

An Adaptive Type-2 Input Based Nonsingleton Type-2 Fuzzy Logic System for Real World Applications

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Abstract—A Fuzzy Logic System (FLS) is generally credited with being an adequate methodology for real world applications which are subject to high uncertainty levels. Recent works have shown that interval type-2 FLSs can outperform type-1 FLSs in the applications which encompass high uncertainty levels. However, the majority of interval type-2 FLSs handle the linguistic and input numerical uncertainties using singleton interval type-2 FLSs that mix the numerical and linguistic uncertainties to be handled only by the linguistic labels type-2 fuzzy sets. This ignores the fact that if input numerical uncertainties were present, they should affect the incoming inputs to the FLS. Even in the papers that employed nonsingleton type-2 FLSs, the input signals were assumed to have a predefined shape (mostly Gaussian or triangular) which might not reflect the real uncertainty distribution which can vary with the associated measurement. In our previous work, we have presented some of the theoretical basis for generating an adaptive type-2 fuzzy input which is better able to represent the encountered uncertainty at a given measurement. The nonsingleton type-2 fuzzy inputs are dynamic and they are automatically generated from data and they do not assume a specific shape about the uncertainty distribution associated with the given sensor. In this paper, we will present an overview on how the adaptive type-2 input based nonsingleton interval type-2 FLS can operate in real time. We will present real world experiments using a mobile robot which will show how under high input uncertainty levels, the nonsingleton type-2 FLS can give a good performance and outperform its singleton type-2 and type-1 FLSs counterparts.

Keywords- *type-2 nonsingleton, interval type-2 fuzzy logic systems, uncertainty, measurement data, mobile robots*

I. INTRODUCTION

The Fuzzy Logic System (FLS) is credited with being an adequate methodology for dealing with the uncertainties and imprecision present in real world applications. While traditionally type-1 FLSs have been widely employed in real world applications, recent years have shown a rapidly growing interest in the research and application of type-2 FLSs. This is because, it has become apparent that type-1 FLSs cannot fully handle the large amounts of uncertainties encountered in real world environments and applications. Interval type-2 FLSs

have shown to provide better performance when compared to type-1 FLSs in applications with high levels of uncertainty. The difference in performance has been attributed to the nature of interval type-2 fuzzy sets which can better account for the uncertainties as they incorporate a Footprint of Uncertainty (FOU) and they can be assumed to embed a large number of type-1 fuzzy sets. However, the majority of the type-2 FLSs handle the linguistic and input numerical uncertainties using singleton interval type-2 FLSs that mix the numerical and linguistic uncertainties to be handled only by the linguistic labels type-2 fuzzy sets. Input numerical uncertainties are associated with the noise, imprecision and uncertainty associated with sensors and input devices. However, the linguistic uncertainties are associated with human words and perceptions. Hence, it seems paradoxical to use singleton type-2 FLSs to handle the input numerical uncertainties, as this ignores the fact that if input numerical uncertainties were present, they should affect the incoming inputs to the FLS. Thus we cannot treat the incoming FLS inputs as perfect signals as in the case of singleton FLSs. Hence, there is a need to employ nonsingleton FLSs to handle the input numerical uncertainties by modelling the inputs as fuzzy inputs rather than crisp values.

Although, there are many papers on type-1 FLSs, limited number of papers considered the usage of type-1 nonsingleton FLSs such as [1-6] and even fewer number considered type-2 nonsingleton FLS [7-11]. The work done so far in (type-1 and type-2) nonsingleton FLSs use predefined shapes for the uncertainty distribution affecting the FLSs inputs (mainly Gaussian and triangular) which might not reflect the real uncertainty distribution associated with the given sensor [12-14]. For example in the case of a sonar sensor, the numerical input uncertainties depend on many factors such as the shape and type of objects reflecting the sonar signal as well as wind speed, humidity and temperature. Moreover, the uncertainty distribution also varies with the measured values where for the case of a sonar sensor, the amount of uncertainty affecting measurement readings at 20cm distance could be much less than the uncertainties affecting the measurement at large distances such as 3m. Hence, it is not fair to consider that there

is specific shape for the uncertainty distribution (triangular, Gaussian, etc.) with fixed parameters (average, standard deviation, etc.) for all the measured values.

In our previous work [15], we have presented some of the theoretical basis for generating an adaptive type-2 fuzzy input which is better able to represent the encountered uncertainty at a given measurement. The nonsingleton type-2 fuzzy inputs are dynamic and they are automatically generated from data and they are piece-wise linear type-2 fuzzy sets which are changing for different measurements to model the encountered uncertainty. Piece-wise linear fuzzy sets were used in [16] for the representation of membership functions. However, we use piece-wise linear fuzzy sets to represent the type-2 fuzzy input variables. In this paper, we will present an overview on how the adaptive type-2 input based nonsingleton interval type-2 FLS can operate in real time. We will present real world experiments using a mobile robot which will show how under high input uncertainty levels, the nonsingleton type-2 FLS can give a good performance and outperform its singleton type-2 and type-1 FLSs counterparts.

Section II will give a summary of the structure of the real – time type-2 nonsingleton type-2 FLS. Section III will present the experiments and results. Finally Section IV provides conclusions and future work.

II. THE TYPE-2 NONSINGLETON TYPE-2 FLS

A. The Structure of Type-2 Nonsingleton Type-2 FLS

The structure of the type-2 nonsingleton type-2 FLS [15] is illustrated in Fig.1. The type-2 nonsingleton type-2 FLS employs interval type-2 fuzzy sets to handle the encountered linguistic uncertainty associated with the linguistic labels while the nonsingleton type-2 fuzzy input variables handle the encountered numerical input uncertainties.

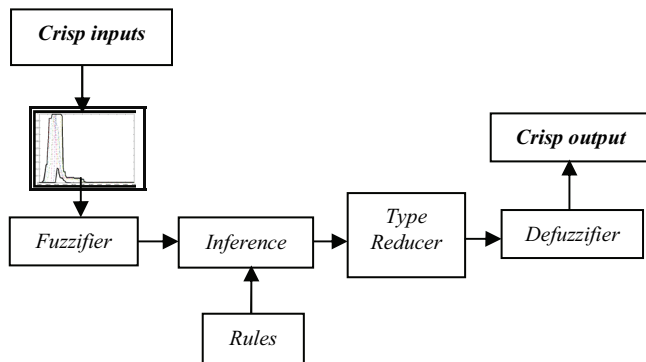


Figure 1. The structure of type-2 nonsingleton type-2 FLS.

B. Modeling Type-2 Fuzzy Input Variables

Fig.2 shows a high level overview on the steps taken to model the type-2 fuzzy input variables at given measurement values. The first step involves sampling the given input at different measurements by logging many readings at the same measurement value (for example for a sonar sensor, collecting 10,000 readings at 20cm, 40cm, etc.) which will create a data distribution (or a histogram). The data distributions are then converted into piece-wise linear type-1 fuzzy sets that are convex and normal as explained in [15] and [17]. We then

repeat the same process under various uncertainty conditions (for the sonar sensors, subjecting it to humidity, heat, wind, different surfaces, etc.) which will create different type-1 fuzzy sets for the different uncertainty conditions [15], [17]. We then combine the various type-1 fuzzy sets at the given measurement value to form a type-2 fuzzy set that models the uncertainty at the given measurement value.

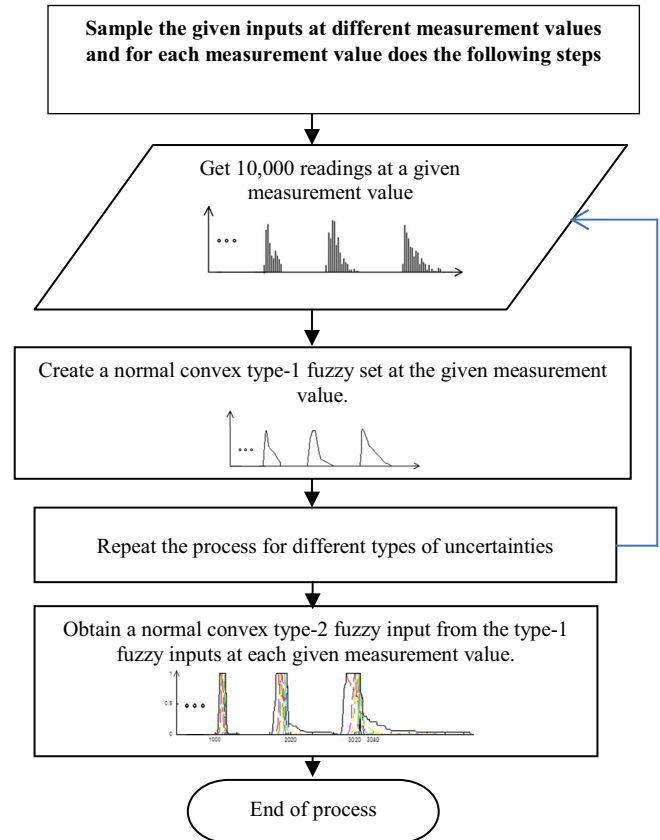


Figure 2. The off-line process of obtaining the type-2 fuzzy input at certain measurement values.

As we cannot generate the type-2 fuzzy sets for all the measurements values, hence, we need to interpolate the type-2 fuzzy set for an incoming measurement value by interpolating the type-2 fuzzy sets representing the measurements to the left and right of the incoming measurement value. For each control cycle (as shown in Fig.3), we have a measurement window which takes 5 consecutive crisp measurements from the sensor at the given instance and takes the average to form an incoming measurement value which interpolates type-2 fuzzy sets at the right and left of the incoming measurement to generate an interpolated type-2 fuzzy input as explained in [17]. The type-2 fuzzy input is then fuzzified and then it triggers the inference engine and the rule base to generate a type-2 fuzzy output which is then type-reduced and defuzzified to generate the final crisp output of the FLS.

Fig.4-6 present the real-world shapes of the nonsingleton type-2 fuzzy inputs generated at given measurements of a given sonar sensor at 1m, 2m, and 3m respectively. The top figures of each pair set of figures (Fig.4-Fig.6) are the type-2 fuzzy input variable for the front sonar sensor and the figures below are

related to back sonar sensor of the mobile robot shown in Fig.11. For each given measurement, we log 10,000 readings from the sensor. In the case of a sonar sensor for the robot shown in Fig.11, we keep the distance fixed and read 10,000 readings at the same fixed distance which will result in a data distribution or a histogram. Following the steps in Fig. 2, we then create a normal convex type-1 fuzzy set from the given data distribution, which will model the uncertainties at the given measurement conditions. We then apply various environmental conditions, which could affect the input measurements. As for the case of a sonar sensor, we can consider changing shapes and materials of the objects as well as changing temperature, sound noise and wind. Each condition will then have an associated type-1 fuzzy set which models the uncertainty at the given measurement (fixed distance). We then combine the various type-1 fuzzy sets to create a convex type-2 fuzzy set (as explained in [15] and [17]) which models the various uncertainties encountered at each measurement value.

The upper and lower membership functions of type-2 fuzzy input for the considered sonar sensors are shown by black lines in Fig.4-Fig.6, while the related type-1 fuzzy input for each case of uncertainty is shown by dashed lines. Fig.4-Fig.6 illustrate that not only the shape of type-2 fuzzy sets change for different range of distances, but also the sonar sensors may not be effected in a same way.

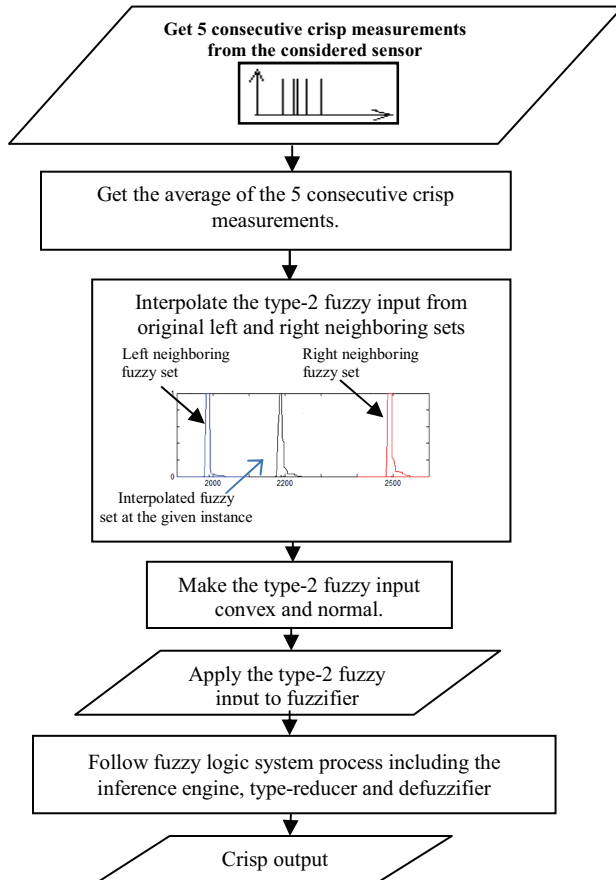


Figure 3. The process of the on-line nonsingleton type-2 FLS

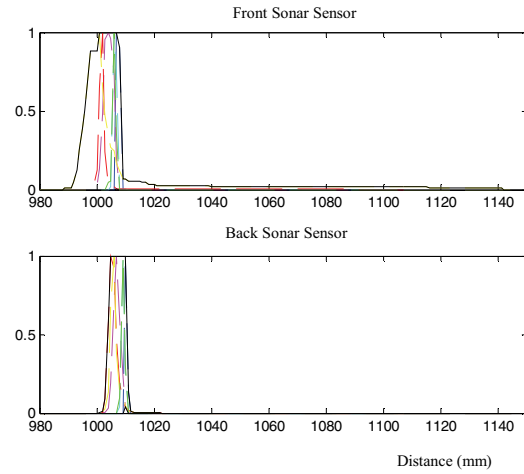


Figure 4. The generated input type-2 fuzzy sets for the front sonar sensor (top figure) and the back sonar sensor (bottom figure) of the robot in Fig.11 when the edge is set around 1 m from the robot.

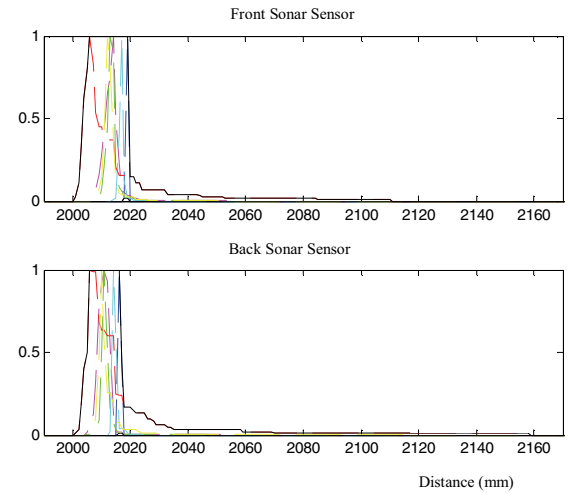


Figure 5. The generated input type-2 fuzzy sets for the front sonar sensor (top figure) and the back sonar sensor (bottom figure) of the robot in Fig.11 when the edge is set around 2m from the robot.

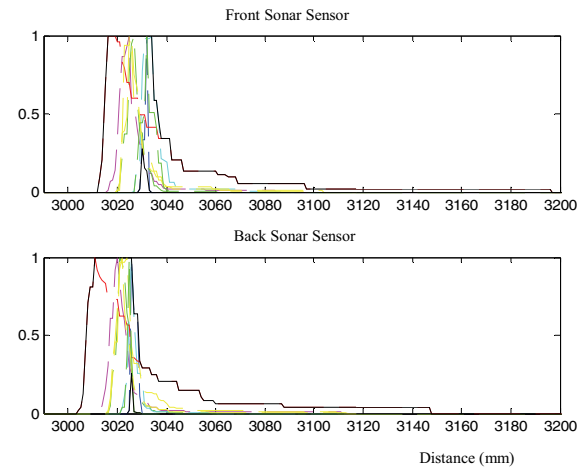


Figure 6. The generated input type-2 fuzzy sets for the front sonar sensor (top figure) and the back sonar sensor (bottom figure) of the robot in Fig.11 when the edge is set around 3m from the robot.

C. The Type-2 Nonsingleton Type-2 Fuzzification

The fuzzifier converts the fuzzy inputs into linguistic labels with given membership values to activate the rule base and inference engine. The fuzzification operation between the type-2 fuzzy input variables and the type-2 fuzzy sets assigned for linguistic labels (the antecedent type-2 fuzzy set specified by F_k^l) to obtain lower and upper membership values of input k within rule l are shown in Equations (1) and (2) respectively:

$$\bar{\mu}_k(x_k) = \sup_{x_k \in X_k} (\bar{\mu}_{x_k}(x_k) * \bar{\mu}_{F_k^l}(x_k)) \quad (1)$$

$$\underline{\mu}_k(x_k) = \sup_{x_k \in X_k} (\underline{\mu}_{x_k}(x_k) * \underline{\mu}_{F_k^l}(x_k)) \quad (2)$$

The $\bar{\mu}_{x_k}(x_k)$ and $\underline{\mu}_{x_k}(x_k)$ are the upper and lower membership functions of the type-2 fuzzy input variable and $\bar{\mu}_{F_k^l}(x_k)$ and $\underline{\mu}_{F_k^l}(x_k)$ are the upper and lower membership functions of the antecedent type-2 fuzzy sets (representing the linguistic labels) respectively. The sup operation has been considered as maximum t-conorm, and the $*$ operation is minimum t-norm. In [17], we described the theory of type-2 nonsingleton type-2 FLS, and how the fuzzification output is calculated for the upper membership functions. The process of finding the output of the fuzzifier follows Equations (1) and (2) to find the upper and lower membership values to the given respective linguistic label. Due to the limited space, in this subsection, we will concentrate on presenting the process followed to find the lower membership value to the given respective linguistic label. The process is quite complicated as the input type-2 fuzzy variable is a piece-wise linear function (as shown in Fig.7).

According to Equation (2), to get lower bound of the membership to the given respective linguistic label, we need to apply the t-norm (minimum operation) between the type-1 membership functions $\underline{\mu}_{x_k}(x_k)$ and $\underline{\mu}_{F_k^l}(x_k)$. This will result in a type-1 fuzzy set (shown by thick line in Fig.8 - Fig.10), where we need to find its sup (maximum) to find $\underline{\mu}_k(x_k)$.

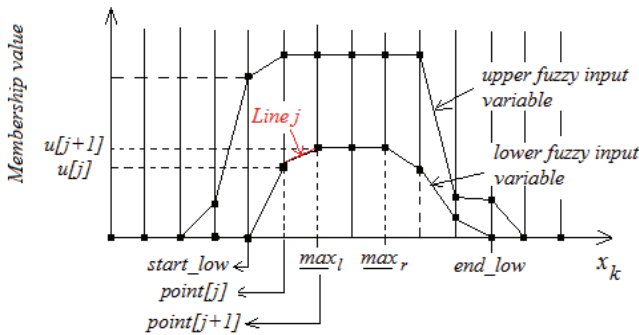


Figure 7. The piece-wise linear type-2 fuzzy input variable.

Due to the limited space in this paper, we will present a sample for the operation between the antecedent type-2 fuzzy set and the type-2 fuzzy input variable for the lower bound when the starting point *start_low* (shown in Fig.8 - Fig.10) (of the lower membership function for type-2 fuzzy input variable)

is smaller than d_{m2} (of the lower membership function antecedent type-2 fuzzy set) and the ending point *end_low* (of the lower membership function for type-2 fuzzy input variable) is greater than a_{m2} (of the lower membership function of antecedent type-2 fuzzy set).

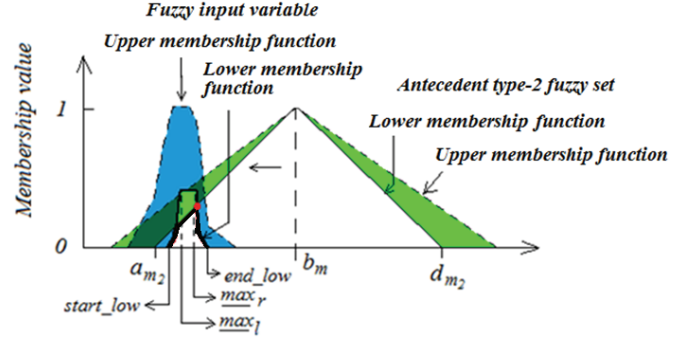


Figure 8. Finding $\underline{\mu}_k(x_k)$ for Case 1.

The case 1 in Fig.8 happens when the intersection between the lower membership function of the type-2 fuzzy input and the lower membership function of the antecedent is located on the antecedent line from b_m to a_{m2} , but the intersection does not happen on the maximum of the lower membership function of the type-2 fuzzy input. In that case, we need to obtain the first intersection point from right to left starting from the pieces of line j ending at *end_low*. As the slope of the antecedent line from b_m to a_{m2} is positive, the first intersection from right side has the highest membership value, which will be the result of max-min operation between the two fuzzy sets to lead to $\underline{\mu}_k(x_k)$.

The given line j (of fuzzy input) ranges from *point[j]* to *point[j+1]* in the x -axis line where $j+1$ is the line connecting to the line j and located at its right in the piece-wise linear function. The $u[j]$ and $u[j+1]$ are also membership value at *point[j]* and *point[j+1]* respectively. The membership value for any point x_k located on line j is obtained from Equation (3) as follows:

$$\begin{aligned} \frac{\mu_{x_k}(x_k) - u[j]}{x_k - \text{point}[j]} &= \frac{u[j+1] - u[j]}{\text{point}[j+1] - \text{point}[j]} \\ \mu_{x_k}(x_k) &= \frac{u[j+1] - u[j]}{\text{point}[j+1] - \text{point}[j]} x_k + \frac{\text{point}[j+1]u[j] - \text{point}[j]u[j+1]}{\text{point}[j+1] - \text{point}[j]} \quad (3) \end{aligned}$$

The membership value $\mu_{F_k^l}(x_k)$ of any point x_k on a line of the antecedent fuzzy set (starting from an extreme left point "left" (a_{m2} in our case) and ending at an extreme right point "right" (b_m in our case)) is obtained as follows:

$$\begin{aligned} \frac{\mu_{F_k^l}(x_k) - \mu_{F_k^l}(\text{left})}{x_k - \text{left}} &= \frac{\mu_{F_k^l}(\text{right}) - \mu_{F_k^l}(\text{left})}{\text{right} - \text{left}} \\ \mu_{F_k^l}(x_k) &= \frac{\mu_{F_k^l}(\text{right}) - \mu_{F_k^l}(\text{left})}{\text{right} - \text{left}} x_k + \frac{\text{right} \mu_{F_k^l}(\text{left}) - \text{left} \mu_{F_k^l}(\text{right})}{\text{right} - \text{left}} \quad (4) \end{aligned}$$

At the intersection point $x_{k,c}$ between an antecedent line and a line j of the fuzzy input variable, $\mu_{x_k}(x_k)$ in Equation (3) is equal to $\mu_{F_k^l}(x_k)$ in Equation (4). Hence $x_{k,c}$ could be found as follows:

$$x_{k,c} = \frac{\begin{bmatrix} (\text{right}\mu_{F_k^l}(\text{left}) - \text{left}\mu_{F_k^l}(\text{right}))(\text{point}[j+1] - \text{point}[j]) + \\ (\text{right} - \text{left})(\text{point}[j]u[j+1] - \text{point}[j+1]u[j]) \end{bmatrix}}{\begin{bmatrix} (\text{right} - \text{left})(u[j+1] - u[j]) + \\ (\mu_{F_k^l}(\text{left}) - \mu_{F_k^l}(\text{right}))(\text{point}[j+1] - \text{point}[j]) \end{bmatrix}} \quad (5)$$

Substituting with the value of $x_k = x_{k,c}$ in Equation (3) or (4), we can find $\underline{\mu}_k^l(x_k) = \mu_k^l(x_k)$.

The case 2 in Fig.9 happens when at least one intersection between lower membership function of the type-2 fuzzy input and the lower membership function of the antecedent happens on the maximum of the lower membership function of the type-2 fuzzy input (or it can happen when the fuzzy input variable is within a_{m2} to d_{m2} domain but no intersection occurs). In that case, $\underline{\mu}_k^l(x_k)$ is equal to the maximum membership value of the lower membership function of type-2 fuzzy input.

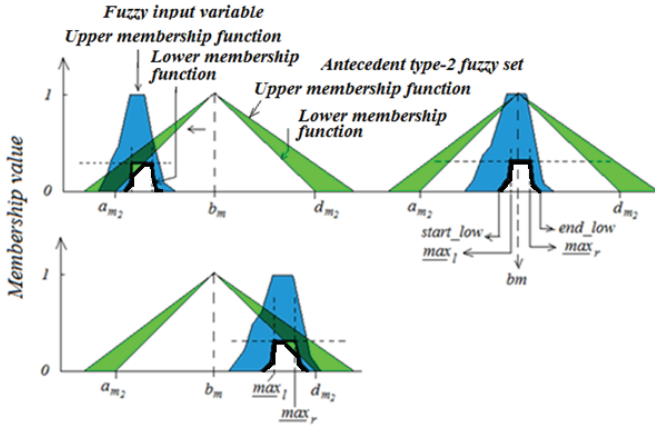


Figure 9. Finding $\underline{\mu}_k^l(x_k)$ for Case 2.

The case 3 in Fig.10 happens when the intersection between lower membership function of the type-2 fuzzy input and the lower membership function of the antecedent is located on the antecedent line from b_m to d_{m2} , but the intersection does not happen on the maximum of the lower membership function of the type-2 fuzzy input. In that case, we need to obtain the first intersection point from left to right side starting from the pieces of line j starting at start_low . As the slope of the antecedent line from b_m to d_{m2} is negative, the first intersection from left has the highest membership value. Hence $\underline{\mu}_k^l(x_k)$, could be found using the same approach followed in Equations (3 –5).

Following the same procedure the upper and lower membership values ($\underline{\mu}_k^l(x_k)$ and $\bar{\mu}_k^l(x_k)$) could be found for the various cases as explained in [17].

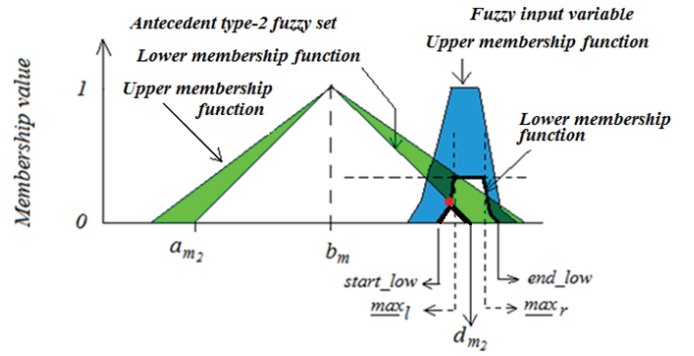


Figure 10. Finding $\underline{\mu}_k^l(x_k)$ for Case 3.

D. Rule Base

The rule base of singleton and nonsingleton FLS are similar. Considering a FLS having inputs $x_1 \in X_1, \dots, x_p \in X_p$ and one output $y \in Y$, the rule l is defined as [12]:

$$\text{If } x_1 \text{ is } \tilde{F}_1^l \text{ ... and } x_p \text{ is } \tilde{F}_p^l, \text{ then } y \text{ is } \tilde{G}^l \quad (6)$$

Where $l=1, \dots, M$ and M is the total number of rules in the rule base. The $\tilde{F}_k^l$ is the antecedent membership function for input k and rule l , and $\tilde{G}$ is consequent membership function for rule l .

E. Inference Engine

We have shown in the fuzzifier subsection, how to find the upper and lower membership values of a given type-2 fuzzy input variable to a given linguistic label. The process needs to be done for all inputs ($k=1, \dots, p$). At the end, the firing strength of each rule $F^l = [\underline{f}^l, \bar{f}^l]$ could be found where $\underline{f}^l$ and $\bar{f}^l$ will be obtained as follows:

$$\underline{f}^l = T_{k=1}^p \{\underline{\mu}_k^l(x_k)\} \quad (7)$$

$$\bar{f}^l = T_{k=1}^p \{\bar{\mu}_k^l(x_k)\} \quad (8)$$

F. Type-Reduction and Defuzzification

After obtaining the lower and upper firing strengths for each rule from the inference engine and obtaining the centroid of consequent membership function, the center of sets type-reducer [12] is used to produce the type-reduced sets. We then take the average of the type-reduced set to provide the final defuzzified crisp output.

III. EXPERIMENTS AND RESULTS

A. The Robots and the Experimental Setup

In our experiments, we have employed a four-wheel mobile robot (shown in Fig. 11) with two sonar sensors to follow an edge made of cylindrical objects. In order to test the robot under unseen input uncertainties, we have applied various sources of uncertainties to the input sonar sensors as shown in Fig.11a where we have used a hair dryer stuck on robot to blow hot air through the path of front sonar sensor. We have also employed a fan fixed at a certain position on ground to create cold wind at the starting robot position. These employed

uncertainties are introduced during the online operation of the robot and they have not been seen by the robot during the FLS design. We chose changing the wind speed (through the fan and dryer) and temperature (through the hair dryer) to introduce input uncertainties to the sonar sensors. This is because the wind, temperature variations and atmospheric turbulences are all meteorological conditions [18] that have an effect on sound propagation (as we used sonar sensors). The speed of sound in air is proportional to the square root of absolute temperature as follows:

$$c = 20.05\sqrt{T + e/p} \quad (9)$$

Where T is absolute temperature in Kelvin, and e is partial pressure of water vapor, and p is barometric pressure. Wind is also another factor affecting the sound travel. The wind in the same direction of sound will cause the wave propagation to be bent downward, and in the upwind direction, sound waves move upward which cause no direct sound penetration [18].

To obtain the position of the robot in a precise way, the VICON system is used in which six cameras are employed. To be able to recognize the robot, four markers are used (as shown in Fig.11). The top of the cylinders are also marked to be able to check if the edges of the cylinders are in a same line where the starting and ending cylinders are modeled in VICON to be able to distinguish the certain position of wall (one of them is shown in Fig.11b).

B. The FLSs Design

We have designed a type-1 FLS which is able to give a good performance under the normal lab conditions. The performance of the type-1 FLS is considered to be acceptable when it produces small overshoots/undershoots and small steady state error in a usual room temperature without applying external uncertainty. The antecedent membership functions for the type-1 FLS are shown in Fig.12 (drawn with thin line). We have then blurred the type-1 fuzzy sets by a given uncertainty value to generate the FOU of the type-2 fuzzy sets (shown in Fig.12 by thick line) for the interval (singleton and nonsingleton) type-2 FLS. The consequent membership functions (for all FLSs) are type-1 fuzzy sets as there are no major uncertainties associated with robot outputs. The rule-base (employed for all FLSs) is shown in Table I.

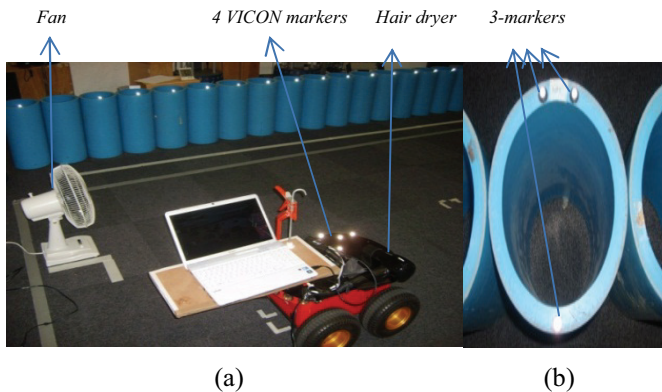


Figure 11. (a) The Experimental setup where the robot is marked to be tracked by VICON camera system in robot arena. (b) One of the cylindars modeled in VICON system using three markers to know the exact position of wall.

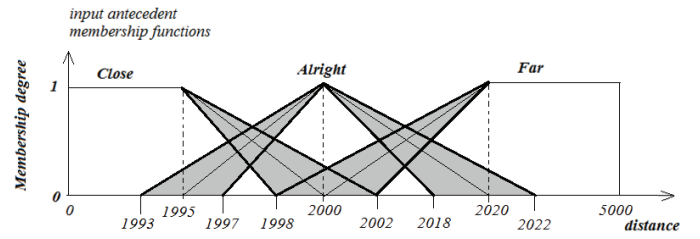


Figure 12. Antecedent membership functions for both the front and back sensors. The type-1 fuzzy sets are shown in thin line while the interval type-2 fuzzy sets are shown in thick lines.

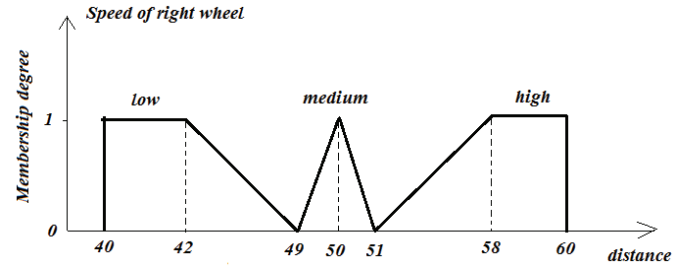


Figure 13. The consequent type-1 fuzzy sets for the used FLSs

TABLE I. THE RULE BASE FOR ALL THE FLSs. THE RULE BASE OUTPUT IS THE SPEED OF RIGHT WHEEL.

Front sonar	Back sonar		
	Close	Alright	Far
Close	low	low	low
Alright	low	medium	low
Far	high	high	high

C. Control Surface Analysis

The control surface graphically represents the unknown function articulated by the FLS. Fig. 14 shows the control surfaces for the singleton type-1 and type-2 FLSs and the type-2 nonsingleton type-2 FLS. The control surfaces show the front and back sensors plotted against the speed of right wheel. Note that a smooth shape of a control surface translates to a smooth control response that can deal with uncertainty and imprecision.

As seen from Fig.14a, the control surface of the type-1 FLS is rather steep which means it can be affected by high levels of uncertainties in the inputs where small change in the inputs can result in large output changes which might cause high overshoots/undershoots and steady state errors in addition to moving towards an unstable situation. On the other hand the control surface of the singleton type-2 FLS (shown in Fig.14b) is smoother than the type-1 FLS, which means it can handle higher level of uncertainties. However, the control surface of the type-2 nonsingleton type-2 FLS (shown in Fig.14c) is much smoother where there are smooth transition at all parts of the control surface where any small change in inputs will

result in a small change in the outputs. Thus using the type-2 input based nonsingleton type-2 FLS result in a control surface which is smoother than the singleton type-1 and type-2 FLSs. This smooth response will consequently give a very good control performance that can handle the high levels of uncertainties and disturbances where small variations in inputs will not cause significant changes to outputs. Thus, it is expected that the type-2 nonsingleton type-2 FLS will be able to handle higher levels of uncertainties than its singleton type-1 and type-2 FLSs counterparts. This will be evidenced in the real world experiments reported in the following subsection.

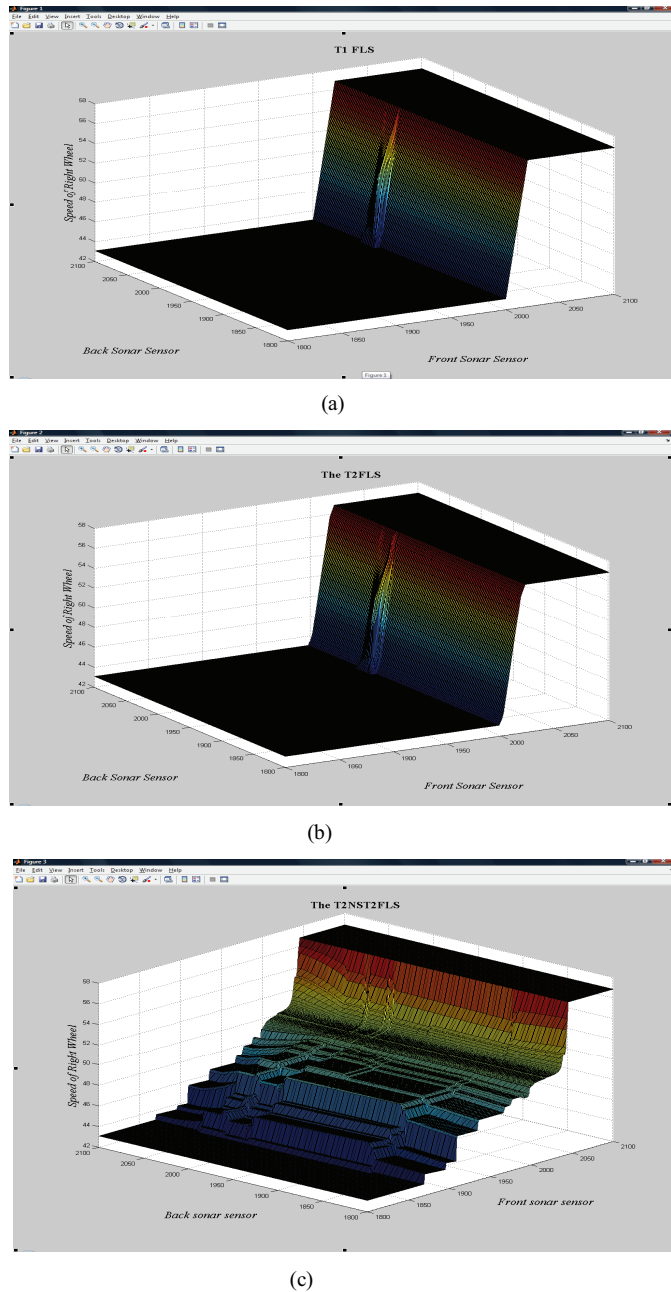


Figure 14. The control surface of (a) The singleton type-1 FLS. (b) The singleton interval type-2 FLS. (c) The type-2 nonsingleton interval type-2 FLS.

D. Real World Experiments

Fig.15 and Fig.16 show the results of some robotic experiments, which compare the performance of singleton type-1, singleton type-2 and type-2 input based nonsingleton type-2 FLS to realize an edge following behavior at a desired distance of 2m. The robot starts from the distance of 1.8m from the nearest edge. Fig. 15 shows the robots paths of singleton type-1, type-2 and nonsingleton type-2 FLS under no induced uncertainty. As shown in Fig.15 and Table II (as expected under no uncertainty), the performance of all the FLSs will be quite similar where the type-1 FLS will be slightly better. However, as high levels of uncertainties are introduced (by blowing hot wind through the way of one sonar) as shown in Fig.16 and Table II, the type-2 nonsingleton type-2 FLS gives the best performance in terms of the Root Mean Squared Error (RMSE) (which could represent the average error) and the Standard Deviation (STD) (which could represent the size of the overshoots/undershoots from the average). In terms of control performance, the type-2 nonsingleton type-2 FLS provides the best performance followed (by a large margin) by the singleton type-2 FLS and then the singleton type-1 FLS which performs poorly.

Fig.16 evidences the control surface analysis in Fig.14 where the type-2 nonsingleton type-2 FLS recovers quickly from the uncertainties with relatively smaller overshoot/undershoot.

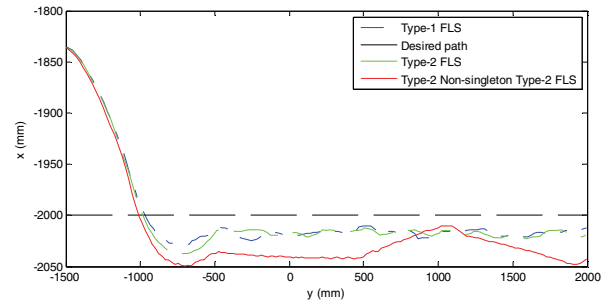


Figure 15. The robot's paths when employing singleton type-1, type-2 and nonsingleton type-2 FLS under no uncertainty.

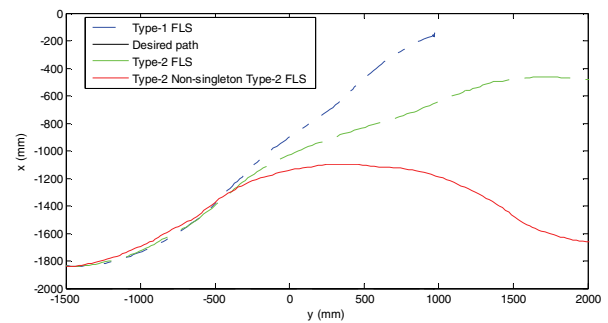


Figure 16. The robot's paths when employing singleton type-1, type-2 and nonsingleton type-2 FLS under high uncertainty levels.

TABLE II. SUMMARY RESULTS OF WALL FOLLOWING FOR THE THREE TYPES OF FLSs.

Different Conditions	Different Types of FLS		
	Type-1 FLS	Type-2 FLS	Type-2 nonsingleton Type-2
No uncertainty	STD =45.7 RMSE=45.6	STD =46.7 RMSE=46.6	STD =48.1 RMSE=51.2
Large uncertainty	STD =626.7 RMSE=1268.2	STD=478.7 RMSE=1037.8	STD =245.0 RMSE=634.4

IV. CONCLUSION

In this paper, we have presented an adaptive type-2 fuzzy input based nonsingleton type-2 FLS which is better able to handle the encountered input uncertainties. The nonsingleton type-2 fuzzy inputs are dynamic and they are automatically generated from data and they do not assume a specific shape about the uncertainty distribution associated with the given sensor. We have presented an overview on how the adaptive type-2 input based nonsingleton interval type-2 FLS can operate in real time. We have shown through control surface analysis, how the nonsingleton type-2 FLS can produce much smoother control surface than the singleton type-1 and type-2 FLSs. This should result in a better control response that handles high uncertainty levels. This was evidenced through the real world experiments using mobile robots where under high uncertainty levels, the nonsingleton type-2 FLS can outperform its singleton type-2 and type-1 FLSs counterparts. This work has shown the benefits achieved when using type-2 nonsingleton type-2 FLSs in applications characterized by high input uncertainty levels. In our future work, we aim to allow the various parameters of the type-2 nonsingleton type-2 FLSs to be learnt directly from data online.

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