

Generalized Projective Synchronization of Time-Delayed Chaotic Systems via Sliding Adaptive Fuzzy Control

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Abstract— In this paper, generalized projective synchronization (GPS) of two time-delayed chaotic systems using sliding adaptive fuzzy control (SAFC) is investigated. The proposed method combines the advantages of the adaptive control, fuzzy systems and sliding mode control theory. Lyapunov stability theory is employed to guarantee the stability of error dynamics. Assuming that the parameters of the chaotic drive system are unknown, recursive least square (RLS) method is applied to estimate these unknown parameters. Simulation results show the validity and effectiveness of the proposed method for synchronization of two identical and nonidentical time-delayed chaotic systems.

Keywords: Generalized Projective Synchronization, Time-Delayed Chaotic Systems, Adaptive Fuzzy Control

I. INTRODUCTION

Since Pecora and Carroll observed the synchronization of two identical chaotic systems under different initial conditions in 1990 [1], chaos synchronization has attracted many interests, especially in secure communication. Moreover, it has been widely explored in a variety of fields such as physical, chemical and ecological process [2]. In previous researches, different types of synchronization have been found in interacting chaotic systems, such as complete synchronization (CS) [1,3,4,5,6], projective synchronization (PS)[2], generalized synchronization (GS) [7,8,9,10], phase synchronization [11,12], anti-phase synchronization [13,14,15] and so on. In PS, the drive-response system synchronizes up to a constant scaling factor α . CS is a special case of PS at $\alpha = 1$, as well as anti-phase synchronization at $\alpha = -1$ [16].

Many various synchronization methods such as backstepping design [17], active control [18], nonlinear control [19], adaptive control [20] and adaptive fuzzy control [21] have been applied for chaos synchronization in low-

dimensional systems such as Lorenz and Duffing systems [22]. Since low-dimensional chaotic signals can be easily detected, they are not suitable for applications such as secure communications [22]. But, because of the infinite-dimensional of time-delayed chaotic systems, they are safety for secure communications.

Since Mackey and Glass first observed chaos in time-delayed system [23], many researchers interested in time-delayed chaotic systems. After that, the synchronization of time-delayed chaotic systems is also investigated by many researchers.

A few works have been done on synchronization of time-delayed chaotic systems. The problem of CS [24,25] and GS [26] of time-delayed systems have been studied for some special models. PS has been proposed to synchronize identical Logistic delayed models [27]. Fuzzy delayed output feedback synchronization for time-delayed chaotic systems is studied, too [28]. Sliding mode controller is applied for projective synchronization of time-delayed systems in [29].

In this paper, sliding adaptive fuzzy control (SAFC) is proposed for generalized projective synchronization (GPS) of two identical and nonidentical time-delayed chaotic systems. Furthermore, synchronization of time-delayed chaotic systems with unknown parameters is studied. It is shown that the speed of synchronization is appropriate and the performance of the proposed controller is well when the response system designed with the estimated parameters of the drive system.

This paper is organized as follows: in section 2, the model of time-delayed chaotic systems and its properties are described. The structure of proposed controller constitutes section 3. Stability analysis is studied in section 4. Simulation

results and conclusion are given in section 5 and 6, respectively.

II. MODEL OF SYSTEM

A class of time-delayed systems is considered as follows:

$$\dot{x}(t) = -ax(t) + f_b(x(t-\tau)) + u(t) \quad (1)$$

where, $x, u \in R$ are the state variable and control input, respectively. a, b, τ are the system parameters, $\tau > 0$. Time-delayed system in (1) can show chaotic behavior when the nonlinear function $f_b(\cdot)$ satisfies the following conditions [30,31]:

- (i) $f_b(0) = 0$;
- (ii) $\forall x \quad xf_b(x) \geq 0$;
- (iii) $\exists x_{eq} > 0 \quad f_b(x_{eq}) = x_{eq}$;
- (iiii) $\exists r \in (0, x_{eq}) : \forall x \quad f_b(x) \leq f_b(r) = M, M > 0$.

Some time-delayed chaotic systems satisfied above conditions are Mackey-Glass, Ikeda and Logistic models. These models are given as

Mackey-Glass Model [30]

$$f_b(z) = \frac{bz}{1 + |z|^C} \quad (C > 1) \quad (2)$$

Ikeda model [30]

$$f_b(z) = \begin{cases} b \sin(z) & |z| \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Logistic model [31]

$$f_b(z) = \begin{cases} bz(1-z) & 0 \leq z \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

In this paper, Mackey-Glass and Logistic models are studied.

III. GENERALIZED PROJECTIVE SYNCHRONIZATION VIA SLIDING ADAPTIVE FUZZY CONTROL

In this section, SAFC is designed to chaos synchronization of time-delayed chaotic systems.

Now, the drive and response systems are considered as follows:

$$\text{Drive: } \begin{cases} \dot{x}(t) = -ax(t) + f_b(x(t-\tau)) \\ x(t) = \varphi_1 \quad -\tau \leq t \leq 0 \end{cases} \quad (5)$$

$$\text{Response: } \begin{cases} \dot{y}(t) = -ay(t) + g_\beta(y(t-\theta)) + u \\ y(t) = \varphi_2 \quad -\theta \leq t \leq 0 \end{cases} \quad (6)$$

where $f_b(\cdot)$ and $g_\beta(\cdot)$ have the properties described in section 2. a, b, β are systems parameters, and τ, θ are time delays. φ_1, φ_2 are initial values of drive and response systems, respectively. Notice that, if synchronization of two identical systems is considered, then $f = g$. Also f is not equal to g for synchronization of two nonidentical systems. For GPS, the synchronization error is defined as follows:

$$e = \alpha x - y \quad (7)$$

where α is the scaling factor of GPS, then

$$\dot{e} = -\alpha ax(t) + ay(t) - g_\beta(y(t-\theta)) + \alpha f_b(x(t-\tau)) - u \quad (8)$$

It is assumed that $f_b(\cdot)$ and $g_\beta(\cdot)$ are bounded and α is known. The control signal is defined as

$$u = u_1 + u_2 \quad (9)$$

Where u_2 is the output of fuzzy controller and u_1 is determined as follows

$$u_1 = -\alpha ax(t) + ay(t) - g_\beta(y(t-\theta)) + \alpha f_b(x(t-\tau)) \quad (10)$$

To design the proposed controller u_2 , GD is applied to train the parameters of fuzzy controller. In the proposed method, minimizing the first derivation of Lyapunov function is considered, so that the stability is achieved. This derivation is applied as a cost function to determine unknown parameters utilized to design of the control system.

Sliding surface is defined as follows:

$$s = e = \alpha x - y \quad (11)$$

The fuzzy system used in this study has product inference engine, singleton fuzzifier, center average defuzzifier, and Gaussian membership function. So, the fuzzy system is given as

$$u_2 = \frac{\sum_{l=1}^E w^l \left[\frac{\prod_{i=1}^n \exp(-(\frac{s-c_i^l}{\sigma})^2)}{\sigma} \right]}{\sum_{l=1}^E \left[\frac{\prod_{i=1}^n \exp(-(\frac{s-c_i^l}{\sigma})^2)}{\sigma} \right]} = \hat{W}^T \phi \quad (12)$$

where $\hat{W} = [w^1, \dots, w^n]^T$, E is fixed, w^l , c_i^l and σ are free parameters. $s \in R^n$ is the input of the fuzzy system, and $u_2 \in R$ is the output of the fuzzy system. w^l is the center of fuzzy set. Parameters c_i^l and σ are centers and radius of basis function, respectively. In this paper, c_i^l and σ are chosen arbitrarily. Since this fuzzy system is considered with one input, n is equal to 1. To design the controller, the parameter w^l must be determined based on the reaching condition, $\dot{s} < 0$. Thus, the GD method is applied to determine it as follows:

$$\dot{w}^l = -\gamma \frac{\partial s(t) \dot{s}(t)}{\partial w^l(t)} \quad (13)$$

Equation (13) can be rewritten as

$$\begin{aligned} \dot{w}^l &= -\gamma \frac{\partial s(t) \dot{s}(t)}{\partial w^l(t)} = -\gamma \frac{\partial s(t) \dot{s}(t)}{\partial u(t)} \frac{\partial u(t)}{\partial w^l(t)} \\ &= -\gamma \frac{\partial s(t) \dot{s}(t)}{\partial u_2(t)} \frac{\partial u_2(t)}{\partial w^l(t)} \\ &= \gamma s(t) \frac{\exp(-(\frac{s-c^l}{\sigma})^2)}{\sum_{l=1}^E \exp(-(\frac{s-c^l}{\sigma})^2)} = \gamma s(t) \phi_l(s) \end{aligned} \quad (14)$$

where γ is the learning rate parameter. Finally, w_i can be on-line adjusted as it is shown in (14).

Now, with defined control signal the synchronization is achieved. In the next section, the stability of the control system is investigated.

IV. STABILITY ANALYSIS

Lyapunov stability analysis is applied to prove the stability of error dynamics. From the theory of sliding mode control and (8), a perfect control law is obtained as

$$u_{eq} = -\dot{e} + \dot{s} + \lambda s \quad (15)$$

where is presented in (11).

First, to analyze the stability, (9) and (10) are substituted in (8):

$$\dot{e} = -u_2 \quad (16)$$

If u_2 is substituted with u_{eq} then $\dot{s}(t) + \lambda s(t) = 0$. Since λ is a positive constant parameter, the sliding surface variable s will converge to zero, gradually. So, $\dot{e}$ will converge to zero. From (15) and (16):

$$\dot{s}(t) = -\lambda s(t) + (u_{eq} - u_2) \quad (17)$$

Theorem [32]. Suppose that the input universe of discourse U is a compact set in R^n . Then, for any given real continuous function $g(z)$ on U and arbitrary $\xi > 0$, there exists a fuzzy system $F(z)$ in the form of (10) such that

$$\sup_{z \in U} \|g(z) - F(z)\| \leq \xi$$

That is, the fuzzy systems with product inference engine, singleton fuzzifier, center average defuzzifier, and Gaussian membership functions are universal approximators.

Therefore, the control law u_2 can be obtained via the optimal weighting vector $\bar{W}$ of the fuzzy system. The designed control is applied to estimate the u_{eq} . This approximation can be achieved with the error which is smaller than ξ

$$\max |\bar{u}(x, \bar{W}) - u_{eq}(x)| < \xi$$

where $\bar{u}(x, \bar{W}) = \sum_{k=1}^n \bar{w}_k \phi_k = \bar{W}^T \phi$ and $u_{eq}(x) = \bar{W}^T \phi + \xi$.

Define $\tilde{W} = \bar{W} - \hat{W}$ as the weighting error, and then (17) can be rewritten as

$$\dot{s}(t) = -\lambda s(t) + (\tilde{W}^T \phi + \xi) \quad (18)$$

Notice that $\bar{W}$ and $\hat{W}$ are the optimal and estimated values of W , respectively.

Define the Lyapunov function as follows:

$$V(s, \tilde{W}) = \frac{1}{2} s^2 + \frac{1}{2\gamma} \tilde{W}^T \tilde{W} \quad (19)$$

The first derivation of (19) with respect to time is obtained as

$$\dot{V} = s\dot{s} + \frac{1}{\gamma}\tilde{W}^T\dot{\tilde{W}} = s\dot{s} + \frac{1}{\gamma}\tilde{W}^T(-\dot{\tilde{W}}) \quad (20)$$

Substituting (14) and (18) in to (20), then

$$\dot{V} = s(-\lambda s + \tilde{W}^T\phi + \xi) + \frac{1}{\gamma}\tilde{W}^T(-\gamma s\phi) = -\lambda s^2 + s\xi \quad (21)$$

Thus

$$\dot{V} \leq |s|(-\lambda|s| + \xi) \quad (22)$$

So, if $|s| \geq \xi/\lambda$, then $\dot{V} \leq 0$ which means that this Lyapunov function will decrease gradually. It can be concluded that the error of state variable will converge into a boundary layer ξ/λ around $e = s = 0$ in the phase plane.

Hence, the SAFC is stable and the error of state variable will converge into a small value.

V. SIMULATION RESULTS

A. Synchronization of two identical systems with known parameters

To synchronize two identical Mackey-Glass systems, the parameters of drive and response systems are chosen as $a = 1, b = 2, \phi_1 = 0.1, \phi_2 = 0.01, \tau = 10, \theta = 10$, and it is assumed that $\alpha = 0.5$. Fig. 1a,b shows the state trajectory of drive and response systems. Fig. 1c shows the phase plane ($\alpha x, y$) and Fig. 1d shows the synchronization error.

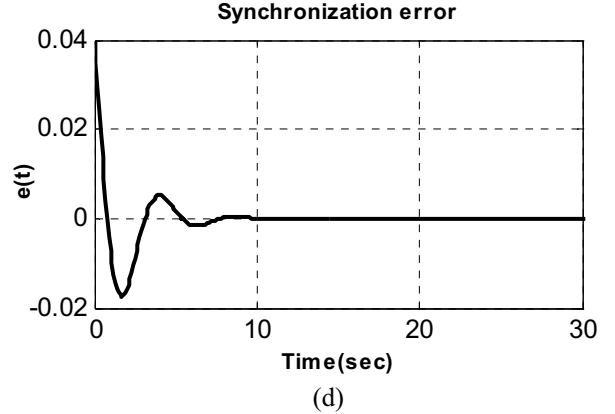
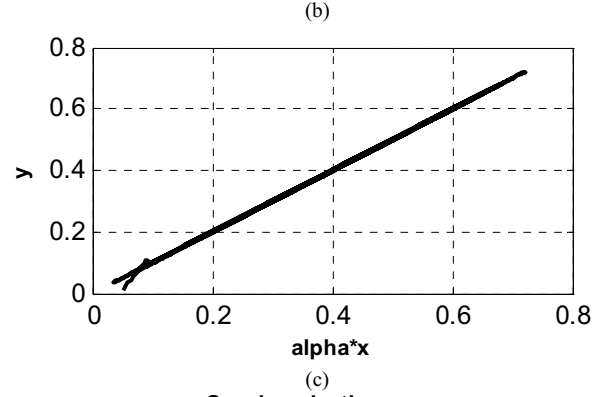
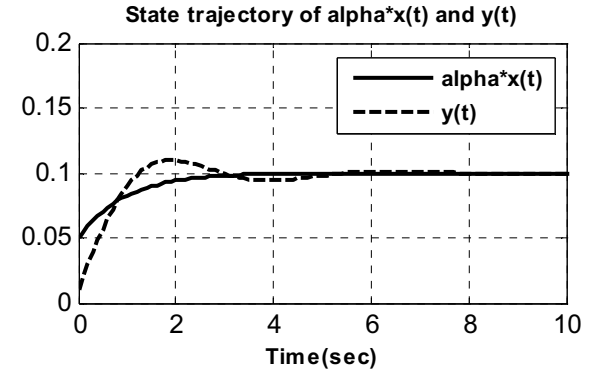
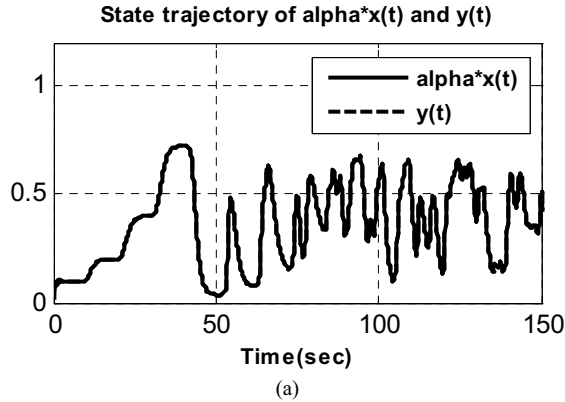


Figure 1. The state trajectory of Mackey-Glass system: (a, b) drive and response systems; (c) the phase plane; (d) the synchronization error.

B. Synchronization of two nonidentical systems with known parameters

In this section, Mackey-Glass and Logistic models are chosen as drive and response systems, respectively. The parameters of two considered systems are $a = 1, b = 2, \phi_1 = 0.1, \phi_2 = 0.01, \tau = 20, \theta = 20$, and it is assumed that $\alpha = 0.5$. Fig. 2a,b shows the state trajectory of drive and response systems, Fig. 2c shows the phase plane ($\alpha x, y$) and Fig. 2d shows the synchronization error.

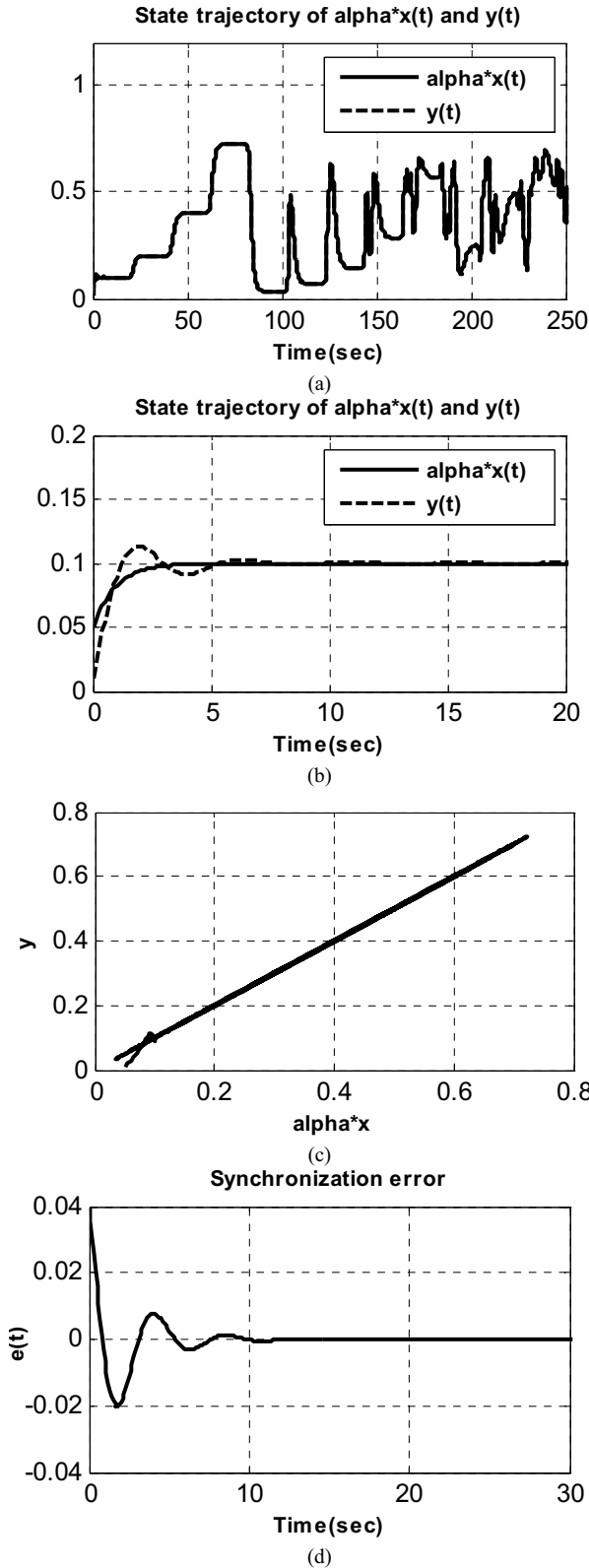


Figure 2. The state trajectory of Mackey-Glass and Logistic systems: (a, b) drive (Mackey-Glass) and response (Logistic) system; (c) the phase plane; (d) the synchronization error.

C. Synchronization of two identical systems with unknown parameters

In this section, synchronization is studied when the drive system has unknown parameters with known nonlinear part. To design a response system, the estimated parameters of the drive system is used. Mackey-Glass model is chosen as drive and response systems. The parameters of drive system are $a = 1, b = 2, \varphi_1 = 0.1, \tau = 10$, and for response system, the parameters are considered as $\varphi_2 = 0.01, \theta = 10$ and it is assumed that $\alpha = 0.5$.

The recursive least square (RLS) method is applied to estimate $\hat{a}, \hat{b}$. Note that the estimated parameters affect the $s(t)$. The initial values of $\hat{a}, \hat{b}$ are considered as 0.1 and 0.5, respectively. Fig. 3a,b shows the state trajectory of drive and response systems, Fig. 3c shows the phase plane $(\alpha x, y)$, and Fig. 3d shows the synchronization error. The convergence of parameters $\hat{a}, \hat{b}$ to their actual values are shown in Figs. 5a and 5b.

In this section, the performance of SAFC for synchronization of delayed chaotic systems is compared with fuzzy model-based adaptive synchronization of time-delayed chaotic systems proposed in [22]. In this method, to design the fuzzy controller, the chaotic system is modeled by Takagi Sugeno fuzzy system considering the properties of the nonlinear part of the system [22].

Define $h_1(x)$ and $h_2(x)$ as

$$h_1(x) = \begin{cases} \frac{f(x)}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} \quad (23)$$

$$h_2(x) = 1 - h_1(x) \quad (24)$$

The adaptation laws for $\hat{a}_i$ and $\hat{b}_i$ are chosen as

$$\dot{\hat{a}}_i = -\frac{h_1 e(t) y(t)}{p_i} \quad (25)$$

$$\dot{\hat{b}}_i = \frac{h_i e(t) y(t - \tau)}{q_i} \quad (26)$$

where $p_i, q_i > 0$ (Vasegh and Johari Majd, 2009).

The parameters of Mackey-Glass model with the both controllers are assumed as it is considered in this section. Furthermore, we choose $p_i = q_i = 1/220$, $a_s = 10$. The initial values of $\hat{a}, \hat{b}$ are chosen as 0.1 and 0.5, respectively. Simulation results are shown in Figs.4 and 5. From Figures.3,4 and 5, it is concluded that SAFC is better than the other considered method for chaos synchronization of two identical systems when the drive system has unknown parameters. The convergence of parameters with SAFC is faster and smoother than that with fuzzy model-based adaptive control. In summary, it is concluded that the speed of synchronization with the proposed method is better than that with fuzzy model-based adaptive control and it is an advantage of SAFC. Notice that the speed of synchronization with SAFC can be controlled and improved.

For better comparison, the performance of two considered controllers is compared with the following criteria:

1. Synchronization error variance after 10τ (EV).
2. Synchronization error settling time. It is the time at which $|e| < 0.005$ and e remains in this bound (EST).

As we know, the less the EV, the better synchronization is achieved. The less the EST, the sooner is the convergence. The results of these comparisons are given in Table 1.

It is resulted that SAFC performs better than fuzzy model-based adaptive control. Notice that the best performance of fuzzy model-based adaptive control was considered.

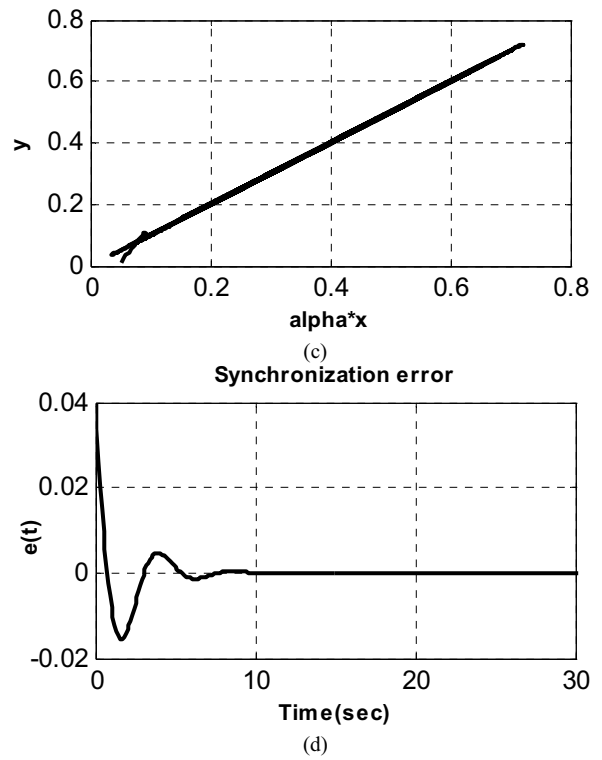
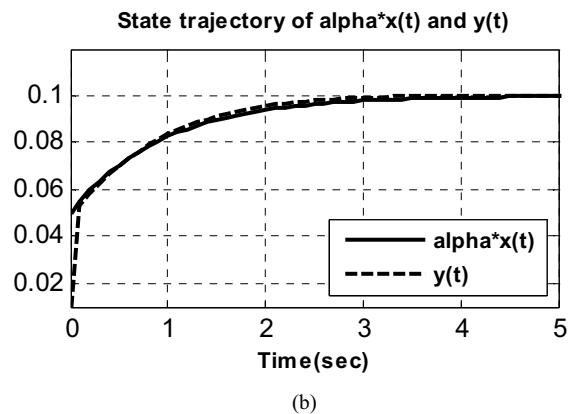
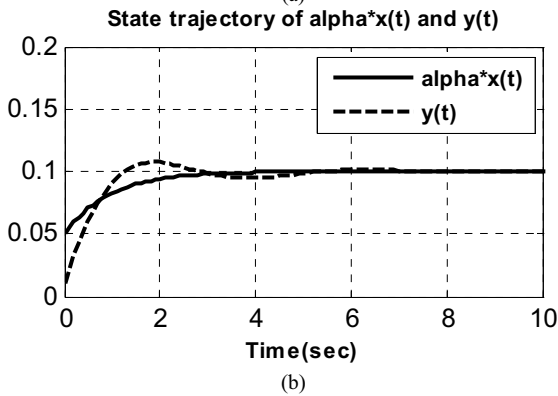
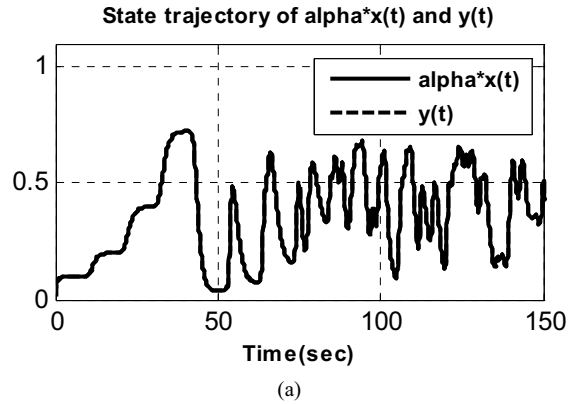
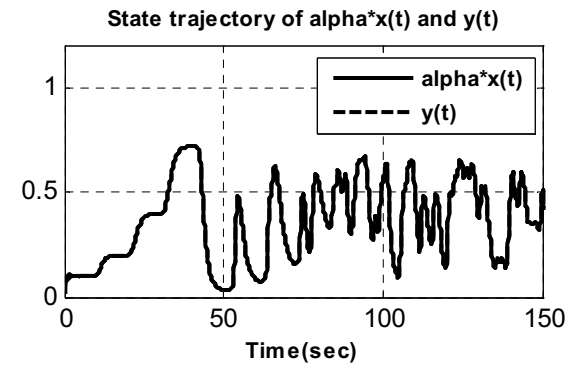


Figure 3. The state trajectory of two identical Mackey-Glass systems with SAFC: (a, b) drive and response systems; (c) the phase plane; (d) the synchronization error.



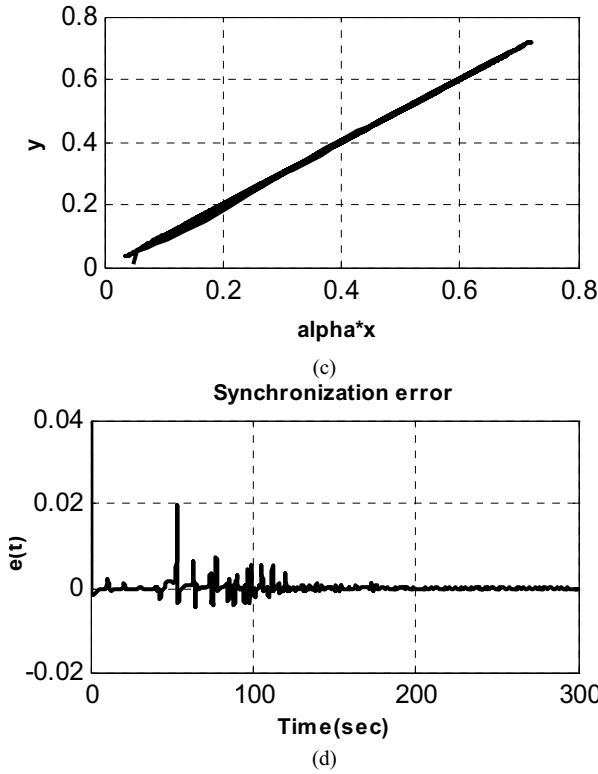


Figure 4. The state trajectory of two identical Mackey-Glass systems with FMBAC: (a, b) drive and response systems; (c) the phase plane; (d) the synchronization error.

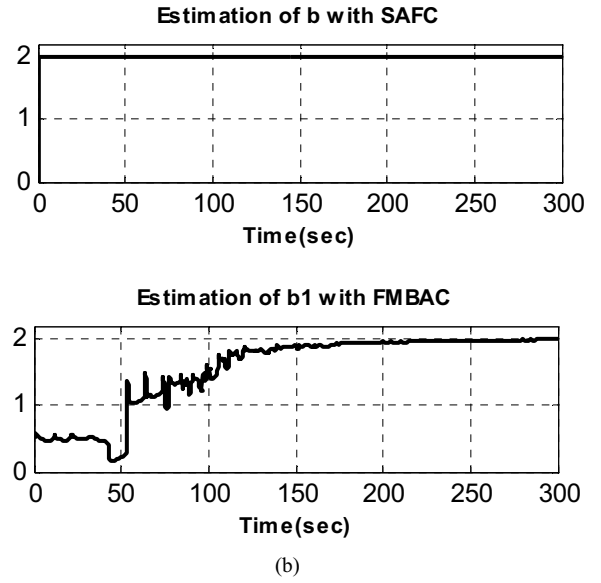
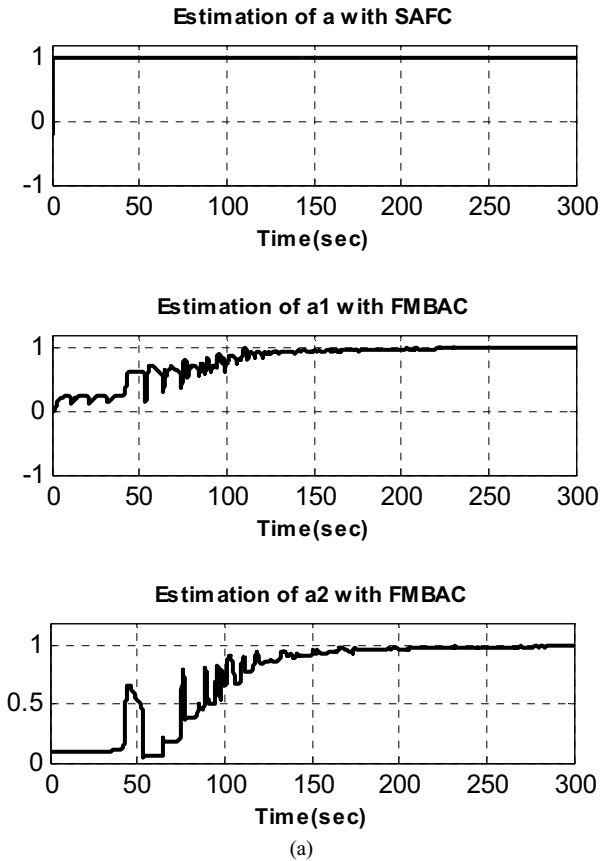


Figure 5. Estimation of parameters: (a) $\hat{a}, \hat{a}_1, \hat{a}_2$ (b) $\hat{b}, \hat{b}_1$ for Mackey-Glass model.

TABLE I. COMPARISONS BETWEEN DIFFERENT METHODS OF SYNCHRONIZATION

Criterion	Control Method	
	SAFC	FMBAC
EV	1.21e-15	2.18e-7
EST(sec)	2.7	112.1

VI. CONCLUSION

In this paper, an effective control method was used for synchronization of scalar time-delayed chaotic systems. Using the Lyapunov stability theory, a sliding mode controller based on the fuzzy system was applied for synchronization of two identical and nonidentical chaotic systems. When synchronization with unknown parameters was considered, the drive system had unknown parameters with known nonlinear part. To design a response system, the estimated parameters of the drive system was used. Numerical simulations for Mackey-Glass and Logistic models are provided to show the effectiveness and feasibility of the proposed controller. The convergence of parameters to their actual values with SAFC is faster than that with FMBAC. These results are obtained from comparisons of these two methods with two criteria which show the quality of the performance of the control system.

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