

Mixed-Time T-S Fuzzy Optimal Estimator for Target Tracking

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Abstract—This paper is concerned with the mixed-time fuzzy optimal estimator for the target tracking. The proposed method takes account of behaviour for target motion in continuous time and system measurement in discrete time. The overall dynamic of system in continuous-time is discretised by using Takagi-Sugeno(T-S) fuzzy models. Based on the fuzzy model the fuzzy estimation is studied. Then the error state of the filtering process is means square bounded. A basis dependent Lyapunov function approach is developed to design the fuzzy filter and a filter gain is minimized by using some linear matrix inequalities(LMIs) constraints. A numerical example is provided to demonstrate various aspects of theoretical results.

Index Terms—T-S Fuzzy, Mixed-Time, LMI, Fuzzy Optimal Estimator, Lyapunov function

I. INTRODUCTION

The purpose of target tracking is an accurate state estimation of a moving object. Estimating as accurately as possible of the state of a dynamic system have been a very important topic. Among the various approaches proposed in this topic, the estimation processes a relative states between a moving platform and a target as well as estimation of the target states[1], [2], [3]. In most cases, whether the target states are described in inertial, spherical, or modified spherical coordinates in[5]. The model-based adaptive filtering method was presented in [7], [9], [10], [11] that consider the motion of a target by using multiple sub-filters. Almost all maneuvering target tracking methods are model based. Research continues on techniques and applications to the target tracking problem each of these approaches has its own drawbacks. And all are approximate from sense in discrete-time domain and some do not apply to the tracking problem[6], [7], [9], [10]. They assume that the target motion and its observation can be represented by some known mathematical models accurately[8]. The most of thesis used target motion and measurement model in continuous-time(CT) domain or discrete-time(DT) domain.

However, the target motion is more accurately modeled in CT domain. They should not depend on how and when samples are taken. And measurements are usually available only at discrete time instants. To design the target motion in DT domain, the discretisation of it is usually first required. The discretisation methods of the fuzzy system have been studied[12], [13]. This method is directly discretise the global dynamics of

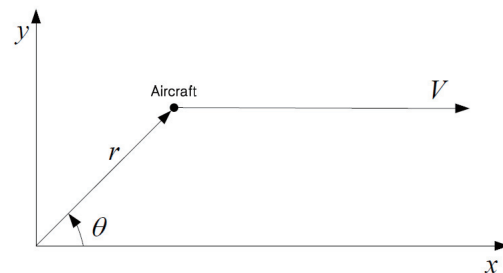


Fig. 1. Radar tracking geometry

the fuzzy model. And Takagi-Sugeno(T-S) fuzzy model have the important advantage of its universal approximation.

In this paper, we concerned with the mixed-time fuzzy optimal estimator for the target tracking. We propose that take account of behavior for target motion in CT domain and system measurement in DT domain. The overall dynamic of system in CT is discretised by using T-S fuzzy models. Based on the fuzzy model, the fuzzy estimation is studied. Then the error state of the filtering process is mean square bounded. A basis dependent Lyapunov function approach is developed to design the fuzzy filter and a filter gain is minimized by using some linear matrix inequalities(LMIs) constraints. A numerical example is provided to demonstrate various aspects of theoretical results. This paper is organized as follows: Section 2 describes the conventional discretisation method of the continuous-time T-S fuzzy model. Section 3 describes the fuzzy filter design by using the discretizing system and measurement. In Section 4, a numerical example is provided to design the fuzzy filter. Finally, the conclusions are given in Section 5.

II. PRELIMINARIES AND PROBLEM DESCRIPTION

We assume here that the target moves in a plane that is the two-dimensional radar tracking problem illustrated in Figure 1. In fact, the target motion is more accurately modeled in CT domain while observations are usually available only at DT instants. It is often more appropriate to use the following mixed-time models for most tracking problem.

A. State and measurement models

The moving target[14] can be described as

$$\begin{aligned} r^2 &= x^2 + y^2 \\ 2r\dot{r} &= 2x\dot{x} + 2y\dot{y} \\ \dot{r} &= \frac{x}{r}\dot{x} + \frac{y}{r}\dot{y} \end{aligned} \quad (1)$$

where $x = r\cos\theta$, $\dot{x} = V$, $\dot{y} = 0$, V is velocity, r is the radar measurement range and θ is azimuth, therefore

$$\dot{r} = V\cos\theta \quad (2)$$

Assume that the aircraft is not accelerating so that $\dot{V} = 0$. If r and θ do not change much during each scan of the radar (about 1-10s), we have $\ddot{r} \approx 0$ and $\ddot{\theta} \approx 0$.

Differentiating

$$\begin{aligned} \frac{d}{dt}\dot{r} &= \dot{V}\cos\theta - V\dot{\theta}\sin\theta \\ \frac{d}{dt}\dot{\theta} &= -\left(\frac{\dot{V}r - V\dot{r}}{r^2}\right)\sin\theta - \frac{V}{r}\dot{\theta}\cos\theta \end{aligned} \quad (3)$$

We have the state equation as

$$\frac{d}{dt} \begin{bmatrix} r \\ \dot{r} \\ \theta \\ \dot{\theta} \end{bmatrix} = A \begin{bmatrix} r \\ \dot{r} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ w_1(t) \\ 0 \\ w_2(t) \end{bmatrix} \quad (4)$$

where A is system matrix and $w_1(t), w_2(t)$ are random disturbances. From (4), we have the continuous-time state model.

$$\dot{X}(t) = AX(t) + BW(t) \quad (5)$$

$$y(k) = CX(k) + v(k) \quad (6)$$

where $X(t) = [r \ \dot{r} \ \theta \ \dot{\theta}]^T$ denotes the state vector, $y(k)$ denotes measurement equation, A, B, C denote the system matrix, $W(t)$ is maneuver noise, $v(k)$ is measurement noise.

Remark 1: Measurements are available only at discrete-time instants because they are in fact from sensor. It is often more appropriate to use the following mixed-time models for most tracking problems.

The general solution to (5) excited by the initial state $X(t_0)$ is given by

$$\dot{X}(t) = \Phi(t, t_0) + \int_{t_0}^t \Phi(t, \tau)Bw(\tau)d\tau \quad (7)$$

where the state transition matrix $\Phi(t, t_0)$ satisfies $\Phi(t, t_0) = \Phi(t, t_1)\Phi(t_1, t_0)$ with initial condition $\Phi(t, t_0) = I$. The solution to (5) at $t = (k+1)T$ with $t_0 = kT$ is

$$\begin{aligned} X(kT+T) &= \exp(AT)X(kT) \\ &+ \int_{kT}^{kT+T} \exp(A(kT+T-\tau))BW(kT)d\tau \\ &= GX(kT) + HW(kT) \end{aligned} \quad (8)$$

where $G = \exp(AT)$ and $H = (G - I)BA^{-1}$.

B. The discretizing the continuous-time T-S fuzzy model

The discretising the continuous-time T-S fuzzy model is to apply the discretisation method for system to the consequent parts of the fuzzy rules. The rule of the T-S fuzzy system is formulated in the following form

$$\begin{aligned} R_i : &\text{IF } z_1(t) \text{ is about } \Gamma_{i1} \text{ and } \dots \text{ and } z_g(t) \text{ is about } \Gamma_{ig}, \\ &\text{THEN } \dot{X}(t) = A_iX(t) + B_iW(t) \end{aligned} \quad (9)$$

where $i \in \mathcal{R}_g = \{1, 2, \dots, g\}$, denotes the i th fuzzy rule; $z_h(t)$, $h \in \mathcal{R}_p = \{1, 2, \dots, p\}$, the h th premise variable; $\Gamma_{ih}, (i, h) \in \mathcal{R}_g \times \mathcal{R}_p$, the fuzzy set. If $z_h(t)$ for $t \in [kT, kT+T)$ is $z_h(kT)$, the rule of the corresponding discrete-time T-S fuzzy model is obtained as follows

$$\begin{aligned} R_i : &\text{IF } z_1(kT) \text{ is about } \Gamma_{i1} \text{ and } \dots \text{ and } z_g(kT) \text{ is about } \Gamma_{ig}, \\ &\text{THEN } X(kT+T) = G_iX(kT) + H_iW(kT) \\ &y(kT) = C_iX(kT) + v(kT) \end{aligned} \quad (10)$$

where $G_i = \exp(A_iT)$ and $H_i = (G_i - I)B_iA_i^{-1}$. Using the center-average defuzzification, product inference and singleton fuzzifier, the overall dynamics of this discrete-time T-S fuzzy model is described by

$$X(kT+T) = \sum_{i=1}^g \theta_i(z(kT))(G_iX(kT) + H_iW(kT)) \quad (11)$$

$$y(kT) = \sum_{i=1}^g \theta_i(z(kT))(C_iX(kT) + v(kT)) \quad (12)$$

where $\theta_i(z(kT)) = \frac{\mu_i(z(kT))}{\sum_{i=1}^g \mu_i(z(kT))}$, $\mu_i(z(kT)) = \prod_{j=1}^g \Gamma_{ij}(z_j(kT))$ and $\Gamma_{ih}(z_h(kT))$ is the grade of membership of $z_h(kT)$ in Γ_{ih} .

Remark 2: In the target tracking, a target is usually treated as a point object a shape in tracking. The state is often approximated piecewise constant with $X(k) = X(t), t(k) \leq t < t(k+1)$ when discretizing a continuous time system. Note that $W(k) \neq W(t_k), X(k) \neq X(t_k)$.

We denote that

$$\begin{aligned} X(kT) &= G_{if}X(kT) + H_{if}W(kT) \\ y(kT) &= C_{if}X(kT) + v(kT) \end{aligned} \quad (13)$$

where

$$\begin{aligned} G_{if} &= \sum_{i=1}^g \theta_i(z(kT))G_i, & H_{if} &= \sum_{i=1}^g \theta_i(z(kT))H_i, \\ C_{if} &= \sum_{i=1}^g \theta_i(z(kT))C_i. \end{aligned} \quad (14)$$

In this paper, we use the discretizing fuzzy system for accurately model of the target motion. Because the target motion modeling is one of the first tasks for maneuvering target tracking.

III. T-S FUZZY MODEL BASED FUZZY FILTER DESIGN

This Section presents a fuzzy filter design method based on Lyapunov function. We present the proposed discretised version of the continuous-time T-S fuzzy system. The discretising fuzzy model is used to deal with the state estimation. Before proceeding, let us introduce the following lemma which will be essential for the proof of our main result.

emma 1: (Schur complement [4]) Given constant symmetric matrices N, O and L of appropriate dimensions, the following two inequalities are equivalent:

$$O > 0, \quad N + L^T O L < 0, \quad (15)$$

$$\begin{bmatrix} N & L^T \\ L & -O^{-1} \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -O^{-1} & L \\ L^T & N \end{bmatrix} < 0$$

The following fuzzy filter is proposed to deal with a state estimation.

Filter Rule R_j

IF $z_1(kT)$ is about Γ_{j1} and $\dots$ and $z_g(kT)$ is about Γ_{jg} ,
THEN $\hat{X}(kT + T) = G_j \hat{X}(kT) + L_j(y(kT) - \hat{y}(kT))$ (16)

where $\hat{X}(kT)$ is an estimate of $X(kT)$, $G_j = \exp(A_j T)$ and L_j is the filter parameter to be determined, $\hat{y}(kT) = \sum_{k=1}^q \theta(z(kT)) C_k(kT) \hat{X}(kT)$ and L_j is the filter gain with j th filter rule. The defuzzified outputs are given by

$$\begin{aligned} \hat{X}(kT + T) &= \sum_{j=1}^q \theta_j(z(kT)) \sum_{k=1}^q \theta(kT)(z(kT)) \\ &\quad \times [G_j \hat{X}(kT) + L_j(y(kT) - \hat{y}(kT))] \\ &= \sum_{i=1}^q \theta_i(z(kT)) \sum_{j=1}^q \theta_j(z(kT)) \sum_{k=1}^q \theta_k(z(kT)) \\ &\quad \{L_j C_i X(kT) + L_j v(kT) \\ &\quad + (G_j - L_j C_k(kT)) \hat{X}(kT)\} \end{aligned} \quad (17)$$

For the notational convenience in the following discussions, we denote that

$$\begin{aligned} \hat{X}(kT + T) &= L_{jf} C_{if} X(kT) \\ &\quad + (C_{jf} - L_{jf}) \hat{X}(kT) + L_j v(kT) \end{aligned} \quad (18)$$

where

$$\begin{aligned} L_{jf} &= \sum_{j=1}^q \theta_j(z(kT)) L_j, \quad G_{jf} = \sum_{j=1}^q \theta_j(z(kT)) G_j, \\ C_{kf} &= \sum_{k=1}^q \theta_j(z(kT)) C_k. \end{aligned}$$

From (11) and (17), the augmented system can be expressed as

$$\eta(kT + T) = G_m \eta(kT) + B_m d(kT + T) \quad (19)$$

where

$$\begin{aligned} \eta(kT + T) &:= [X(kT + T) \quad \hat{X}(kT + T)]^T \\ d(kT + T) &:= [W(kT + T) \quad \hat{v}(kT + T)]^T \\ G_m &:= \begin{bmatrix} G_{if} & 0 \\ L_{if} C_{if} & (C_{jf} - L_{jf}) \end{bmatrix} \\ B_m &:= \begin{bmatrix} H_{if} & 0 \\ 0 & L_{jf} \end{bmatrix} \end{aligned} \quad (20)$$

Before proceeding further, we define the estimation error as

$$\begin{aligned} e(kT + T) &:= X(kT + T) - \hat{X}(kT + T) \\ &= [I \quad -I] \times \begin{bmatrix} X(kT + T) \\ \hat{X}(kT + T) \end{bmatrix} \\ &= D_m \eta(kT). \end{aligned} \quad (21)$$

Then, the optimal fuzzy filter is concerned. We assume that the filter system is stochastically stable in the mean square with filter gain L_{jf} performance. The filter gain is to specify L_{jf} such that the following minimal problem is achieved in (22)

$$\min \sup_{d(kT) \in L_\infty} \frac{\|e(kT)\|_\infty}{\|d(kT)\|_\infty}. \quad (22)$$

However it is difficult to directly solve the minimax problem in (22). In this paper, the suboptimal approach via minimizing the upper bounded of the L_{jf} gain is obtained. The optimal L_{jf} gain fuzzy filter problem is the following minimax problem.

Theorem 1:

$$\begin{bmatrix} P & * & * & * \\ 0 & -\alpha^2 I & * & * \\ \left(\frac{G_{m_{ijk}} D_m + G_{m_{ikj}} D_m}{2} \right) & B_m D_m & P^{-1} & * \\ -D_m & 0 & 0 & I \end{bmatrix} \geq 0 \quad (23)$$

where P is symmetric positive-definite matrix and α is positive constant.

Proof: Using by the Tanaka disturbance rejection [15]; there exists a quadratic function $V(e(kT)) = e(kT)^T P e(kT)$, $P > 0$, and $\alpha \geq 0$, such that for all $t \in [kT, kT + T)$,

$$\begin{aligned} V(e(kT)) &= e^T(kT) P e(kT), \\ \Delta V(e(kT)) &\geq 0 + e^T(kT) e(kT) - \alpha^2 d^T(kT) d(kT) \leq 0 \\ \xi(k) &= [\eta(kT)^T \quad d(kT)^T], \end{aligned} \quad (24)$$

$$\begin{aligned}
& \triangle V(e(kT)) \\
&= E \{V(e(kT+T))|\xi(k)\} - V(kT) \\
&= E \{ (D_m \eta(kT+T))^T P (D_m \eta(kT+T)) |\xi(kT)\} \\
&\quad - \eta^T(kT) D_m^T P D_m \eta(kT) \\
&= E \{ [(G_m \eta(kT+T) + B_m d(kT+T))^T D_m^T P \\
&\quad \times D_m [G_m \eta(kT+T) + B_m d(kT+T)] |\xi(kT)\} \\
&\quad - \eta^T(kT) D_m^T P D_m \eta(kT) \\
&= E \left\{ \xi(kT)^T \begin{bmatrix} G_m^T D_m^T \\ B_m^T D_m^T \end{bmatrix} P \begin{bmatrix} D_m G_m & D_m B_m \end{bmatrix} \xi(kT) |\xi(kT) \right\} \\
&\quad - \eta^T(kT) D_m^T P D_m \eta(kT) \\
&= \xi(kT)^T \left(\begin{bmatrix} G_m^T D_m^T \\ B_m^T D_m^T \end{bmatrix} P \begin{bmatrix} D_m G_m & D_m B_m \end{bmatrix} \right) \xi(kT) \\
&\quad - \eta^T(kT) D_m^T P D_m \eta(kT). \tag{25}
\end{aligned}$$

Since $V(e(kT)) \geq 0$, this implies

$$\frac{\|e(kT)\|}{\|d(kT)\|} \leq \alpha \tag{26}$$

and

$$\begin{aligned}
& e^T(kT)e(kT) - \alpha^2 d^T(kT)d(kT) \\
&= \xi^T(kT) \begin{bmatrix} D_m^T D_m & 0 \\ 0 & -\alpha^2 I \end{bmatrix} \xi(kT). \tag{27}
\end{aligned}$$

Then we have

$$\begin{aligned}
& \xi^T(kT) \left(\begin{bmatrix} G_m^T D_m^T \\ B_m^T D_m^T \end{bmatrix} P \begin{bmatrix} D_m G_m & D_m B_m \end{bmatrix} \right. \\
&\quad \left. - \begin{bmatrix} D_m^T P D_m & 0 \\ 0 & \alpha^2 I \end{bmatrix} \right) \xi(kT) \leq 0. \tag{28}
\end{aligned}$$

Form the Schur complement [4], we obtain the LMIs condition;

$$\begin{bmatrix} P & * & * & * \\ 0 & -\alpha^2 I & * & * \\ \left(\frac{G_{m_{ijk}} D_m + G_{m_{ikj}} D_m}{2} \right) & B_m D_m & P^{-1} & * \\ -D_m & 0 & 0 & I \end{bmatrix} \geq 0 \tag{29}$$

By multiplying both sides of (29) by block-diag $[X \ I \ I \ I]$, (23) is obtained, where $X = P^{-1}$, the subscript $\{i, j, k\}$ is fuzzy filter rule each. The proof is completed. ■

IV. SIMULATION

In this section, we demonstrate the results in Section 3 to discretise the continuous-time T-S system for the moving target like the Van der Pol oscillator. We first the dynamic state equations in CT are given by

$$\begin{aligned}
& \frac{d}{dt} x_1(t) = x_2(t) \\
& \frac{d}{dt} x_2(t) = -x_1(t) + (1 - x_1^2(t))x_2(t) + w(t) \\
& y(t) = (1 + 0.25x_1(t)) + v(t) \tag{30}
\end{aligned}$$

The corresponding T-S fuzzy model of the system in (30) is expressed as follows:

$$\begin{aligned}
& R_1 : \text{IF } x_1(t) \text{ is about } \Gamma_{11}, \\
& \quad \text{THEN } \dot{x}(t) = A_1 x(t) + B_1 w(t), \\
& \quad \quad y(t) = C_1 x(t) + v(t). \\
& R_2 : \text{IF } x_1(t) \text{ is about } \Gamma_{12}, \\
& \quad \text{THEN } \dot{x}(t) = A_2 x(t) + B_2 w(t), \\
& \quad \quad y(t) = C_2 x(t) + v(t). \tag{31}
\end{aligned}$$

where A_i, B_i, C_i are by the following as

$$\begin{aligned}
& A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 1 - k^2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, \\
& C_1 = [1 + 0.2k^2 \quad 0], \quad C_2 = [1 \quad 0], \\
& B_i = [0 \quad 1]^T, \quad i = 1, 2, \tag{32}
\end{aligned}$$

and the membership function are

$$\Gamma_{11} = x_1^2/k^2, \quad \Gamma_{12} = 1 - x_1^2/k^2 \tag{33}$$

where $k = 2.5$.

First, we simulate the continuous-time T-S fuzzy system. To obtain the corresponding discretised T-S fuzzy system for the sampling time $T = 0.01(s)$, Theorem 1 is applied. The discretised version of the continuous-time T-S fuzzy system is

$$\begin{aligned}
& R_1 : \text{IF } x_1(kT) \text{ is about } \Gamma_{11} \\
& \quad \text{THEN } x(kT+T) = \Phi_1 x(kT) + B_1 w(kT) \\
& \quad \quad y(kT+T) = C_1 x(kT) + v(kT) \\
& R_2 : \text{IF } x_2(kT) \text{ is about } \Gamma_{12} \\
& \quad \text{THEN } x(kT+T) = \Phi_2 x(kT) + B_2 w(kT) \\
& \quad \quad y(kT+T) = C_2 x(kT) + v(kT) \tag{34}
\end{aligned}$$

By using Theorem 1 and solving the corresponding LMIs constraints, we obtain the filter gains L and P :

$$\begin{aligned}
& P = \begin{bmatrix} 0.0277 & -0.0050 \\ -0.0050 & 0.0036 \end{bmatrix}, \\
& L_1 = \begin{bmatrix} 0.1496 \\ 1.3159 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.2187 \\ 1.7408 \end{bmatrix}. \tag{35}
\end{aligned}$$

V. CONCLUSION

In this paper, we has been proposed for the mixed-time fuzzy optimal estimator design for the target tracking. The proposed method has taken account of behavior for target motion in CT domain and system measurement in DT domain. The overall dynamic system in DT domain has discretized by using T-S fuzzy models. Based on the fuzzy model, the fuzzy estimation has been studied. The error state of the filtering process have mean square bounded. In addition, a basis dependent Lyapunov function approach has developed to design the fuzzy filter and a filter gain has minimized by

using some LMIs constraints. A numerical example have been provided to demonstrate various aspects of theoretical results.

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