

On Extension of Consequent Parts of T-S Inference Model

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Abstract—This paper discusses the extension of consequent parts of T-S inference model. First, it proposes an extended T-S inference model in which the coefficients of consequent parts of the fuzzy rules are extended from constants to functions. It also shows the property of the proposed model from the point of view of the equivalence and the monotonicity, by using simplified model of the proposed model.

Index Terms—Fuzzy inference, Takagi-Sugeno (T-S) fuzzy inference model, Single Input Connected (SIC) fuzzy inference model, Equivalence, Monotonicity.

I. INTRODUCTION

The fuzzy inference models have powerful capability in nonlinear systems, control, and so on since Zadeh introduced fuzzy sets and Mamdani [1] applied the concept of fuzzy inference to a steam engine experimental device. Especially, researches on the T-S inference model [2], which are widely used as fuzzy control method and so on, are reported in many papers.

The extension of the consequent parts of the T-S model is very difficult. For example, the single input connected fuzzy inference model (SIC model) by Hayashi et al. [4]–[6] can be transformed to the extended simplified fuzzy inference model [3].

From this reason, in this paper, we first propose an extended T-S inference model in which the coefficients of consequent parts of the fuzzy rules are extended from constants to functions, and show that this model can obtain complex inference results.

Second, we consider the extended simplified fuzzy inference model as a simple model of extended T-S inference model. As previously stated, the SIC model can be transformed to the extended simplified fuzzy inference model [3]. However, the extended simplified fuzzy inference model can not be generally transformed to the SIC model, and the equivalence conditions between the extended simplified fuzzy inference model and the SIC model were not shown. Therefore, we discuss the equivalence conditions of these models.

Finally, we show the monotonicity [6]–[8] of the extended simplified fuzzy inference model by using the equivalence conditions.

II. FUZZY INFERENCE MODELS

In this section we review the T-S inference model [2] and SIC model [4]–[6].

A. T-S Inference Model

Rules in the Takagi-Sugeno (T-S) inference model [2] are constituted as

$$\text{Rule } R_j = \begin{cases} x_1 = A_j^1, x_2 = A_j^2, \dots, x_n = A_j^n \\ \longrightarrow y = a_0^j + a_1^j x_1 + \dots + a_n^j x_n \end{cases} \quad (1)$$

where $x_1, x_2, \dots, x_n$ are variables of the antecedent part, $A_j^1, A_j^2, \dots, A_j^n$ fuzzy sets, $a_0^j + a_1^j x_1 + \dots + a_n^j x_n$ functions of consequent part, and a_i^j 's are real number. The inference result y^0 for inputs $x_1^0, x_2^0, \dots, x_n^0$ is given as

$$y^0 = \frac{\sum_{j=1}^M h_j (a_0^j + a_1^j x_1^0 + \dots + a_n^j x_n^0)}{\sum_{j=1}^M h_j}, \quad h_j = \prod_{i=1}^n A_j^i(x_i^0) \quad (2)$$

where h_j is the degree of the j th rule and M is the total number of the fuzzy rules.

Example 1: We first consider the following T-S rules as a simple example.

$$\begin{aligned} x_1 = A_1^1, x_2 = A_1^2 &\longrightarrow y = 0.45 - 0.2x_1 + 0.35x_2 \\ x_1 = A_1^1, x_2 = A_2^2 &\longrightarrow y = -0.1 - 0.2x_1 - 0.1x_2 \\ x_1 = A_1^1, x_2 = A_3^2 &\longrightarrow y = 0.05 - 0.2x_1 + 0.25x_2 \\ x_1 = A_2^1, x_2 = A_1^2 &\longrightarrow y = 0.4 - 0.3x_1 + 0.35x_2 \\ x_1 = A_2^1, x_2 = A_2^2 &\longrightarrow y = -0.15 - 0.3x_1 - 0.1x_2 \\ x_1 = A_2^1, x_2 = A_3^2 &\longrightarrow y = -0.3x_1 + 0.25x_2 \\ x_1 = A_3^1, x_2 = A_1^2 &\longrightarrow y = 0.25 + 0.1x_1 + 0.35x_2 \\ x_1 = A_3^1, x_2 = A_2^2 &\longrightarrow y = -0.3 + 0.1x_1 - 0.1x_2 \\ x_1 = A_3^1, x_2 = A_3^2 &\longrightarrow y = -0.15 + 0.1x_1 + 0.25x_2 \end{aligned} \quad (3)$$

where fuzzy sets, as shown in Fig. 1, are used. From (2), the inference result of (3) is obtained as shown in Fig. 2. $\square$

B. Single Input Connected (SIC) Fuzzy Inference Model

We briefly review the single input connected fuzzy inference model (SIC model) by Hayashi et al. [4]–[6]. The SIC model has n rule modules, and the final inference result is obtained by the weighted average of the degrees of the antecedent part

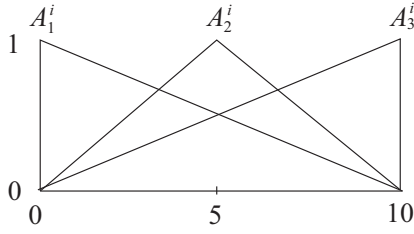


Fig. 1. Fuzzy sets.

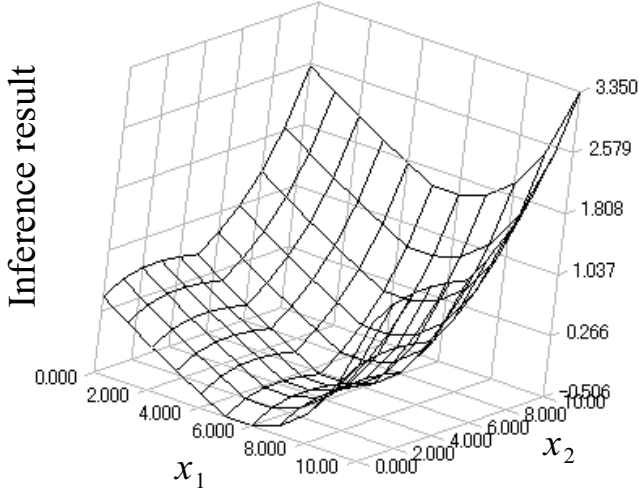


Fig. 2. Inference result by the T-S model of (3).

and consequent part of each rule module. The rule modules of the SIC model are given as

$$\begin{aligned}
 \text{Rules-1} : \{x_1 = A_j^1 \longrightarrow y_1 = y_j^1\}_{j=1}^{m_1} \\
 \vdots \\
 \text{Rules-}i : \{x_i = A_j^i \longrightarrow y_i = y_j^i\}_{j=1}^{m_i} \\
 \vdots \\
 \text{Rules-}n : \{x_n = A_j^n \longrightarrow y_n = y_j^n\}_{j=1}^{m_n}
 \end{aligned} \quad (4)$$

where Rules- i stands for the i th single input rule module, the i th input item x_i is the sole variable of the antecedent part of the Rules- i , and y_i stands for the variable of its consequent part. A_j^i means the fuzzy set of the j th rule of the Rules- i , y_j^i is real value of consequent part, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m_i$, and m_i is the number of rules in Rules- i .

The degree of the antecedent part in the j th rule of Rules- i is obtained by (5) for input x_i^0 .

$$h_j^i = A_j^i(x_i^0) \quad (5)$$

The final inference result y_0 is given as follows by using degrees of antecedent part and consequent part from each rule

module.

$$\begin{aligned}
 y_0 &= \frac{\sum_{j=1}^{m_1} h_j^1 y_j^1 + \dots + \sum_{j=1}^{m_n} h_j^n y_j^n}{\sum_{j=1}^{m_1} h_j^1 + \dots + \sum_{j=1}^{m_n} h_j^n} \\
 &= \frac{\sum_{i=1}^n \sum_{j=1}^{m_i} h_j^i y_j^i}{\sum_{i=1}^n \sum_{j=1}^{m_i} h_j^i} \quad (6)
 \end{aligned}$$

III. EXTENSION OF COEFFICIENTS OF CONSEQUENT PARTS IN T-S INFERENCE MODEL

In this section, we propose an extended T-S inference model in which the coefficients of consequent parts of the fuzzy rules are extended from constants to functions. The rules of proposed model are obtained as follows.

$$\text{Rule } R_j = \begin{cases} x_1 = A_j^1, x_2 = A_j^2, \dots, x_n = A_j^n \longrightarrow \\ y = a_0^j(\mathbf{x}) + a_1^j(\mathbf{x})x_1 + \dots + a_n^j(\mathbf{x})x_n \end{cases} \quad (7)$$

where $x_1, x_2, \dots, x_n$ are variables of the antecedent part, $A_j^1, A_j^2, \dots, A_j^n$ fuzzy sets, and $a_i^j(\mathbf{x})$'s are functions of coefficients of consequent part, where $a_i^j(\mathbf{x})$ is accorded a considerable degree. The inference result y^0 for inputs $x_1^0, x_2^0, \dots, x_n^0$ is given as

$$y^0 = \frac{\sum_{j=1}^M h_j (a_0^j(\mathbf{x}) + a_1^j(\mathbf{x})x_1^0 + \dots + a_n^j(\mathbf{x})x_n^0)}{\sum_{j=1}^M h_j} \quad (8)$$

where h_j is obtained in the same way as the conventional T-S inference model.

Example 2: We consider the rules of (9) of an extended T-S inference model, as a simple extension of (3), which is shown at the bottom of the next page, where fuzzy sets of Fig. 1 are used. From (8), the inference result of (9) is obtained as shown in Fig. 3. From the result, we can find out that the above extended T-S model can obtain complex inference results. $\square$

IV. PROPERTY OF EXTENDED T-S INFERENCE MODEL BY USING EQUIVALENCE

Generally speaking, decision of the coefficient functions of the proposed T-S model is very difficult. Therefore, in this section, we discuss the extended simplified fuzzy inference model which reduce a part of the functions of the consequent parts to real number, as a simple version of extended T-S model.

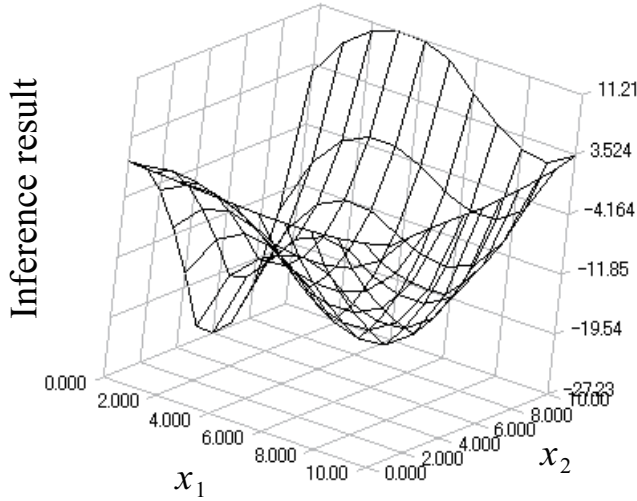


Fig. 3. Inference result by the extended T-S model of (9).

A. Equivalence

Reference [3] has shown that the SIC model can be transformed to the simplified fuzzy inference model which extends the coefficients of consequent parts to a compatibility function, but not vice versa. Therefore, we first show that the extended simplified fuzzy inference model can be transformed to the SIC model under a condition.

In order to distinguish subscript j of Rules- i in (4), j_i is used, and thus (4) is rewritten as

$$\text{Rules-}i : \{x_i = A_{j_i}^i \longrightarrow y_i = y_{j_i}^i\}_{j_i=1}^{m_i} \quad (10)$$

We can obtain the following rules of the extended simplified fuzzy inference model which corresponds to (10) of the rules of the SIC model [3]:

$$\begin{aligned} & x_1 = A_{j_1}^1, x_2 = A_{j_2}^2, \dots, x_n = A_{j_n}^n \\ \longrightarrow y &= \frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_{j_1}^1 + \dots + \frac{\sum_{j_i=1}^{m_n} h_{j_i}^n}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_{j_n}^n \end{aligned} \quad (11)$$

where $j_i = 1, 2, \dots, m_i$.

Now, we consider the rules of the extended simplified fuzzy inference model as follows.

$$x_1 = A_1^1, x_2 = A_1^2 \longrightarrow y = f_1(x_1, x_2) = a \quad (12)$$

$$x_1 = A_1^1, x_2 = A_2^2 \longrightarrow y = f_2(x_1, x_2) = b \quad (13)$$

$$x_1 = A_2^1, x_2 = A_1^2 \longrightarrow y = f_3(x_1, x_2) = c \quad (14)$$

$$x_1 = A_2^1, x_2 = A_2^2 \longrightarrow y = f_4(x_1, x_2) = d \quad (15)$$

Suppose that the rules of the SIC model corresponding to (12)–(15) are given as

$$\text{Rules-1} = \begin{cases} x_1 = A_1^1 \longrightarrow y_1 = y_1^1 \\ x_1 = A_2^1 \longrightarrow y_1 = y_1^2 \end{cases} \quad (16)$$

$$\text{Rules-2} = \begin{cases} x_2 = A_1^2 \longrightarrow y_2 = y_2^1 \\ x_2 = A_2^2 \longrightarrow y_2 = y_2^2 \end{cases}$$

Then, according to (10) and (11), (17)–(20), which are shown at the bottom of the next page, can be deduced from (16). From the correspondence of (12)–(15) and (17)–(20), we have the following relations:

$$\frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_1^1 + \frac{\sum_{j_i=1}^{m_n} h_{j_i}^n}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_1^n = a \quad (21)$$

$$\frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_1^1 + \frac{\sum_{j_i=1}^{m_n} h_{j_i}^n}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_2^n = b \quad (22)$$

$$\frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_2^1 + \frac{\sum_{j_i=1}^{m_n} h_{j_i}^n}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_1^n = c \quad (23)$$

$$\frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_2^1 + \frac{\sum_{j_i=1}^{m_n} h_{j_i}^n}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_2^n = d \quad (24)$$

$$\begin{aligned} x_1 = A_1^1, x_2 = A_1^2 &\longrightarrow y = 0.45 - 0.2x_1 + 0.35x_2 \\ x_1 = A_1^1, x_2 = A_2^2 &\longrightarrow y = -0.1(-0.2x_1 + 5)x_1 - (0.1x_2 - 10)x_2 \\ x_1 = A_1^1, x_2 = A_3^2 &\longrightarrow y = 0.05 - 0.2x_1 + 0.25x_2 \\ x_1 = A_2^1, x_2 = A_1^2 &\longrightarrow y = 0.4 - 0.3x_1 + 0.35x_2 \\ x_1 = A_2^1, x_2 = A_2^2 &\longrightarrow y = -0.15 - (0.3x_1 - 10)x_1 - 0.1x_2 \\ x_1 = A_2^1, x_2 = A_3^2 &\longrightarrow y = (-0.3x_1 - 2)x_1 + (0.25x_2 + 1)x_2 \\ x_1 = A_3^1, x_2 = A_1^2 &\longrightarrow y = 0.25 + 0.1x_1 + 0.35x_2 \\ x_1 = A_3^1, x_2 = A_2^2 &\longrightarrow y = -0.3 + 0.1x_1 - 0.1x_2 \\ x_1 = A_3^1, x_2 = A_3^2 &\longrightarrow y = -0.15 + 0.1x_1 + 0.25x_2 \end{aligned} \quad (9)$$

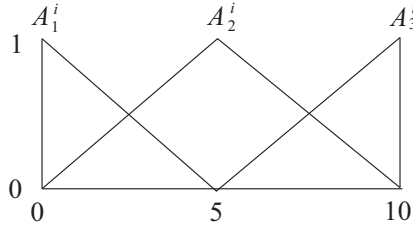


Fig. 4. Fuzzy sets with fuzzy partition.

From (21)–(24), the relation (25) is obtained

$$a + d = b + c \quad (25)$$

Under the conditions of (25), the rules of the SIC model of (16) can be obtained if $\frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i}$, $\frac{\sum_{j_i=1}^{m_2} h_{j_i}^2}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i}$, y_1^1 , y_2^1 , y_1^2 , and y_2^2 of (21)–(24) are fixed.

Example 3: We consider an example of the rules of the SIC model used in [3] as follows:

$$\begin{aligned} \text{Rules-1} &= \begin{cases} x_1 = A_1^1 \longrightarrow y_1 = 1 \\ x_1 = A_2^1 \longrightarrow y_1 = 2 \\ x_1 = A_3^1 \longrightarrow y_1 = 1 \end{cases} \\ \text{Rules-2} &= \begin{cases} x_2 = A_1^2 \longrightarrow y_2 = 1 \\ x_2 = A_2^2 \longrightarrow y_2 = 3 \\ x_2 = A_3^2 \longrightarrow y_2 = 5 \end{cases} \end{aligned} \quad (26)$$

where fuzzy sets A_1^i, A_2^i, A_3^i ($i = 1, 2$) in Fig. 4 are used. The inference results of (26) by the SIC model are obtained as shown in Fig. 5.

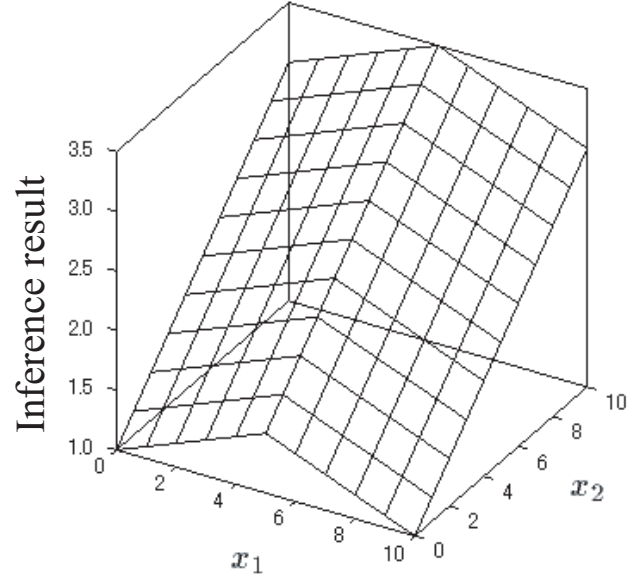


Fig. 5. Inference result by the SIC model of (26).

On the other hand, based on the above discussion, we obtain the rules of the extended simplified fuzzy inference model as (27), as shown at the bottom of the next page.

Note that $y_1^1 = 1, y_2^1 = 2, y_3^1 = 1$, and $y_1^2 = 1, y_2^2 = 3, y_3^2 = 5$ from (26), and the functional coefficients are as follows because fuzzy sets of antecedent parts constitute fuzzy

$$x_1 = A_1^1, x_2 = A_2^2 \longrightarrow y = \frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_1^1 + \frac{\sum_{j_i=1}^{m_2} h_{j_i}^2}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_1^2 = a \quad (17)$$

$$x_1 = A_1^1, x_2 = A_2^2 \longrightarrow y = \frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_1^1 + \frac{\sum_{j_i=1}^{m_2} h_{j_i}^2}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_2^2 = b \quad (18)$$

$$x_1 = A_2^1, x_2 = A_1^2 \longrightarrow y = \frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_2^1 + \frac{\sum_{j_i=1}^{m_2} h_{j_i}^2}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_1^2 = c \quad (19)$$

$$x_1 = A_2^1, x_2 = A_2^2 \longrightarrow y = \frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_2^1 + \frac{\sum_{j_i=1}^{m_2} h_{j_i}^2}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot y_2^2 = d \quad (20)$$

TABLE I
RULES OF EXTENDED SIMPLIFIED FUZZY INFERENCE MODEL IN TABLE FORM

		x_2			
		A_1^2	A_2^2	$\dots$	A_n^2
x_1	A_1^1	$f(x_1, x_2)_{(1,1)}$	$f(x_1, x_2)_{(1,2)}$	$\dots$	$f(x_1, x_2)_{(1,n)}$
	A_2^1	$f(x_1, x_2)_{(2,1)}$	$f(x_1, x_2)_{(2,2)}$	$\dots$	$f(x_1, x_2)_{(2,n)}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	A_m^1	$f(x_1, x_2)_{(m,1)}$	$f(x_1, x_2)_{(m,2)}$	$\dots$	$f(x_1, x_2)_{(m,n)}$

partition.

$$\frac{\sum_{j_i=1}^{m_1} h_{j_i}^i}{\frac{n}{m_i}} = \frac{1}{2} \quad (28)$$

From (27), the extended simplified fuzzy inference model can obtain the same result of the SIC model as shown in Fig. 5.

In the case of (27), considering the elicitation process of (25), the equivalence conditions between the extended simplified fuzzy inference model and SIC model are obtained as follows:

$$f_1(x_1, x_2) + f_5(x_1, x_2) = f_2(x_1, x_2) + f_4(x_1, x_2) \quad (29)$$

$$f_2(x_1, x_2) + f_6(x_1, x_2) = f_3(x_1, x_2) + f_5(x_1, x_2) \quad (30)$$

$$f_4(x_1, x_2) + f_8(x_1, x_2) = f_5(x_1, x_2) + f_7(x_1, x_2) \quad (31)$$

$$f_5(x_1, x_2) + f_9(x_1, x_2) = f_6(x_1, x_2) + f_8(x_1, x_2) \quad (32)$$

□

(25) and (29)–(32) show that two diagonal sums of rules in table form should be equal.

Therefore, as the generalization of the above results, we have the following Theorem regarding the equivalence between the extended simplified fuzzy inference model and the SIC model.

Theorem 1: In the extended simplified fuzzy inference model, when the rules of the extended simplified fuzzy inference model are given in table form, as shown in Table I, and the following conditions are satisfied, the inference result

is equal to that of the SIC model.

$$\begin{aligned} & f(x_1, x_2)_{(i,j)} + f(x_1, x_2)_{(i+1,j+1)} \\ &= f(x_1, x_2)_{(i+1,j)} + f(x_1, x_2)_{(i,j+1)} \end{aligned} \quad (33)$$

B. Monotonicity

It is important that inference results have the property of monotonicity; this is necessary when fuzzy inference is applied to evaluation problems, engineering problems, etc. However, deriving the monotonic conditions of the fuzzy inference model is very difficult [7], [8]. In this subsection, we show the monotonicity of extended simplified fuzzy inference model by using equivalence conditions as shown in previous subsection.

We have the following Theorem regarding the monotonicity of the extended simplified fuzzy inference model.

Theorem 2: For the extended simplified fuzzy inference model, the inference results monotonically increase, if the antecedent parts and consequent parts of each input item are monotonically increasing, and the fuzzy sets constitute fuzzy partitions, when the condition (33) is satisfied. □

proof: Inference result y^0 of the extended simplified fuzzy inference model becomes as follows from (11):

$$y^0 = \frac{\sum_{j=1}^M \left\{ h_j \left(\frac{y_{j_1}^1 \sum_{j_i=1}^{m_1} h_{j_i}^1}{\frac{n}{m_i}} + \dots + \frac{y_{j_n}^n \sum_{j_i=1}^{m_n} h_{j_i}^n}{\frac{n}{m_i}} \right) \right\}}{\sum_{j=1}^M h_j} \quad (34)$$

$$\begin{aligned} x_1 = A_1^1, x_2 = A_1^2 &\longrightarrow y = f_1(x_1, x_2) = 0.5 \cdot y_1^1 + 0.5 \cdot y_1^2 = 1 \\ x_1 = A_1^1, x_2 = A_2^2 &\longrightarrow y = f_2(x_1, x_2) = 0.5 \cdot y_1^1 + 0.5 \cdot y_2^2 = 2 \\ x_1 = A_1^1, x_2 = A_3^2 &\longrightarrow y = f_3(x_1, x_2) = 0.5 \cdot y_1^1 + 0.5 \cdot y_3^2 = 3 \\ x_1 = A_2^1, x_2 = A_1^2 &\longrightarrow y = f_4(x_1, x_2) = 0.5 \cdot y_2^1 + 0.5 \cdot y_1^2 = 1.5 \\ x_1 = A_2^1, x_2 = A_2^2 &\longrightarrow y = f_5(x_1, x_2) = 0.5 \cdot y_2^1 + 0.5 \cdot y_2^2 = 2.5 \\ x_1 = A_2^1, x_2 = A_3^2 &\longrightarrow y = f_6(x_1, x_2) = 0.5 \cdot y_2^1 + 0.5 \cdot y_3^2 = 3.5 \\ x_1 = A_3^1, x_2 = A_1^2 &\longrightarrow y = f_7(x_1, x_2) = 0.5 \cdot y_3^1 + 0.5 \cdot y_1^2 = 1 \\ x_1 = A_3^1, x_2 = A_2^2 &\longrightarrow y = f_8(x_1, x_2) = 0.5 \cdot y_3^1 + 0.5 \cdot y_2^2 = 2 \\ x_1 = A_3^1, x_2 = A_3^2 &\longrightarrow y = f_9(x_1, x_2) = 0.5 \cdot y_3^1 + 0.5 \cdot y_3^2 = 3 \end{aligned} \quad (27)$$

From Reference [3] and the equivalence condition, as shown in this paper, we can transform the inference results of extended simplified fuzzy inference model from (34) into

$$\begin{aligned}
y^0 &= \frac{\sum_{j_1=1}^{m_1} h_{j_1}^1}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot \frac{\sum_{j_1=1}^{m_1} h_{j_1}^1 y_{j_1}^1}{\sum_{j_1=1}^{m_1} h_{j_1}^1} + \dots \\
&\quad + \frac{\sum_{j_n=1}^{m_n} h_{j_n}^n}{\sum_{i=1}^n \sum_{j_i=1}^{m_i} h_{j_i}^i} \cdot \frac{\sum_{j_n=1}^{m_n} h_{j_n}^n y_{j_n}^n}{\sum_{j_n=1}^{m_n} h_{j_n}^n} \\
&= \frac{\sum_{i=1}^n \sum_{j=1}^{m_i} h_{j_i}^i y_{j_i}^i}{\sum_{i=1}^n \sum_{j=1}^{m_i} h_{j_i}^i} \quad (35)
\end{aligned}$$

(Confer [3] for detail of proof). (35) is equal to the inference results of the SIC model of (6). Reference [6] has shown that the monotonicity of SIC model is satisfied, if the antecedent parts and consequent parts of each input item are monotonically increasing, and the fuzzy sets constitute fuzzy partitions. Therefore, Theorem 2 holds. $\square$

Note that the monotonicity of the extended T-S inference model can be also argued as well as the extended simplified fuzzy inference model, if the consequent functions are positive value and monotonically increasing, the antecedent parts are monotonically increasing, and the fuzzy sets constitute fuzzy partitions.

V. CONCLUSION

In this paper, we proposed an extended T-S inference model in which the coefficients of consequent parts of the fuzzy rules are extended from constants to functions.

To decide the coefficient functions of the model is generally very difficult. However, we showed an example of the coefficient functions and its property, by using simplified model of this model.

We have also shown the equivalence conditions between the extended simplified fuzzy inference model and the SIC model. Since the SIC model is applied to PID control, etc., and good results are obtained, the proposed model will be also useful. Moreover, we clarified to obtain the monotonicity condition of this model easily, by using the above equivalence conditions.

Since the number of the parameters of the general proposed model, as shown in (7), is large compared with the conventional T-S inference model, it may obtain good results in various fields. However, how to select and optimization of its parameters should be investigated.

Further studies are required to find out other applicative coefficient functions, and to apply this model to real systems.

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