

Robust Stabilization For Continuous Fuzzy Systems With Time Varying Delay

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Abstract—This paper focuses on the problem of robust control for Takagi-Sugeno (T-S) fuzzy systems with time-delay. The less conservative conditions are developed by considering a parameter dependent Lyapunov-Krasovskii functional and slack matrix variables. The design conditions are formulated as an LMI (linear matrix inequality) feasibility problem, that can be efficiently solved using convex optimization numerical techniques. Numerical examples are presented to illustrate the effectiveness of the proposed approach with the comparisons to other conditions from the literature.

Index Terms—Takagi-Sugeno (T-S), Stability, Stabilization, Linear Matrix Inequalities (LMIs), Fuzzy Lyapunov-Krasovskii function(LKF)

I. INTRODUCTION

In the last two decades, the stability analysis and control design of nonlinear systems described by Takagi-Sugeno (T-S) fuzzy model with time delay have been extensively studied (see [2], [4], [5], [7], [8], [12], [13], [14], [16], [18]) and references therein. Two main reasons are contributed to this fact : 1) Ability of T-S fuzzy models [17] to approximate large class of nonlinear systems; 2) The existence of delays in many industrial processes such as transportation systems, communication systems, chemical processing systems, environmental systems and power systems ... and its consequences on the stability and the performance degradation propagation/transportation of material, information, or energy [15].

Stability analysis and control design for T-S systems have been widely and systematically handled in the literature via Linear Matrix Inequalities (LMIs) approach (see [3], [9]). In this framework, several strategies have been used to overcome mathematical and numerical difficulties, promoting less-conservative conditions. The standard quadratic Lyapunov approach is the most popular one with the use of slack matrix variables into the LMI formulation [9], [10]. As an interesting alternative type of Lyapunov function, the fuzzy weighting-dependent Lyapunov function (FWLF) has been addressed for continuous-time fuzzy systems in [11].

In the context of stabilization problem of delayed systems, a very effective strategy consists on the use of the multiple Lyapunov-Krasovskii functional approach based on bounding techniques. It is well known that some weighted cross products may arise in the analysis of the delay-dependent stabilization

problem. Thus, getting better bounds of them will play a key role in reduction conservatism.

Therefore, in this paper, new less-conservative conditions for stabilization of TS fuzzy systems subject to uncertainties and time delay are proposed.

This paper is organized as follows: system and some preliminaries are presented in Section 2. Section 3 contains the main result where a new delay dependent stabilization conditions are established. In Section 4, two numerical examples are given to demonstrate the effectiveness and the benefits of the proposed method. Some conclusions are drawn in Section 5.

Notations: $W + W^T$ is denoted as $Sym(W)$ for simplicity.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider a nonlinear system with state-delay which could be represented by a (TS) fuzzy time-delay model described by Plant Rule i

IF $\theta_1(t)$ is N_{i1} and... $\theta_p(t)$ is N_{ip} THEN

$$\begin{aligned} \dot{x}(t) = & (A_i + \Delta A_i)x(t) + (A_{1i} + \Delta A_{1i})x(t - \sigma(t)) \\ & + (B_i + \Delta B_i)u(t), \quad i = 1, \dots, r \end{aligned} \quad (1)$$

Where $\theta_j(t)$ and N_{ij} ($i = 1, \dots, r, j = 1, \dots, p$) are, respectively, the premise variables and the fuzzy sets ; $x(t) \in R^n$ is the state ; $u(t) \in R^m$ is the control input ; r is the number of IF-THEN rules; the time-delay, $\sigma(t)$, is a time-varying continuous function that satisfies

$$0 \leq \sigma(t) \leq \bar{\sigma}, \quad \dot{\sigma}(t) \leq \beta \quad (2)$$

The parametric uncertainties ΔA_i , ΔA_{1i} , ΔB_i are time-varying matrices that are defined as follows: $\Delta A_i = D_A F_i(t) E_{Ai}$, $\Delta A_{1i} = D_{A1} F_i(t) E_{A1i}$, $\Delta B_i = D_B F_i(t) E_{Bi}$. Let $\bar{A}_i = A_i + \Delta A_i$; $\bar{A}_{1i} = A_{1i} + \Delta A_{1i}$; $\bar{B}_i = B_i + \Delta B_i$.

Where A_i , A_{1i} , B_i , D_A , D_{A1} , D_B , E_{Ai} , E_{A1i} , E_{Bi} are constant known matrices and $F_i(t)$ is unknown and possibly time-varying matrix satisfying $F_i^T(t)F_i(t) \leq I$ for any given t . It is assumed the element of $F_i(t)$ are Lebesgue measurable. By using the commonly used center-average defuzzifier, product inference and singleton fuzzifier, the T-S fuzzy systems

can be inferred as

$$\dot{x}(t) = \sum_{i=1}^r h_i(\theta(t)) \left\{ \bar{A}_i x(t) + \bar{A}_{1i} x(t - \sigma(t)) + \bar{B}_i u(t) \right\} \quad (3)$$

where $\theta(t) = [\theta_1(t), \dots, \theta_p(t)]$ and $v_i(\theta(t)) = \prod_{j=1}^p N_{ij}(\theta_j(t))$ are the membership functions of the system with respect to i th plan rule. Denote

$$h_i(\theta(t)) = v_i(\theta(t)) / \sum_{i=1}^r v_i(\theta(t)). \quad (4)$$

$h_i(\theta(t))$ is the averaging weight for each rule, representing the normalized grade of membership, and satisfies

$$h_i(\theta(t)) \geq 0, \quad \text{and} \quad \sum_{i=1}^r h_i(\theta(t)) = 1, \quad (i = 1, 2, \dots, r) \quad (5)$$

In order to obtain the main results in this paper, the following lemma is needed:

Lemma 2.1: [5] Given matrices $D, E, F(t)$ with compatible dimensions and $F(t)$ satisfying $F(t)^T F(t) \leq I$. Then, the following inequality holds for any $\epsilon > 0$:
 $DF(t)E + E^T F(t)^T D^T \leq \epsilon DD^T + \epsilon^{-1} E^T E$

III. MAIN RESULTS

A. Time delay dependent stabilization conditions

We consider the problem of delay-dependent stabilization of system (3). Based on the Parallel Distributed Compensation (PDC), the following fuzzy control law is employed to deal with the problem of stabilization via state feedback.

Controller Rule i :

IF $\theta_1(t)$ is N_{i1} and... $\theta_p(t)$ is N_{ip} THEN

$$u(t) = K_i x(t), \quad i = 1, 2, \dots, r. \quad (6)$$

Hence, the overall fuzzy control law is represented by

$$u(t) = \sum_{i=1}^r h_i(\theta(t)) K_i x(t), \quad (7)$$

where $K_i (i = 1, 2, \dots, r)$ are the local control gain matrices. In the sequel, for brevity we use h_i to denote $h_i(\theta(t))$. Consider system (3) associated with the control law (7), then the resulting closed-loop system can be expressed as follows:

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j \left\{ \hat{A}_{ij} x(t) + \bar{A}_{1i} x(t - \sigma(t)) \right\} \quad (8)$$

Where $\hat{A}_{ij} = \bar{A}_i + \bar{B}_i K_j$.

Theorem 3.1: Case of nominal System:

System (8) is asymptotically stable, if there exist some matrices $\bar{P}_i > 0$, $\bar{H} > 0$, $\bar{S} > 0$ and $\bar{T} > 0$, any matrices Θ_i , Y_i , U , $\bar{P}_{\Phi, i}$, $\bar{L}$, $\bar{W}$, μ_1 , μ_2 and μ_3 are positive scalars satisfying the following set of LMIs:

$$P_l + \Theta_i > 0 \quad (9)$$

$$\Lambda_{ii} < 0, \quad (i = 1, \dots, r) \quad (10)$$

$$\Lambda_{ij} + \Lambda_{ji} < 0, \quad i < j \quad (11)$$

where

$$\Lambda_{ij} = \begin{bmatrix} \Gamma_{1ij} & * & * & * & * & * \\ \Gamma_{2ij} & \Gamma_{3i} & * & * & * & * \\ \Gamma_{4ij} & \Gamma_{5i} & \Gamma_6 & * & * & * \\ -\bar{W}_1 & -\bar{W}_2^T & 0 & -\bar{T} & * & * \\ \bar{L}_1^T & \bar{L}_2^T & 0 & 0 & -\bar{\sigma}^{-1} \bar{H} & * \\ \bar{W}_1^T & \bar{W}_2^T & 0 & 0 & 0 & -\bar{\sigma}^{-1} \bar{H} \end{bmatrix} \quad (12)$$

$$P_{\Phi, i} = \sum_{l=1}^r \Phi_l (P_l + \Theta_i) \quad (13)$$

where

$$\begin{aligned} \Gamma_{1ij} &= \text{Sym}(A_i U + B_i Y_j + \bar{L}_1) + \bar{P}_{\Phi, i} + \bar{S} + \bar{T} \\ \Gamma_{2ij} &= \mu_1 (A_i U + B_i Y_j) + U^T A_{1i}^T + \bar{L}_2 - \bar{L}_1^T + \bar{W}_1^T \\ \Gamma_{3i} &= \mu_1 \text{Sym}(A_{1i} U - \bar{L}_2 + \bar{W}_2) - (1 - \beta) \bar{S} \\ \Gamma_{4ij} &= \mu_2 (A_i U + B_i Y_j) - U^T + \bar{P}_i^T \\ \Gamma_{5i} &= \mu_2 (A_{1i} U) - \mu_1 U^T \\ \Gamma_6 &= -\mu_2 \text{Sym}(U) + \bar{\sigma} \bar{H} \end{aligned}$$

Proof: For simplicity, we set $x_\sigma = x(t - \sigma(t))$, $x_{\bar{\sigma}} =$

$$x(t - \bar{\sigma}), \quad L = \begin{bmatrix} L_1 \\ L_2 \\ 0 \\ 0 \end{bmatrix}, \quad W = \begin{bmatrix} W_1 \\ W_2 \\ 0 \\ 0 \end{bmatrix}, \quad M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ 0 \end{bmatrix}$$

$$\text{and } \Psi(t) = \begin{bmatrix} x(t) \\ x_\sigma \\ \dot{x}(t) \\ x_{\bar{\sigma}} \end{bmatrix}$$

In this part we consider that ΔA_i , ΔB_i and $\Delta A_{1i} = 0$.

Let us choose a candidate Lyapunov-Krasovskii function as follows:

$$\begin{aligned} V &= V_1(t) + V_2(t) + V_3(t) \\ V_1(t) &= x^T(t) P(t) x(t) \\ V_2(t) &= \int_{t-\sigma(t)}^t x^T(s) S x(s) ds + \int_{t-\bar{\sigma}}^t x^T(s) T x(s) ds \\ V_3(t) &= \int_{-\bar{\sigma}}^0 \int_{t+\alpha}^t x^T(s) H \dot{x}(s) ds d\alpha \end{aligned} \quad (14)$$

where $P(t)$ is a fuzzy weighting-dependent Lyapunov function

$$P(t) = \sum_{i=1}^r h_i P_i \quad (i = 1, 2, \dots, r) \quad (15)$$

Taking the derivative of V_i , $1, \dots, 3$ along the trajectory of system (8) yields

$$\dot{V}_1(x(t)) = 2\dot{x}^T(t) P(t) x(t) + x^T(t) \dot{P}(t) x(t)$$

The derivative of (15) provides information about time derivative of membership functions. Then, we assume that

$$|\dot{h}_l(\theta(t))| \leq \Phi_l \quad (16)$$

where $\Phi_l \geq 0$ ($l = 1, 2, \dots, r$)

Considering the properties in (5) we have

$$\begin{aligned} \dot{P}(t) &= \sum_{l=1}^r \dot{h}_l(t) P_l = \sum_{i=1}^r (h_i(t)) \sum_{l=1}^r \dot{h}_l(t) P_l \\ &= \sum_{i=1}^r \sum_{l=1}^r h_i(t) \dot{h}_l(t) P_l \end{aligned} \quad (17)$$

Once again considering (5) the following equation holds:

$$\sum_{i=1}^r \sum_{l=1}^r h_i \dot{h}_l x^T(t) \Theta_i x(t) = 0 \quad (18)$$

where $\Theta_i > 0$ are slack matrices with appropriate dimensions. Assuming that the assumption (9) and (16) hold, it is easy to have

$$\dot{P}(t) \leq \sum_{i=1}^r \sum_{l=1}^r h_i(t) \Phi_l (P_l + \Theta_i) \quad (19)$$

On the other hand, from the definition of (8) the following equation holds

$$\begin{aligned} 2 \left[x^T(t) M_1 + x_\sigma^T M_2 + \dot{x}(t)^T M_3 \right] \left[-\dot{x}(t) \right. \\ \left. + \sum_{i=1}^r \sum_{j=1}^r h_i h_j (\hat{A}_{ij} x(t) + \bar{A}_{1i} x_\sigma) \right] = 0 \end{aligned} \quad (20)$$

Adding the null terms (20) and (18), we obtain

$$\begin{aligned} \dot{V}_1(x(t)) &\leq \sum_{i=1}^r h_i \left\{ 2x^T(t) P_i \dot{x}(t) \right. \\ &\quad \left. + \sum_{l=1}^r \Phi_l x^T(t) (P_l + \Theta_i) x(t) \right\} \end{aligned} \quad (21)$$

$$\begin{aligned} &+ 2 \left[x^T(t) M_1 + x_\sigma^T M_2 + \dot{x}(t)^T M_3 \right] \\ &\times \left[-\dot{x}(t) + \sum_{i=1}^r \sum_{j=1}^r h_i h_j \hat{A}_{ij} x(t) + \bar{A}_{1i} x_\sigma \right] \end{aligned} \quad (22)$$

$$\begin{aligned} &\leq \sum_{i=1}^r h_i \left\{ 2x^T(t) P_i \dot{x}(t) + x^T(t) P_{\Phi, i} x(t) \right\} \\ &+ 2 \sum_{i=1}^r \sum_{j=1}^r h_i h_j \Psi^T(t) M \begin{bmatrix} \hat{A}_{ij} & \bar{A}_{1i} & -I & 0 \end{bmatrix} \Psi(t) \end{aligned}$$

$$\begin{aligned} \dot{V}_2(x(t)) &= x^T(t) S x(t) - (1 - \dot{\sigma}(t)) x_\sigma^T S x_\sigma \\ &\quad + x^T(t) T x(t) - x_\sigma^T T x_\sigma \end{aligned} \quad (23)$$

$$\begin{aligned} \dot{V}_3(x(t)) &= \bar{\sigma} \dot{x}^T(t) H \dot{x}(t) \\ &\quad - \int_{-\bar{\sigma}}^0 \dot{x}^T(t + \alpha) H \dot{x}(t + \alpha) d\alpha \end{aligned} \quad (24)$$

$$\begin{aligned} &= - \int_{-\bar{\sigma}}^0 \dot{x}^T(t + \alpha) H \dot{x}(t + \alpha) d\alpha = - \int_{t-\bar{\sigma}}^t \dot{x}^T(\alpha) H \dot{x}(\alpha) d\alpha \\ &= - \int_{t-\bar{\sigma}}^{t-\sigma(t)} \dot{x}^T(\alpha) H \dot{x}(\alpha) d\alpha - \int_{t-\sigma(t)}^t \dot{x}^T(\alpha) H \dot{x}(\alpha) d\alpha \\ &\leq \bar{\sigma} \Psi^T(t) L H^{-1} L^T \Psi(t) + 2 \Psi^T(t) L \int_{t-\sigma(t)}^t \dot{x}(\alpha) d\alpha \\ &\quad + \bar{\sigma} \Psi^T(t) W H^{-1} W^T \Psi(t) + 2 \Psi^T(t) W \int_{t-\bar{\sigma}}^{t-\sigma(t)} \dot{x}(\alpha) d\alpha \end{aligned}$$

The desired results are obtained by using the upper bound of the following terms (25) and (26) :

$$\begin{aligned} -2 \Psi^T(t) L \int_{t-\sigma(t)}^t \dot{x}(\alpha) d\alpha &\leq \sigma(t) \Psi^T(t) L H^{-1} L^T \Psi(t) \\ &\quad + \int_{t-\sigma(t)}^t \dot{x}^T(\alpha) H \dot{x}(\alpha) d\alpha \\ &\leq \bar{\sigma} \Psi^T(t) L H^{-1} L^T \Psi(t) + \int_{t-\sigma(t)}^t \dot{x}^T(\alpha) H \dot{x}(\alpha) d\alpha \end{aligned} \quad (25)$$

$$\begin{aligned} -2 \Psi^T(t) W \int_{t-\bar{\sigma}}^{t-\sigma(t)} \dot{x}(\alpha) d\alpha &\leq (\bar{\sigma} - \sigma(t)) \Psi^T(t) W H^{-1} W^T \Psi(t) \\ &\quad + \int_{t-\bar{\sigma}}^{t-\sigma(t)} \dot{x}^T(\alpha) H \dot{x}(\alpha) d\alpha \\ &\leq \bar{\sigma} \Psi^T(t) W H^{-1} W^T \Psi(t) + \int_{t-\bar{\sigma}}^{t-\sigma(t)} \dot{x}^T(\alpha) H \dot{x}(\alpha) d\alpha \end{aligned} \quad (26)$$

where the well-known inequality $-2x^T y \leq x^T H^{-1} x + y^T H y$ with $H > 0$ is used to get the last inequalities. Then, we obtain

$$\begin{aligned} \dot{V}_3(x(t)) &\leq \bar{\sigma} \dot{x}^T(t) H \dot{x}(t) + \bar{\sigma} \Psi^T(t) L H^{-1} L^T \Psi(t) \\ &\quad + \bar{\sigma} \Psi^T(t) W H^{-1} W^T \Psi(t) \\ &\quad + 2 \Psi^T(t) L \begin{bmatrix} I & -I & 0 & 0 \end{bmatrix} \Psi(t) \\ &\quad + 2 \Psi^T(t) W \begin{bmatrix} 0 & I & 0 & -I \end{bmatrix} \Psi(t) \end{aligned} \quad (27)$$

By letting, $M_1 = G^T$, $M_2 = \mu_1 G^T$ and $M_3 = \mu_2 G^T$, with μ_1, μ_2, μ_3 being positive scalars, the derivative of (14) along the trajectory of the closed-loop system (8) can be written as

$$\begin{aligned} \dot{V}(x(t)) &\leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \Psi^T(t) \bar{\Phi}_{ij} \Psi(t) \\ &= \sum_{i=1}^r h_i^2 \Psi(t)^T \bar{\Phi}_{ii} \Psi(t) \\ &\quad + 2 \sum_{i=1}^r \sum_{j>i}^r h_i h_j \Psi(t)^T (\bar{\Phi}_{ij} + \bar{\Phi}_{ji}) \Psi(t) \end{aligned} \quad (28)$$

where $\bar{\Phi}_{ij} = \bar{\lambda}_{ij} + \bar{\sigma} L H^{-1} L^T + \bar{\sigma} W H^{-1} W^T$

$$\bar{\lambda}_{ij} = \begin{bmatrix} \Omega_{1ij} & * & * & * \\ \Omega_{2ij} & \Omega_{3i} & * & * \\ \Omega_{4ij} & \Omega_{5i} & \Omega_6 & * \\ -W_1^T & -W_2^T & 0 & -T \end{bmatrix} \quad (29)$$

with

$$\begin{aligned}\Omega_{1ij} &= \text{Sym}(G^T \hat{A}_{ij} + L_1) + P_{\Phi,i} + S + T, \\ \Omega_{2ij} &= \mu_1(G^T \hat{A}_{ij}) + \bar{A}_{1i}^T G + L_2 - L_1^T + W_1^T, \\ \Omega_{3i} &= \mu_1 \text{Sym}(G^T \bar{A}_{1i} - L_2 + W_2) - (1 - \beta)S, \\ \Omega_{4ij} &= \mu_2(G^T \hat{A}_{ij}) - G + P_i^T, \\ \Omega_{5i} &= \mu_2 G^T \bar{A}_{1i} - \mu_1 G \\ \Omega_6 &= -\mu_2 \text{Sym}(G) + \bar{\sigma} H\end{aligned}$$

From (11) we have $\Gamma_6 < 0$, this imply $0 < \bar{\sigma} H < \mu_2 \text{Sym}(U)$, this indicate that U is non singular when $\mu_2 > 0$. Define $G = U^{-1}$ and $\Upsilon = \text{diag}(G, G, G, G)$. Let $\bar{L}_1 = U^T L_1 U$, $\bar{S} = U^T S U$, $\bar{T} = U^T T U$, $\bar{P}_i = U^T P_i U$, $\bar{H} = U^T H U$, $\bar{L}_2 = U^T L_2 U$, $\bar{W}_1 = U^T W_1 U$, $\bar{W}_2 = U^T W_2 U$, and $Y_i = K_i U$.

Pre-and post multiplying (11) by Υ^T and Υ , respectively, and applying the Schur complement, we obtain $\bar{\Phi}_{ij} + \bar{\Phi}_{ji} < 0$.

By following the similar way it is easy to obtain from (10) that $\bar{\Phi}_{ii} < 0$, then we have $\dot{V}(x(t)) < 0$ when $h_i \geq 0$, therefore, the system (8) is asymptotically stable. ■

Remark 3.1: In some existing literature, for example [6], the term $\int_{t-\bar{\sigma}}^t \dot{x}^T(\alpha) H \dot{x}(\alpha) d\alpha$ in the derivative of $V(t)$ was often estimated as $\int_{t-\sigma(t)}^t \dot{x}^T(\alpha) H \dot{x}(\alpha) d\alpha$ and the term $\int_{t-\bar{\sigma}}^{t-\sigma(t)} \dot{x}^T(\alpha) H \dot{x}(\alpha) d\alpha$ was ignored. This may bring conservativeness. In Theorem 3.1, all these terms are reserved, which can reduce conservativeness.

Corollary 3.2:

System (8) is asymptotically stable, if there exist some matrices $\bar{P}_i > 0$, $\bar{H} > 0$ and $\bar{S} > 0$, any matrices Θ_i , Y_i , U , $\bar{P}_{\Phi,i}$, $\bar{L}$, μ_1 , μ_2 and μ_3 are positive scalars satisfying the following set of LMIs:

$$P_l + \Theta_i > 0 \quad (30)$$

$$\Lambda'_{ii} < 0 (i = 1, 2, \dots, r) \quad (31)$$

$$\Lambda'_{ij} + \Lambda'_{ji} < 0, \quad i < j \quad (32)$$

where

$$\Lambda'_{ij} = \begin{bmatrix} \Gamma'_{1ij} & * & * & * \\ \Gamma'_{2ij} & \Gamma'_{3i} & * & * \\ \Gamma'_{4ij} & \mu_2(A_{1i}U) - \mu_1 U^T & \Gamma'_5 & 0 \\ \bar{\sigma} \bar{L}_1^T & \bar{\sigma} \bar{L}_2^T & 0 & -\bar{H} \end{bmatrix}$$

$$\begin{aligned}\text{where } \Gamma'_{1ij} &= \text{Sym}(A_i U + B_i Y_j + \bar{L}_1) + \bar{P}_{\Phi,i} + \bar{S} \\ \Gamma'_{2ij} &= \mu_1(A_i U + B_i Y_j) + U^T \bar{A}_{1i}^T + \bar{L}_2 - \bar{L}_1^T \\ \Gamma'_{3i} &= \mu_1 \text{Sym}(A_{1i} U - \bar{L}_2) - (1 - \beta) \bar{S} \\ \Gamma'_{4ij} &= \mu_2(A_i U + B_i Y_j) - U^T + \bar{P}_i^T \\ \Gamma'_5 &= -\mu_2 \text{Sym}(U) + \bar{\sigma} \bar{H}.\end{aligned}$$

Theorem 3.2: : Case of uncertain System:

System (8) is asymptotically stable, if there exist some matrices $\bar{P}_i > 0$, $\bar{H} > 0$, $\bar{S}$ and $\bar{T} > 0$, any matrices Θ_i , Y_i , U , $\bar{P}_{\Phi,i}$, $\bar{L}$, $\bar{W}$, ϵ_i , μ_1 , μ_2 and μ_3 are positive scalars satisfying the

following set of LMIs:

$$P_l + \Theta_i > 0 \quad (33)$$

$$\Lambda_{ii} < 0 (i = 1, 2, \dots, r) \quad (34)$$

$$\Lambda_{ij} + \Lambda_{ji} < 0 (i < j = 1, 2, \dots, r) \quad (35)$$

where

$$\Lambda_{ij} = \begin{bmatrix} \Gamma_{1ij} & * & * \\ \Gamma_{2ij} & \Gamma_{3i} & * \\ \Gamma_{4ij} & \mu_2(A_{1i}U) - \mu_1 U^T & \Gamma_5 \\ -\bar{W}_1^T & \bar{W}_2^T & 0 \\ \bar{L}_1^T & \bar{L}_2^T & 0 \\ \bar{W}_1^T & \bar{W}_2^T & 0 \\ E_{A_i}U & 0 & 0 \\ E_{B_i}Y_j & 0 & 0 \\ 0 & E_{A_{1i}}U & 0 \end{bmatrix} \quad (36)$$

$$\begin{aligned}\text{where } \Gamma_{1ij} &= \text{Sym}(A_i U + B_i Y_j + \bar{L}_1) + \bar{P}_{\Phi,i} + \bar{S} + \bar{T} + \epsilon_i D_A D_A^T + \epsilon_i D_B D_B^T + \epsilon_i D_{A1} D_{A1}^T \\ \Gamma_{2ij} &= \mu_1(A_i U + B_i Y_j) + U^T \bar{A}_{1i}^T + \bar{L}_2 - \bar{L}_1^T + \bar{W}_1^T \\ \Gamma_{3i} &= \mu_1 \text{Sym}(A_{1i} U - \bar{L}_2 + \bar{W}_2) - (1 - \beta) \bar{S} + \mu_1^2 \epsilon_i D_A D_A^T + \mu_1^2 \epsilon_i D_B D_B^T + \mu_1^2 \epsilon_i D_{A1} D_{A1}^T \\ \Gamma_{4ij} &= \mu_2(A_i U + B_i Y_j) - U^T + \bar{P}_i^T \\ \Gamma_5 &= -\mu_2 \text{Sym}(U) + \bar{\sigma} \bar{H} + \mu_2^2 \epsilon_i D_A D_A^T + \mu_2^2 \epsilon_i D_B D_B^T + \mu_2^2 \epsilon_i D_{A1} D_{A1}^T \\ \Gamma_6 &= -\bar{T}\end{aligned}$$

Proof:

In this part we consider that ΔA_i , ΔB_i and ΔA_{1i} are $\neq 0$. By using Lemma 2.1 and the Schur complement, we obtain Theorem 3.2. Where the uncertain part is represented as : $\Delta \bar{\Phi}_{ij} = \text{Sym}(\check{D} \check{F}(t) \check{E}) \leq \epsilon_i \check{D} \check{D}^T + \epsilon_i^{-1} \check{E}^T \check{E}$ With

$$\check{D} = \begin{bmatrix} D_A & D_B & D_{A1} & 0 & 0 & 0 \\ \mu_1 D_A & \mu_1 D_B & \mu_1 D_{A1} & 0 & 0 & 0 \\ \mu_2 D_A & \mu_2 D_B & \mu_2 D_{A1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (37)$$

$$\check{F}(t) = \begin{bmatrix} F(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & F(t) & 0 & 0 & 0 & 0 \\ 0 & 0 & F(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & F(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & F(t) & 0 \\ 0 & 0 & 0 & 0 & 0 & F(t) \end{bmatrix} \quad (38)$$

$$\tilde{E} = \begin{bmatrix} E_{A1}U & 0 & 0 & 0 & 0 & 0 \\ E_{B1}Y_j & 0 & 0 & 0 & 0 & 0 \\ 0 & E_{A1i}U & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (39)$$

IV. NUMERICAL EXAMPLES

To illustrate the designed controller law for the T-S fuzzy model with time-varying delay, we consider the following examples.

Example 1. For fuzzy rules as $h_1 = 1 - ((x_2^2)/2.25)$ and $h_2 = x_2^2/2.25$, the following fuzzy T-S model is presented:

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j \left\{ [(A_i + \Delta A_i) + B_i K_j] x(t) + (A_{1i} + \Delta A_{1i}) x(t - \sigma(t)) \right\} \quad (40)$$

where

$$A_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, A_{11} = \begin{bmatrix} 0.5 & -0.9 \\ 0 & 2 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$\Delta A_i = D_A F_i(t) E_{A_i}, \Delta A_{1i} = D_{A1} F_i(t) E_{A_{1i}}, \quad (41)$$

$$D_A = D_B = D_{A1} = \begin{bmatrix} -0.03 & 0 \\ 0 & 1.6 \end{bmatrix}$$

$$E_{A1} = \begin{bmatrix} -0.15 & 0.2 \\ 0 & 0.04 \end{bmatrix}, E_{A2} = \begin{bmatrix} -0.15 & 0.2 \\ 0 & 0.04 \end{bmatrix},$$

$$E_{A11} = \begin{bmatrix} -0.05 & -0.35 \\ 0.08 & -0.45 \end{bmatrix}, E_{A12} = \begin{bmatrix} -0.05 & -0.35 \\ 0.08 & -0.45 \end{bmatrix},$$

$$E_{B1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, E_{B2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The operating domain of the nonlinear plant are assumed to be $x_1(t) \in [-3, 3]$ and $x_2(t) \in [-1, 1]$. Thus, based on the derivative of the membership function, we assume that $\Phi_l = \{3.5, 3.5\}$.

It appears from Table 1 that our result improves the existing ones in the case of uncertain T-S fuzzy model with varying time-delay.

In fact, by solving the LMIs in Theorem 3 in [7], in Theorem 1 in [1], in Corollary 3.2 and Theorem 3.2, the maximum value of the time delay $\bar{\sigma}$ is found in theorem 3.2. Table 1 summarizes the results. By using theorem 3.2 with the choice of $\beta = 0.15$ and $\sigma(t) = 0.379 + 0.15 \sin(t)$ ($\bar{\sigma} = 0.529$, $\mu_1 = \mu_2 = 0.1$) we can obtain the following state-feedback gain matrices:

$$K_1 = 10^3 \cdot \begin{bmatrix} 0.9352 & -3.4038 \end{bmatrix} \quad (42)$$

$$K_2 = 10^3 \cdot \begin{bmatrix} 1.7298 & -6.2924 \end{bmatrix} \quad (43)$$

The simulation was tested under the initial conditions $x(t) = [2 \quad 0]^T$, $t \in [-0.529 \quad 0]$ and uncertainty function

$$F_i(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix} \quad (44)$$

From the simulation results in Figure 1, it can be clearly seen that our method allows to stabilize nonlinear systems represented by uncertain T-S fuzzy model with time-varying delay for the maximum value of delay which can be attain 0.529 s. Despite the presence of uncertainties, the system states converge asymptotically to zero.

TABLE I
COMPARISON AMONG VARYING DELAY-DEPENDENT STABILIZATION METHODS WITH UNCERTAINTIES

Methods	Maximum allowed τ
theorem 1 of [1]	0.1498 s
theorem 3 of [7]	0.4770 s
Corollary 3.2	0.525 s
theorem 3.2	0.529 s

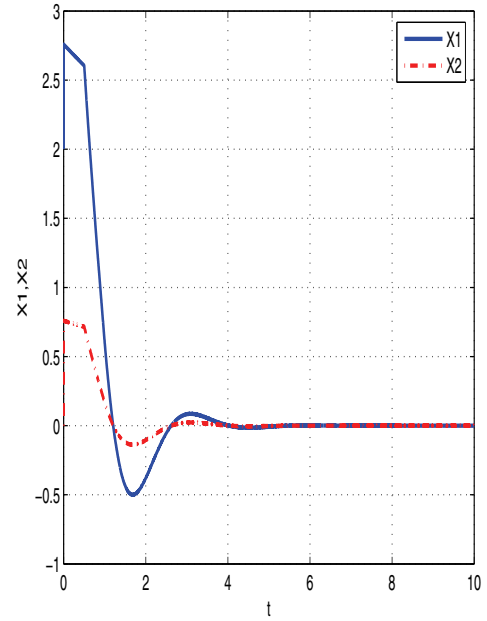


Fig. 1. Control results for system (45) with uncertainties

Example 2. To illustrate the designed controller laws for the T-S fuzzy model with time-varying delay, we consider the following example.

Example . For fuzzy rules as $h_1 = 1/2(1 + (c - x_1(t)/\gamma))$ and $h_2 = 1/2(1 - (c - x_1(t)/\gamma))$ Where $\gamma = 6.3$, $a = 0.5$, $b_1 = b_2 = 0.2$, $c = 5.7$, the following fuzzy T-S model is presented:

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j \left\{ [(A_i + \Delta A_i) + B_i K_j] x(t) + (A_{1i} + \Delta A_{1i}) x(t - \sigma(t)) \right\} \quad (45)$$

where

$$A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & b_1 & 0 \\ 0 & 0 & -\gamma \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & b_1 & 0 \\ 0 & 0 & \gamma \end{bmatrix},$$

$$A_{11} = A_{12} = \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

The operating domain of the nonlinear plant are assumed to be $x_1(t) \in [-10, 12]$, $x_2(t) \in [-12, 8]$ and $x_3(t) \in [0, 25]$. Thus, based on the derivative of the membership functions, we choose $\Phi_i = \{2.15, 2.15\}$.

The free weighting approach gives the best result in the case of uncertain T-S fuzzy model with varying time-delay.

In fact, by solving the LMIs in Theorem 3.1, and in Corollary 3.2, the maximum value of the time delay $\bar{\sigma}$ is found in theorem 3.1. By using theorem 3.1 with the choice of $\beta = 0.15$ and $\sigma(t) = 2.274 + 0.15 \sin(t)$ ($\bar{\sigma} = 2.424$, $\mu_1 = \mu_2 = 1$) we can obtain the following state-feedback gain matrices:

$$K_1 = [14.9314 \quad 0.8607 \quad -2.4751] \quad (46)$$

$$K_2 = [14.9314 \quad 0.8607 \quad -15.0751] \quad (47)$$

The simulation was tested under the initial conditions $x(t) = [1.4 \quad 0.3 \quad -2.4]^T$, $t \in [-2.424 \quad 0]$. From the simulation results in Figure 2, it can be clearly seen that the free weighting matrix approach allows to stabilize nonlinear systems represented by uncertain T-S fuzzy model with time-varying delay for the maximum value of delay which can be attain 2.424 s.

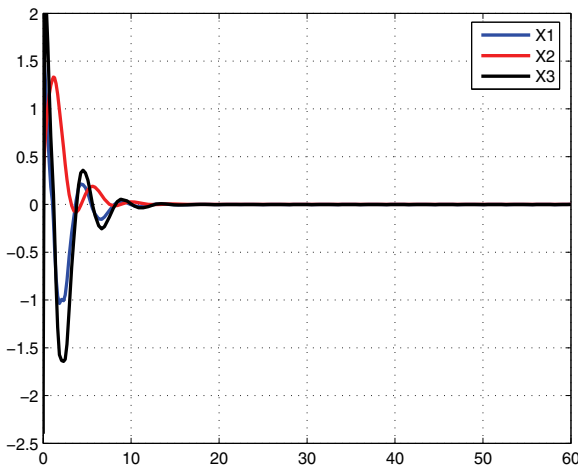


Fig. 2. Control results for system (45) without uncertainties $\bar{\sigma} = 2.424$

V. CONCLUSION

In this paper, we have presented new conditions for robust stabilization for uncertain fuzzy systems with time varying delay. The less conservatism sufficient conditions formulated in LMI terms are established based on the fuzzy weighting-dependent Lyapunov-Krasovskii functional method combined

with the introduction of free weighting matrices. The merits and less conservatism of our method have been illustrated in comparison with existing works.

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