

# General Type-2 Fuzzy Membership Function Design and its Application to Neural Networks

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**Abstract**— Several type-1 fuzzy membership function (T1 FMF) generation methods have been proposed to model the uncertainty of pattern data. However, if we cannot obtain satisfactory results using type-1 fuzzy sets, employment of type-2 fuzzy sets (T2 FSs) for managing uncertainty may allow us to obtain desirable results. In this paper, a general T2 FMF design method and its application to back propagation (BP) neural networks is proposed. The general T2 FMF is designed using data histograms and then type-1 fuzzy membership values which are extracted from the centroid of the T2 FMF are used as inputs to the BP neural network. Applying our proposed membership assignment to the BP neural networks shows improvement of the classification performance since the uncertainty of pattern data are desirably controlled by the T2 fuzzy memberships. Experimental results for several data sets are given.

**Keywords** - type-2 fuzzy membership function generation; neural network; fuzzy input

## I. INTRODUCTION

The generation of appropriate fuzzy membership functions has been an important issue in fuzzy data analysis. Several T1 FMF generation methods have been proposed [1]-[4]. However, in the real world, there are several problems where T1 FMF may not properly represent the uncertainty of the pattern data [5]-[12]. To overcome this issue, many applications of T2 FSs using interval type-2 fuzzy membership functions (IT2 FMF) have been proposed [7]-[13]. However, the generation methods of IT2 FMF have been very rarely discussed. Recently, B. Choi and F. Rhee [7] proposed several IT2 FMF generation methods to effectively represent a pattern set.

Although IT2 FMF has been used primarily due to the computational complexity of general T2 FSs, our interest in using general T2 FMF cannot be overlooked since it may manage the uncertainties of the data more effectively due to the flexibility in its secondary membership. Moreover, the computational complexity can be reduced by employing the  $\alpha$ -plane representation to compute the centroid of T2 FSs using  $\alpha$ -plane computations that are very simple. This is possible since computations can be performed using Karnik-Mendel algorithms that are applied to each  $\alpha$ -plane [14]-[15].

In this paper, we propose a general T2 FMF generation method, where the upper and lower membership functions for

each class and feature are obtained by fitting appropriate parameterized functions to the histograms of the pattern data and the secondary membership functions are estimated by again fitting appropriate parameterized functions to the histograms of the upper and lower membership values for each vertical slice. In Section II, we briefly review histogram based IT2 FMF generation method [7]-[8]. In Section III, we propose our general T2 FMF generation method. In Section IV, we apply our proposed general T2 FMF generation method to back propagation (BP) neural networks. Experimental results are given in Section V. Finally, in Section VI, we present some conclusions.

## II. INTERVAL TYPE-2 FUZZY MEMBERSHIP FUNCTION GENERATION METHODS

In this section, we briefly introduce histogram based IT2 FMF generation method [7]-[8]. The histogram of the feature values describes the distribution of input patterns for each feature. Therefore, the membership function generated from the histogram of the input data can suitably represent the data. To generate the histogram based IT2 FMF, the histogram of the pattern data is obtained for each labeled class and feature by smoothing the domain of each feature using a symmetric window (e.g., a triangular window). In doing so, the vertex of the triangular function is positioned at the first bin of the histogram and the weighted moving average is calculated, then the vertex is moved to the next bin. This procedure is repeated for all of the bins. For simplicity, we use a one class data set. Fig. 1(a) and (b) shows a scatter plot of pattern data and the histogram of the data for feature 1, respectively.

Next, appropriate parameterized functions are fitted to the histogram. To do so, a polynomial function is first fitted to the histogram to determine the number and approximate parameter values for the parameterized function(s). The parameter values for each function are then optimized to fit the histogram using the gradient descent method. In this example, we selected a Gaussian function as our parameterized function. Fig. 1(c) shows a Gaussian function fitted to the histogram of the data.

Then, to estimate an IT2 FMF, Gaussian function fitting to the upper and lower histogram values is needed. Gaussian function fitted to the upper (lower) histogram is used as the

upper (lower) membership function. Fig. 1(d) shows the upper and lower membership function obtained that represent the IT2 FMF. The region between upper and lower membership function represents the FOU of the IT2 FMF and hence represents the IT2 FMF, since its secondary membership values are all 1.

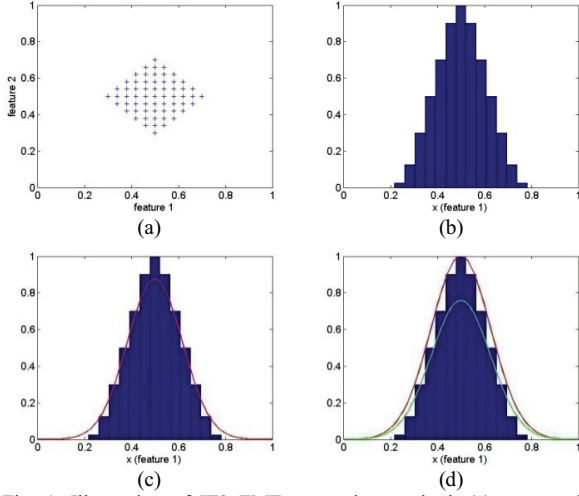


Fig. 1. Illustration of IT2 FMF generation method: (a) scatter plot, (b) histogram for feature 1, (c) Gaussian function fitting for histogram of data, and (d) upper and lower membership function.

### III. GENERAL TYPE-2 FUZZY MEMBERSHIP FUNCTION DESIGN METHOD

In this section, we propose a general T2 FMF design method. For explanation purposes, we use the same data as in Fig. 1. First, to estimate the FOU of a general T2 FMF, we perform Gaussian function fitting to the histogram of data set as in the IT2 FMF generation method. The difference between the general T2 FMF and IT2 FMF generation method is to use window functions of different size for smoothing to obtain the histogram. For example, if we choose to use 6 triangular window functions of size 3, 5, 6, 7, 9, and 13 (i.e., base length), we can obtain 6 smoothed histograms each having an upper and lower membership function. Among all upper and lower membership functions, the smallest value is considered to be the lower membership value and the largest value is considered to be upper membership value for each vertical slice. For illustration purposes, Fig. 2(a) and (b) shows the smoothed histogram and upper/lower membership function for only window size 3 and 7, respectively. Fig. 2(c) represents upper and lower membership functions obtained by fitting Gaussian functions to all 6 smoothed histograms and Fig. 2(d) shows FOU of the general T2 FMF.

Next, the histogram for the upper and lower membership values is obtained at each vertical slice  $x$  domain values. For example, if six windows of different size are used we can get six upper and lower membership values at each vertical slice  $x$  domain value. Then we can obtain the histogram for the membership value domain at each vertical slice. Here, a smoothing window is again used to smooth the histogram. To estimate the secondary membership function, Gaussian function fitting to this histogram is performed. Fig. 2(e)

illustrates the secondary membership function estimated by fitting the histogram of upper and lower membership values at  $x = 0.35$ , which denotes the vertical slice at  $x = 0.35$ . With this procedure, we can estimate the secondary membership function for all  $x$  domain values. Fig. 2(f) shows the 3D version of the general T2 FMF.

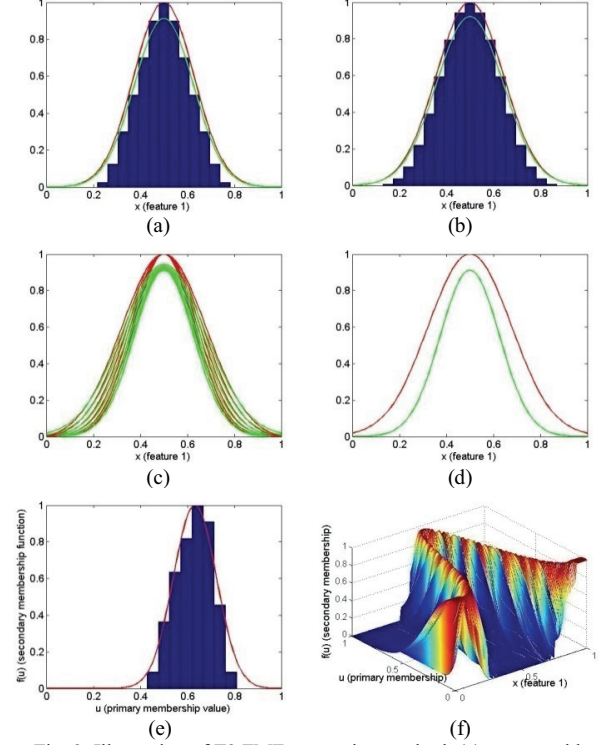


Fig. 2. Illustration of T2 FMF generation method: (a) upper and lower membership functions using 3-window size, (b) upper and lower membership functions using 7-window size, (c) all upper and lower membership functions, (d) FOU, (e) vertical slice at  $x = 0.35$ , and (f) 3D plot

### IV. APPLICATION TO BACKPROPAGATION NEURAL NETWORK

To apply T2 FSs to the BP neural network, we extract type-1 fuzzy membership values using the centroid of each T2 FMF which is calculated by performing type reduction and defuzzification. The centroid of a T2 FS, which is  $C_{\tilde{A}}(x)$ , can be represented by the union of the centroids of all the embedded T2 FSs of  $\tilde{A}$ , i.e., [12]

$$C_{\tilde{A}}(x) = \int_{\forall \theta_1 \in J_{x_1}} \cdots \int_{\forall \theta_N \in J_{x_N}} \min(f_{x_1}(\theta_1), \dots, f_{x_N}(\theta_N)) / \frac{\sum_{i=1}^N x_i \theta_i}{\sum_{i=1}^N \theta_i}, \quad (1)$$

where  $x_i$  is input feature pattern and  $\theta_i$  is the primary membership for  $x_i$ . This centroid can be computed using  $\alpha$ -plane representations [14]-[15].

The  $\alpha$ -plane representation is as follows,

$$C_{\tilde{A}}(x) = \bigcup_{\alpha \in [0,1]} \alpha / C_{\tilde{A}_\alpha}(x), \quad (2)$$

where  $C_{\tilde{A}_\alpha}(x)$  is the centroid of the  $\alpha$ -plane,  $\tilde{A}_\alpha$ . Since each  $\tilde{A}_\alpha$  is an IT2 FS, its centroid is an interval set, i.e.,

$$C_{\tilde{A}_\alpha}(x) = [l_{\tilde{A}_\alpha}, r_{\tilde{A}_\alpha}]. \quad (3)$$

This centroid can be obtained using the enhanced Karnik-Mendel algorithms [13] that are applied to each  $\alpha$ -plane and the defuzzification value of each  $\alpha$ -plane can be calculated as follows,

$$C_\alpha = \frac{l_{\tilde{A}_\alpha} + r_{\tilde{A}_\alpha}}{2}. \quad (4)$$

Then, the centroid of  $\tilde{A}$ ,  $C$ , is obtained by calculating the mean value of defuzzification values of each  $\alpha$ -plane, i.e.,

$$C = \frac{\sum_{\text{All } \alpha} \alpha C_\alpha}{\sum_{\text{All } \alpha} \alpha}. \quad (5)$$

Now, we can extract the type-1 fuzzy membership value of input pattern  $\mathbf{x}$  for class  $j$  according to the distance between the centroid and input pattern, i.e., [7]-[8]

$$f_j(\mathbf{x}) = 1 - |\mathbf{x} - C_j|, \quad (6)$$

where  $C_j$  is the centroid of class  $j$ . It is to be noted that the input patterns are normalized in the interval  $[0,1]$ .

The type-1 fuzzy membership values of input patterns for each class are used as inputs to the BP neural network. Therefore, the dimension of the input to the BP neural network is the same as the number of classes. Fig. 3 illustrates the structure of our proposed general T2 FMF generation method applied to a BP neural network.

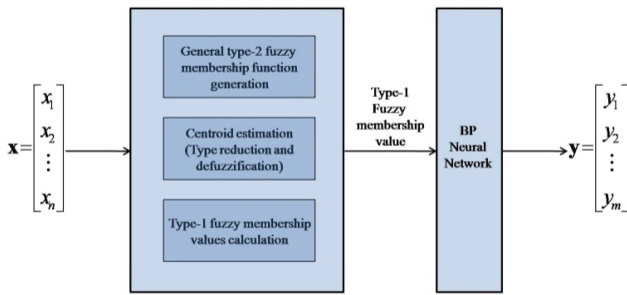


Fig. 3. Structure of the proposed general T2 FMF generation method applied in a BP neural network.

## V. EXPERIMENTAL RESULTS

To show the performance of our proposed general T2 FMF design, BP neural network classification results are given for “T-shape” data sets and segmentation results for real images.

### A. “T-shape” data

For our first example, we use the “T-shape” data set that

consists of 447 patterns (228 and 219 patterns for each class) with 2 features and 2 classes. All pattern features are normalized between 0 and 1. We select 10% patterns for testing and the remaining patterns for training. This procedure is repeated for all the patterns, hence, every pattern will be tested only one time. Fig. 4 shows the scatter plot of the data.

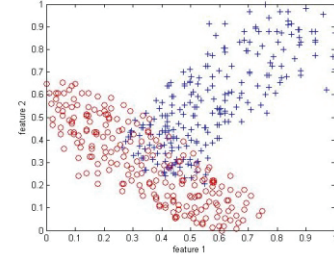
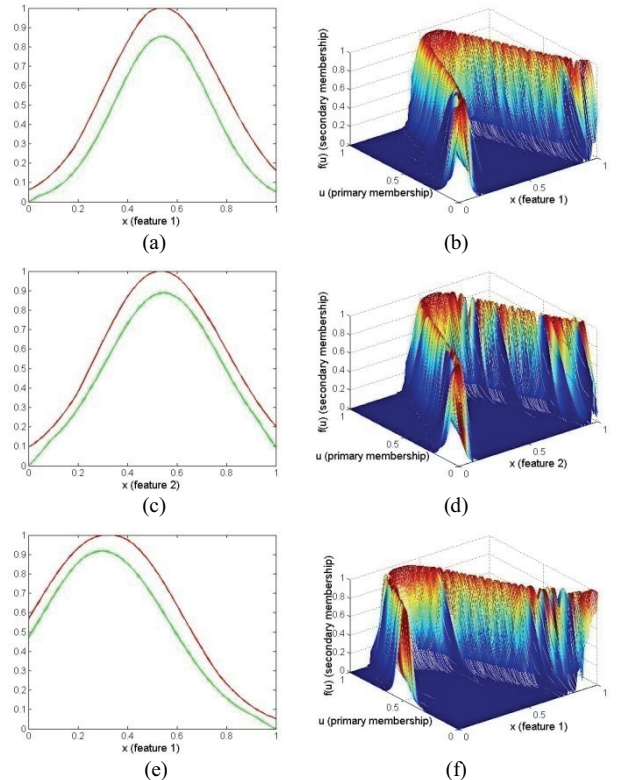


Fig. 4. Scatter plot for “T-shape” data.

As described above, we obtain general T2 FMFs by fitting Gaussian functions to the histograms of the lower and upper membership values at each vertical slice  $x$  domain values for each class and feature, which are obtained from the histograms of the training data. Fig. 5(a) and (b) ((c) and (d)) shows the FOU and 3D plot of the general T2 FMF for class 1 of feature 1(2), respectively. Fig. 5(e) and (f) ((g) and (h)) shows the FOU and 3D plot of the general T2 FMF for class 2 of feature 1(2), respectively.

We next estimate the centroid of the general T2 FSs by performing type reduction and defuzzification procedure using the  $\alpha$ -plane representation and Karnik-Mendel algorithms. We extract type-1 fuzzy membership values for training data using the distance between the centroid and training data.



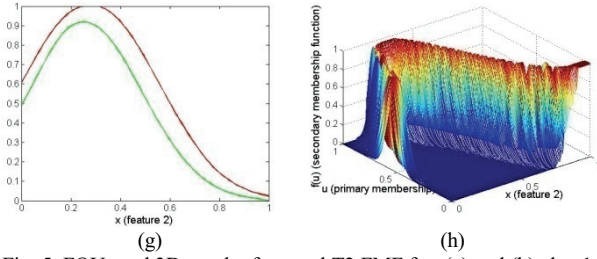


Fig. 5. FOU and 3D graph of general T2 FMF for: (a) and (b) class1 of feature1, (c) and (d) class 1 of feature2, (e) and (f) class2 of feature1, and (g) and (h) class2 of feature2.

Next, we train a BP neural network using the type-1 fuzzy membership values for training data as inputs. The test patterns are then classified using the trained BP neural network. Fig. 6 shows the classification results for the type-1 fuzzy membership values using IT2 FMF and general T2 FMF method, and for crisp input patterns as inputs to the BP neural network according to the number of hidden neurons. As shown in figure, BP neural networks trained with fuzzy inputs outperform the case with crisp inputs.

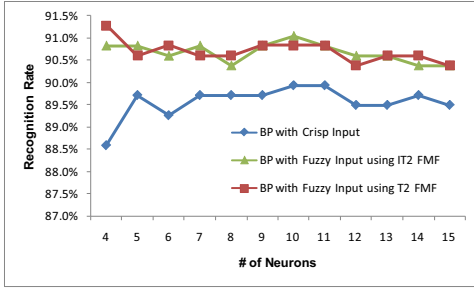


Fig. 6. Classification results for "T-shape" data.

### B. Real image data segmentation

In this example, we show segmentation results for a 200×200 real scene image that consists of three regions, namely road, forest, and sky. The gray levels of median and excess green filtered images were used as feature values. We then randomly select 100 samples in each region and the feature values are used for training data. Fig. 7(a) and (b) shows the original image and the scatter plot of randomly selected sample data, respectively.

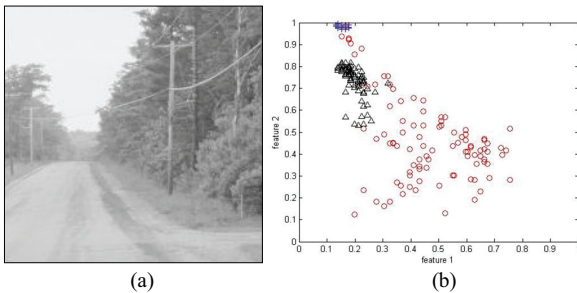


Fig. 7. Real image example: (a) original image and (b) scatter plot of sample data.

As in the previous example, we generate general T2 FMFs and extract type-1 fuzzy membership values for training data.

Next, we train BP neural networks using the type-1 fuzzy membership values for training data. Finally, fuzzy membership values for all feature data of the original image were classified into three classes using the trained BP neural network. Fig. 8(a), (b) and (c) shows the segmentation results for crisp inputs, fuzzy inputs using the IT2 FMF, and fuzzy inputs using the general T2 FMF, respectively, when 10 hidden neurons were used. Fig. 9 shows the recognition rate according to the number of hidden neurons. The recognition rate was improved by 3.2% compared to IT2 FMF method. Table I shows the confusion matrix for the results in Fig. 8. Although the recognition showed little improvement, the segmentation result showed a more significant visual improvement.

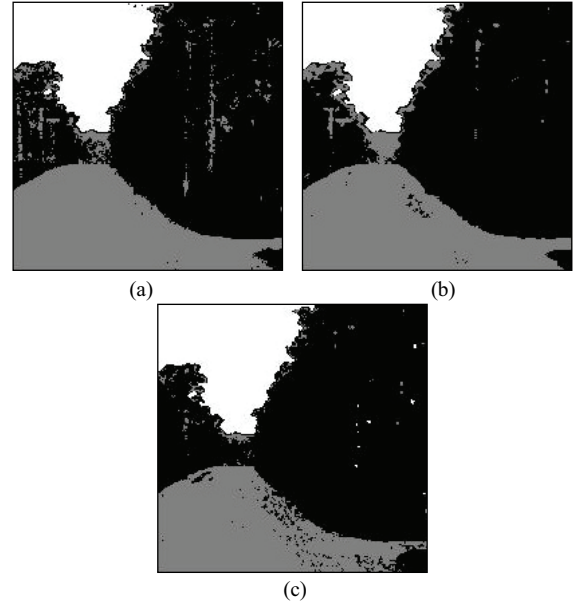


Fig. 8. Segmentation results for BP neural network with 10 hidden neurons: (a) crisp input, (b) fuzzy input using IT2 FMF, and (c) fuzzy input using T2 FMF.

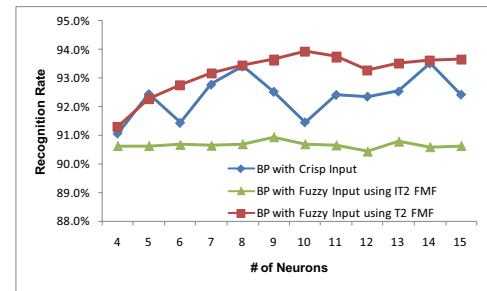


Fig. 9. Recognition rate of real image for various number of neurons.

TABLE I  
CONFUSION MATRIX FOR SEGMENTATION RESULTS IN FIG. 8(B) AND (C)

| Real Image | BP neural network using fuzzy inputs using IT2 FMF<br>Recognition rate = 90.7% |        |      | BP neural network using fuzzy inputs using T2 FMF<br>Recognition rate = 93.9% |        |      |
|------------|--------------------------------------------------------------------------------|--------|------|-------------------------------------------------------------------------------|--------|------|
|            | Road                                                                           | Forest | Sky  | Road                                                                          | Forest | Sky  |
| Road       | 8150                                                                           | 70     | 0    | 7876                                                                          | 344    | 0    |
| Forest     | 3162                                                                           | 21785  | 499  | 1536                                                                          | 23381  | 529  |
| Sky        | 3                                                                              | 16     | 6315 | 3                                                                             | 16     | 6315 |

### C. Noisy image data segmentation

For our final experiment, we apply 5% Gaussian noise to Fig. 7(a). The training and testing procedure are the same as previous example. Fig. 10(a) and (b) shows the noisy image and the scatter plot of sample data, respectively. Fig. 11(a), (b), and (c) illustrates the segmentation results for crisp inputs, fuzzy inputs using IT2 FMF, and fuzzy inputs using general T2 FMF, respectively, when 8 hidden neurons were used in BP neural network training. As shown in Fig. 12, the recognition rate was improved about 4% at most compared to crisp input and the variation of results for various neurons is more stable than the IT2 FMF method.

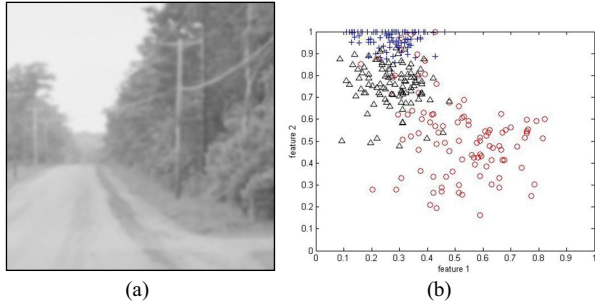


Fig. 10. Real noisy image example: (a) noisy image and (b) scatter plot of sample data.

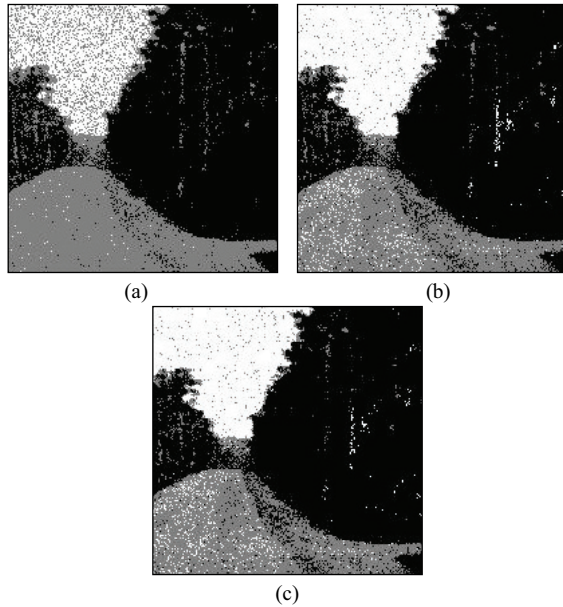


Fig. 11. Segmentation results for BP neural network with 8 hidden neurons: (a) crisp input, (b) fuzzy input using IT2 FMF, and (c) fuzzy input using T2 FMF.

## VI. CONCLUSION

In this paper, we proposed a general T2 FMF design method and applied the type-1 fuzzy membership values, which are estimated from the centroid of the T2 FMF, as inputs to train BP neural networks. The experimental results showed an improvement in classification performance over the case when crisp inputs were trained and IT2 FMF was used.

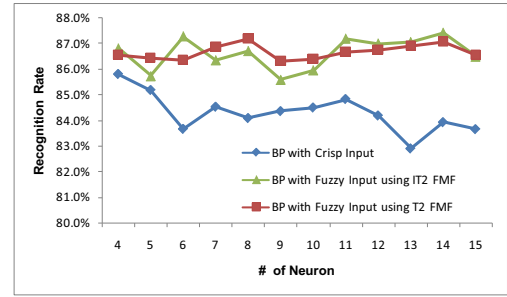


Fig. 12. Recognition rate of noisy image for various number of neurons.

This indicates that our proposed general T2 FMF design method can effectively manage the uncertainty of pattern data. Other general T2 FMF generation methods are currently under investigation.

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