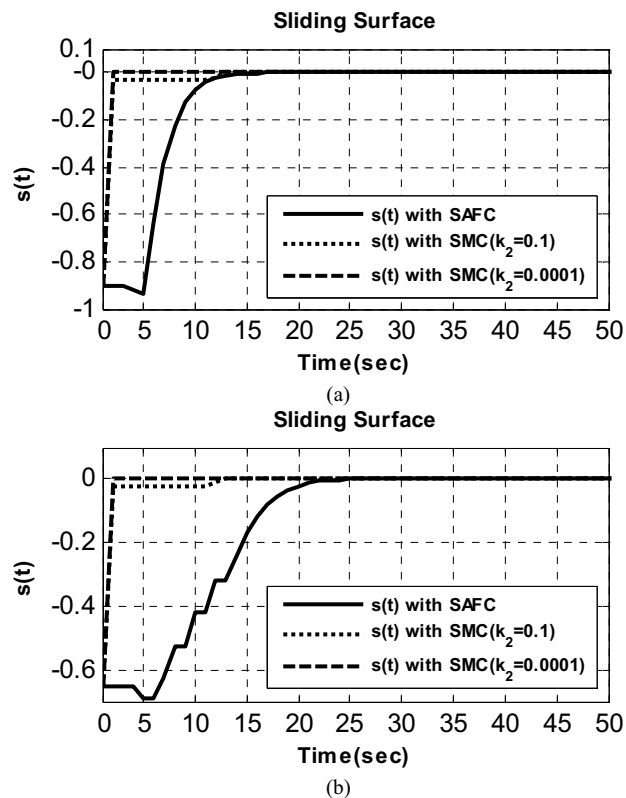


Figure 4. Sliding surface of (a) Mackey-Glass, (b) Logistic model in SAFC and SMC cases.

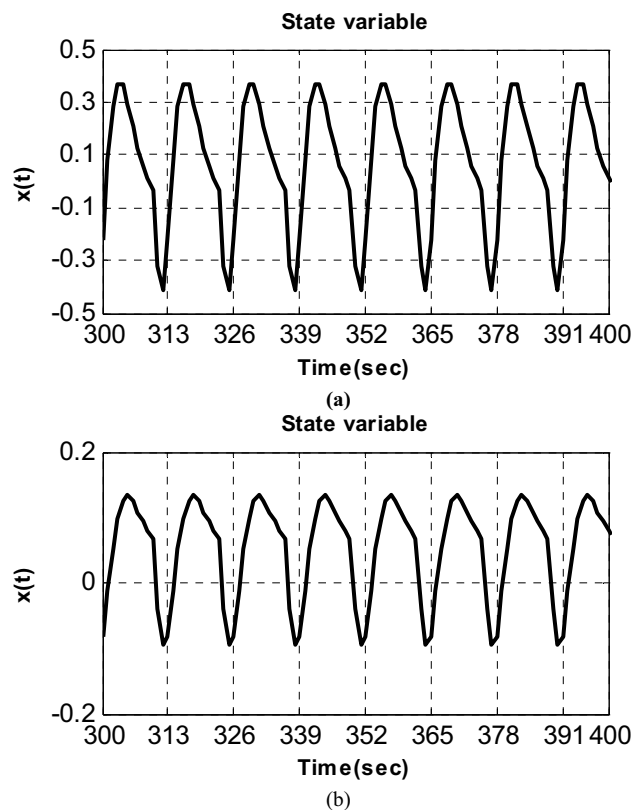


Figure 5. Response of (a) Mackey-Glass, (b) Logistic model to SAFC based delayed feedback with $T = 13$, $\alpha = 1$. The control force was applied at $t = 100$.

For better comparison, the performance of two considered controllers is compared with the following criteria:

1. Mean square error after 10τ (MSE).
2. Output error variance after 10τ (EV).
3. Output error settling time. It is the time at which $|e| < 0.1$ (EST).
4. Control signal variance after 10τ (CV).
5. Maximum of absolute value of control signal (MAC) after 0.1τ .

As we know, the less the MSE and EV, the better control is achieved. The less the EST, the sooner is the convergence. Moreover, less control effort (CV) and lower magnitude of the control signals (MAC) is desired in real-time control applications. The results of these comparisons for Mackey-Glass and Logistic models are given in Table 1, 2 respectively.

TABLE I. COMPARISONS BETWEEN DIFFERENT METHODS OF CHAOS CONTROL FOR MACKEY-GLASS MODEL

Criterion	Control Method		
	SAFC	SMC($k_2 = 0.1$)	SMC($k_2 = 1e-4$)
MSE	0	8.1690e-008	7.2888e-10
EV	0	7.9303e-008	7.3373e-10
EST(sec)	11	2	2
CV	0	7.1373e-005	0.6604
MAC	0.9784	0.9818	2.8349

TABLE II. COMPARISONS BETWEEN DIFFERENT METHODS OF CHAOS CONTROL FOR LOGISTIC MODEL

Criterion	Control Method		
	SAFC	SMC($k_2 = 0.1$)	SMC($k_2 = 1e-4$)
MSE	0	2.6254e-008	5.8916e-010
EV	0	2.4969e-008	5.7386e-010
EST(sec)	10	2	2
CV	4.415e-031	2.2472e-005	0.5165
MAC	0.6936	0.7350	2.7129

These results show that SAFC is better than SMC in all cases except EST.

B. Synchronization of two nonidentical systems with known parameter Implementation with differences in values of a, b, τ s

To study the effect of parameters changing, the parameters of Mackey-Glass and Logistic models are changed to $a = 6, b = 8$ at $t = 400$, so new UFP of Mackey-Glass and Logistic model are 0.89595 and 0.25, respectively and the time delay is changed to $\tau = 105$ at $t = 70$. Figs. 6, 7, 8 Show that the SAFC acts as well as when the parameters are not

changed and SMC performance becomes worse when τ is changed to 105 at $t = 70$.

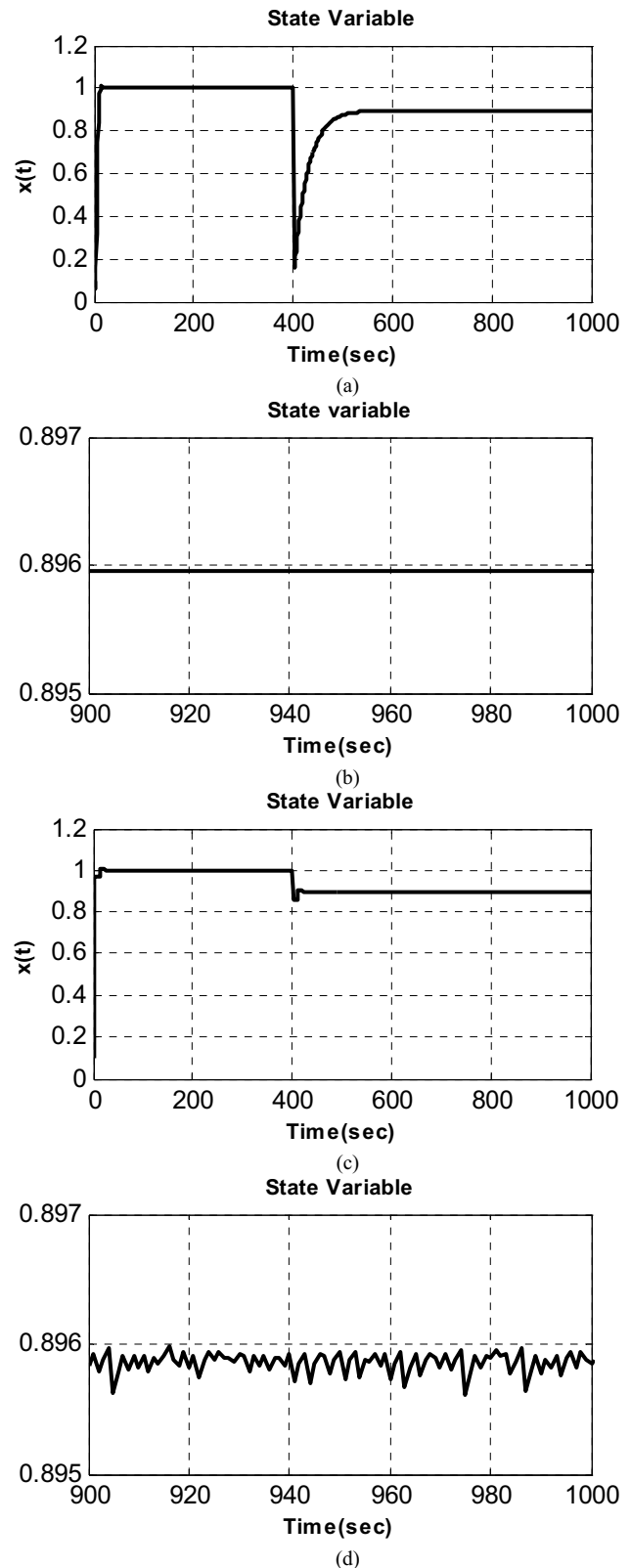
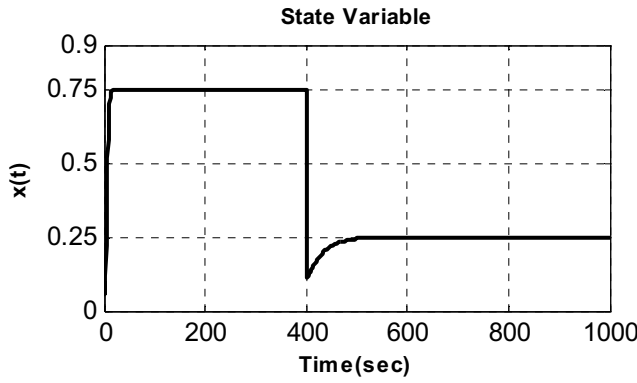
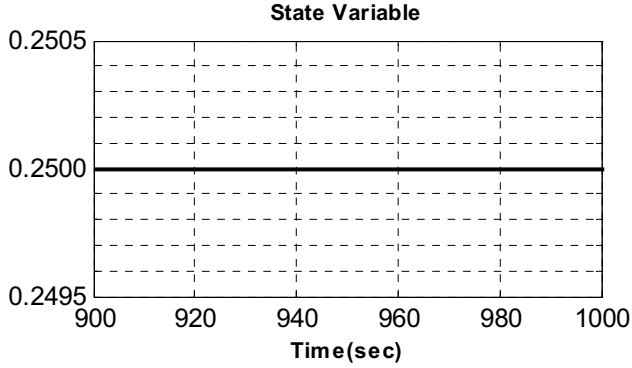


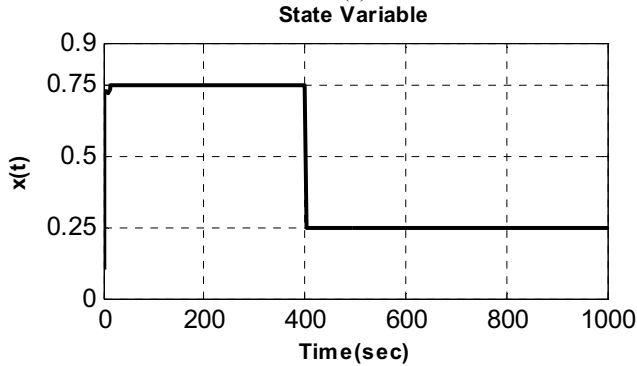
Figure 6. System response of Mackey-Glass model to (a, b) SAFC, (c, d) SMC when the parameters a, b are changed at $t = 400$.



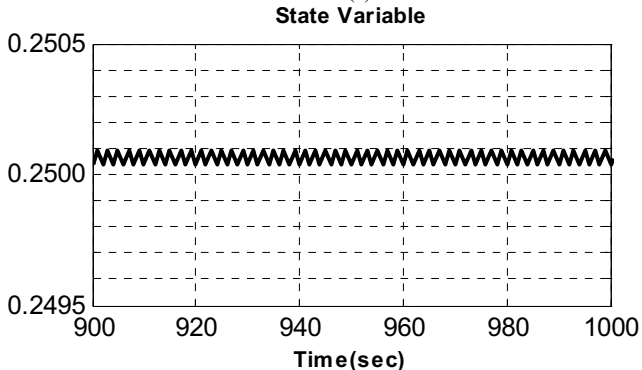
(a)



(b)

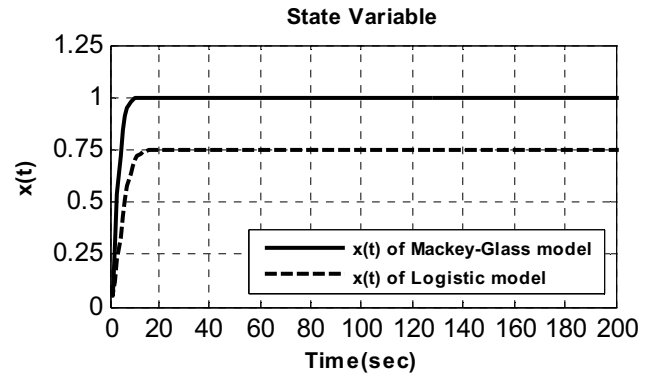


(c)

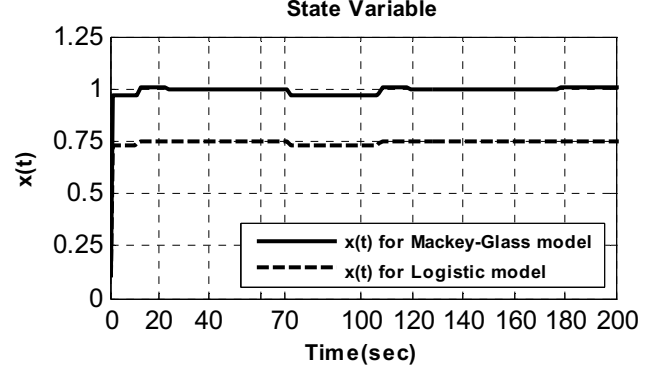


(d)

Figure 7. System response of Logistic model to (a, b) SAFC, (c, d) SMC when the parameters a, b are changed at $t = 400$.



(a)



(b)

Figure 8. System response of Mackey-Glass and Logistic model to (a) SAFC, (b) SMC when the parameter τ is changed at $t = 70$.

In summary, the SAFC performance is better than SMC performance in the cases considered in this section.

VI. CONCLUSION

In this paper, the behavior of two time-delayed chaotic systems is studied; Mackey-Glass and Logistic models. The sliding adaptive fuzzy controller and the sliding adaptive fuzzy controller based delayed feedback are applied to stabilize UFP's and UPO's respectively. Also, the SAFC has been compared with SMC applied to stabilize UFP's. The simulation results show that the SAFC is robust with respect to offline and online changes of parameters. Finally, it is concluded that SAFC is effective to chaos control in time-delayed chaotic systems and its performance is better than the SMC performance in many cases considered in this paper.

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