

An Interval Type 2 Fuzzy Approach to Multilevel Image Segmentation

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Abstract—In this paper, an interval type 2 (IT2) fuzzy entropy based approach is used to compute optimum thresholds for multistage gray scale image segmentation. By finding the maximum IT2 fuzzy entropy of the gray scale image, the optimum thresholds are computed. A termination criterion for multistage segmentation is also proposed based on the range of computed entropy values of the images. Several experimental results are shown to demonstrate the performance of our proposed algorithm which outperforms type 1 (T1) fuzzy based methods.

Keywords—Interval type2 (IT2) fuzzy sets, Footprint of uncertainty (FOU), Fuzzy membership function, Ultrafuzziness.

I. INTRODUCTION

Finding the optimal threshold is a critical step in gray scale image segmentation. Gray scale image segmentation is based on the pixel properties of the image and determining the appropriate threshold can be very challenging. In literature there exist several threshold based image segmentation techniques. According to the information exploitation, threshold techniques have been categorized as histogram shape based, clustering based, entropy based, object attribute based and spatial based [1]–[8].

Several fuzzy based methods have also been proposed. Those methods include fuzzy clustering for finding the optimal threshold and maximum entropy gray level computation to distinguish between the foreground and background [4], [9], [10]. Optimal threshold for separation of the background and object exists at the deepest valley of bimodal histogram of the image [11]. However, it is difficult to obtain optimal threshold when distribution of histogram is complex. To cope with this problem, fuzzy entropy was proposed to quantify the vagueness in images [12]. Huang *et al.* introduced a fuzzy approach to the Shannon entropy and concluded that the gray level point where the fuzziness is minimum; is the optimal threshold [13]. Information theoretic approach was used to maximize (minimize) the image information on basis of fuzziness or crispness [14]. This approach is very simple and commonly used for finding the optimal threshold. Rhee *et al.* proposed a numerical method to quickly locate the optimal threshold using fuzzy sets [15].

Type 2 fuzzy sets have been shown to better handle uncertainties than conventional type 1 fuzzy sets [16]. The ability of type 2 fuzzy sets to represent and manage the

uncertainty that exists in a fuzzy set allows us to model various uncertainties, which cannot be appropriately managed by type 1 fuzzy sets [17]. Tizhoosh introduced a threshold technique based on type 2 fuzzy sets (i.e., ultrafuzzy sets) [18]. In this paper, we address two problems associated with automatic multilevel image segmentation of gray level images. The first problem is to calculate optimal threshold for segmentation. In doing so, we propose a new segmentation method that incorporates interval type 2 (IT2) fuzzy sets. This is achieved by modeling Gaussian interval type 2 (IT2) fuzzy membership functions using embedded type2 fuzziness [19]. Our goal is to decide whether a pixel belongs to the background or foreground. In this case, measurements of uncertainty can be considered as entropy, index of fuzziness and index of non fuzziness. We obtain the optimal threshold by maximizing the amount of ultrafuzziness. A membership value is then assigned to each pixel depending on its distance from the background and foreground center using some parameterized type 1 fuzzy membership function. Based on the membership values we find the threshold by maximizing the index of ultrafuzziness. The second one is to determine the optimum number of thresholds for an image in the segmentation process. Theoretically, there are as many objects in a gray level image as there are gray levels present in the image, but practically it is very difficult to segment it based on the relevant information present. Furthermore, we propose a termination criterion, which calculates and segments objects based on image entropy information. The concept of this termination criterion is based on segmenting the image in different levels using a tree structure and there is no restriction on the number of objects to be segmented.

The remainder of this paper is organized as follows. Section II contains the brief background of IT2 fuzzy sets and membership function generation used in thresholding. In Section III, our proposed segmentation algorithm and termination criterion are discussed. In Section IV, experimental results and discussion for bimodal and multi stage image segmentation are presented to show the effectiveness of our proposed method. Finally, Section V gives the conclusions.

II. BACKGROUND

A. Interval Type 2 Fuzzy sets (IT2 FS)

When we represent uncertainty for a fuzzy set with an interval type 2 fuzzy set, the primary membership J_x' of a

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sample x' can be represented as a membership interval with all secondary grades of the primary memberships equaling to one [20]. We define this interval as the footprint of uncertainty (FOU).

An interval type 2 fuzzy set $\tilde{A}$ can be represented as

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) \mid \forall x \in A, \forall u \in J_x \subseteq [0, 1], \mu_{\tilde{A}}(x, u) = 1\} \quad (1)$$

Fig. 1 illustrates an example of an interval type 2 fuzzy set where the gray shaded region indicates the FOU. In the figure, the membership value for a 1-dimensional sample x' is represented by the interval between upper ($\bar{\mu}_{\tilde{A}}(x')$) and lower ($\underline{\mu}_{\tilde{A}}(x')$) membership [20]. Therefore, each x' has a primary membership interval as

$$J_{x'} = [\underline{\mu}_{\tilde{A}}(x'), \bar{\mu}_{\tilde{A}}(x')] \quad (2)$$

and the vertical slice of x' is shown in Fig. 1(b), where the secondary grade for the primary membership of each x' equals one, according to the property of interval type-2 fuzzy sets.

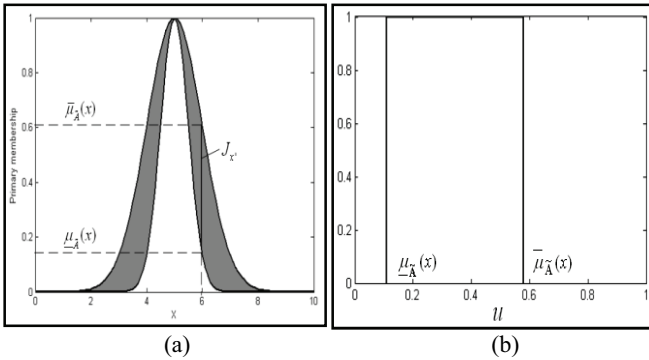


Fig. 1. Illustration of an interval type-2 fuzzy set: (a) interval type-2 fuzzy set (b) vertical slice on x' .

B. Membership Function Generation

Our main concern of this research is to see whether IT2 fuzzy sets can properly handle uncertainties in comparison to T1 and non fuzzy techniques. Fuzziness depends upon the choice of membership function. Fig. 2 shows examples of membership functions used for measurement of fuzziness in an image. In this paper, we consider Gaussian membership functions (see Fig. 3) and model IT2 membership functions using linguistic hedges (see Fig. 4). Existing methods also used linguistic hedges to create IT2 fuzzy membership functions [21], [22]. The linguistic hedges were obtained by conducting experiments to create the FOU, where the values of linguistic hedges depend on statistical properties of the histogram. It is observed that different membership functions exhibit different segmentation results depending upon the gray level distribution of an image.

Let I denotes an image of size $M \times N$ having L numbers of gray levels and $i(x, y)$ is the gray level of pixel at (x, y) in I . Let

$\mu_I(i(x, y))$ denotes the membership value of pixel $i(x, y)$ in image I , then we can define fuzzy image set as

$$I = \{i(x, y), \mu_I(i(x, y))\}, \quad (3)$$

where $0 \leq \mu_I(i(x, y)) \leq 1$, $x = 0, \dots, M-1$ and $y = 0, \dots, N-1$.

Let $h(g)$ denotes the histogram of an image I . For a given threshold value T , the average levels (center) of the background and object, represented by μ_0 and μ_1 , respectively, can be calculated by (4) and (5).

$$\mu_0 = \frac{\sum_{g=0}^T g h(g)}{\sum_{g=0}^T h(g)} \quad \text{if } i(x, y) \leq T \quad (4)$$

and

$$\mu_1 = \frac{\sum_{g=T+1}^{L-1} g h(g)}{\sum_{g=T+1}^{L-1} h(g)} \quad \text{if } i(x, y) > T, \quad (5)$$

The following definitions of fuzziness (entropy) were used to construct type 1 and type 2 fuzzy membership functions [19].

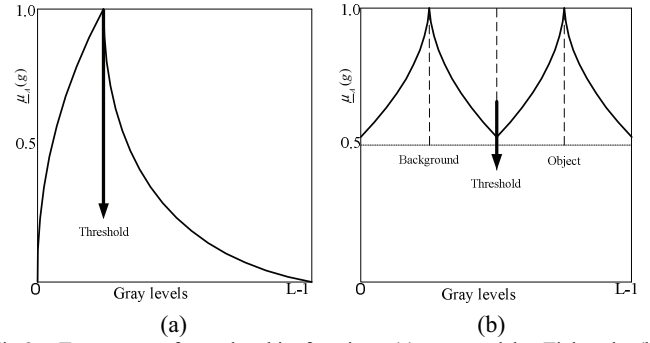


Fig. 2. Two types of membership functions (a) proposed by Tizhoosh .(b) Proposed by Huang .

Definition 1: A general fuzziness measure of a type 1 fuzzy set I , $f(I)$, can be defined as

$$f(I) = k\left(\sum_{i=1}^N q(\mu_I(i(x, y)))\right), \quad (6)$$

where k is monotonically increasing function from R^+ to R^+ and $q: [0, 1] \rightarrow R^+$ is a function associated with each $i(x, y)$. N is the total number of elements in I . Additionally,

- (1) $q(0) = q(1) = 0$;
- (2) $q(0.5)$ is a unique maximum of q ;
- (3) q must be monotonically increasing on $[0, 0.5]$ and monotonically decreasing on $[0.5, 1]$.

Every pixel in input image should belong to either background or object to some degree of membership depending upon the gray level values.

The type 1 fuzzy membership function $\mu(g)$, g is the gray level, is defined as

$$\mu_I(g) = \begin{cases} 1.5 - \frac{1}{1 + e^{-\frac{1}{2}\left(\frac{g-\mu_0}{\sigma}\right)^2}} & \text{if } g_{\min} \leq g \leq T \\ 1.5 - \frac{1}{1 + e^{-\frac{1}{2}\left(\frac{g-\mu_1}{\sigma}\right)^2}} & \text{if } g_{\max} \geq g > T, \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

where $\sigma = |g_{\max} - g_{\min}| / 2 \log(1/TOL)$ is the standard deviation and TOL is suitably selected as 0.01. The minimum and maximum occurring non zero gray levels in an image I are denoted as $g_{\min}$ and $g_{\max}$, respectively.

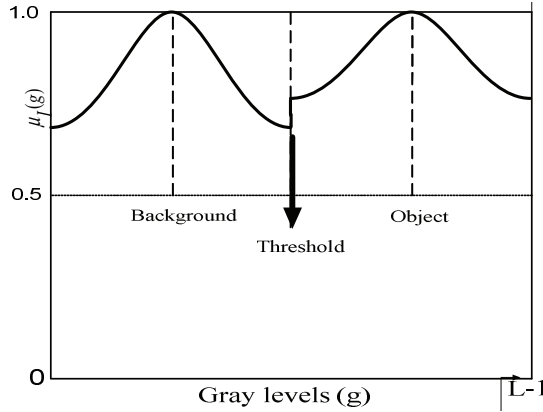


Fig.3. Proposed Gaussian Type 1 fuzzy membership function.

Definition 2: The fuzziness definition of an IT2 fuzzy set $F_{\tilde{A}}$ is the union of the fuzziness of all its embedded type 1 fuzzy sets $\mu_I(g)$, i.e.

$$F_{\tilde{A}} \equiv \bigcup_{\forall \mu_I(g)} f(\mu_I(g)) = [f_l(\tilde{A}), f_r(\tilde{A})], \quad (8)$$

where $f_l(\tilde{A})$ and $f_r(\tilde{A})$ are the minimum and maximum of fuzziness for all $\mu_I(g)$, i.e.

$$f_l(\tilde{A}) = \min_{\forall \mu_I(g)} f(\mu_I(g)) \quad (9)$$

and

$$f_r(\tilde{A}) = \max_{\forall \mu_I(g)} f(\mu_I(g)), \quad (10)$$

Here we provide a simple proof to show that our proposed IT2 MF shown in Fig. 4 follows the above definition. For that we need to show that

$$f_l(\tilde{A}) = \bar{\mu}(g) \quad (11)$$

and

$$f_r(\tilde{A}) = \begin{cases} \underline{\mu}(g); 0.5 \notin [\underline{\mu}(g), \bar{\mu}(g)] \\ 0.5 & \text{otherwise} \end{cases}, \quad (12)$$

Proof: According to definition 1, the fuzziness of a particular gray level g is determined by its deviation from 0.5, the value where it is maximally fuzzy. Membership values that approach 0 or 1 become less fuzzy. The minimum of fuzziness $f_l(\tilde{A})$ is achieved when every value $\mu_I(g)$ is as far away as possible from 0.5 and is satisfied by $\bar{\mu}(g)$ for all gray levels in (11). Similarly, $f_r(\tilde{A})$ is achieved when every $\mu_I(g)$ is as close as possible to 0.5 and is satisfied by $\underline{\mu}(g)$ for all gray levels. When $0.5 \in [\underline{\mu}(g), \bar{\mu}(g)]$, we set $f_r(\tilde{A}) = 0.5$. Therefore, (12) is satisfied.

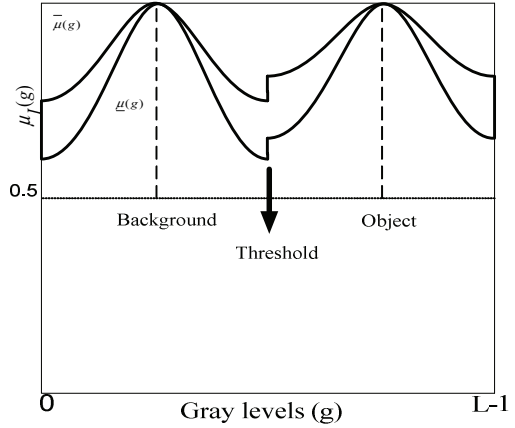


Fig. 4. Modified IT2 modeling of proposed membership function

The upper and lower membership degrees $\bar{\mu}(g)$ and $\underline{\mu}(g)$ can be obtained by linguistic hedges such as deaccentuation and accentuation

$$\bar{\mu}(g) = [\bar{\mu}(g)]^{1/\alpha} \quad (13)$$

And

$$\underline{\mu}(g) = [\underline{\mu}(g)]^\alpha, \quad (14)$$

where $\alpha \in (1, 2]$ [18].

A measure for IT2 ultrafuzziness can be modified for an image as follows

$$\tilde{\gamma}(\tilde{A}) = \frac{1}{MN} \sum_{g=0}^{L-1} h(g) [\bar{\mu}(g) - \underline{\mu}(g)], \quad (15)$$

where $h(g)$ is the histogram of image I [23].

The measure of ultrafuzziness $\tilde{\gamma}(\tilde{A})$ should satisfy following four conditions [24]

- (1) *Minimum ultrafuzziness:* $\tilde{\gamma}(\tilde{A}) = 0$ if $\mu_I(g)$ is a type 1 fuzzy set $\forall g \in X$ $\bar{\mu}(g) = \underline{\mu}(g)$
- (2) *Equal ultrafuzziness:* $\tilde{\gamma}(\tilde{A}) = \tilde{\gamma}(\tilde{A}')$
- (3) *Reduced ultrafuzziness:* $\tilde{\gamma}(\tilde{A}) \geq \tilde{\gamma}(\tilde{A}')$ if $\tilde{A}'$ is a crisper version of $\tilde{A}$

- (4) *Maximum ultrafuzziness*: $\tilde{\gamma}(\tilde{A}) = 1$ if
 $\forall g \in X \quad \bar{\mu}(g) - \underline{\mu}(g) = 1$

When the ultrafuzziness measure for each gray level is computed, these results in a bell shaped curve [15]. We now need to find the gray level that corresponds to the maximum ultrafuzziness (i.e., optimum gray level g_{opt}).

III. PROPOSED ALGORITHM

Following are the steps involved to describe our proposed multilevel algorithm that includes a termination criterion.

Step 1 Compute Ultrafuzziness

Compute IT2 fuzzy entropy of input gray scale image at each gray level using (15) as shown in Fig. 5.

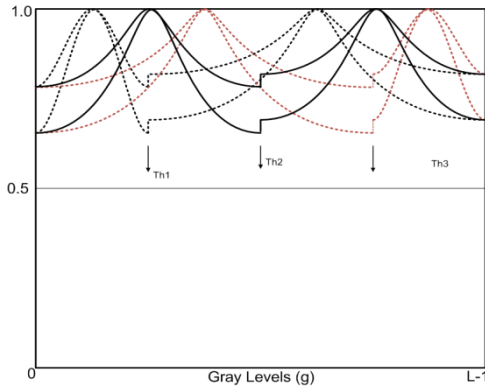


Fig. 5. IT2 MF shifted over gray level histogram range to find maximum fuzziness gray level point. (Shown for proposed Gaussian Membership Function)

Step 2 Selecting Optimum Threshold

Compute optimum threshold gray level g_{opt} at which the ultrafuzziness is maximized by selecting the gray level corresponding to the minimum of the computed entropy curve in the interval.

Step 3 Extensions to Multilevel Thresholding:

Background and foreground threshold images are obtained based on g_{opt} obtained in Step 2. The foreground may have more than one significant objects and an iterative segmentation approach can be applied to separate foreground into two or more regions by computing another g_{opt} (as in Step 2) in the sections of histogram, obtained after each level of segmentation. For determining the number of times we need to compute threshold for a given image, we use a termination criterion based on the shape of entropy curve.

Step 4 Termination Criterion

The termination criterion used in our experiments is obtained from the entropy values computed for a particular section of the histogram during the multilevel threshold procedure. The experiments on the synthetic images show that the thresholded regions of the image have reduced the ultrafuzziness. Since the entropy curve shows the overall ultrafuzziness measure trend for the entire image or sub images, we introduce the termination criterion factor τ as

$$\tau = \frac{\Delta E^{(i)}}{\Psi^{(i)}} \times 255, \quad (16)$$

where $\Delta E^{(i)}$ denotes the total number of pixels and $\Psi^{(i)} = E_{\text{max}} - E_{\text{min}}$ corresponds to the section of the histogram obtained at the i^{th} level of thresholding.

We start our termination algorithm by segmenting the original image and observing the τ -values for the section of original histogram that is obtained at each level of thresholding. For each level, we compare the current τ -value with the τ -value of the previous level. If we observe saturation in the τ -value in a particular node of a segmenting tree as shown in Fig.6, we stop branching from that particular node. This process is continued until we reach a level where all the possible paths become saturated.

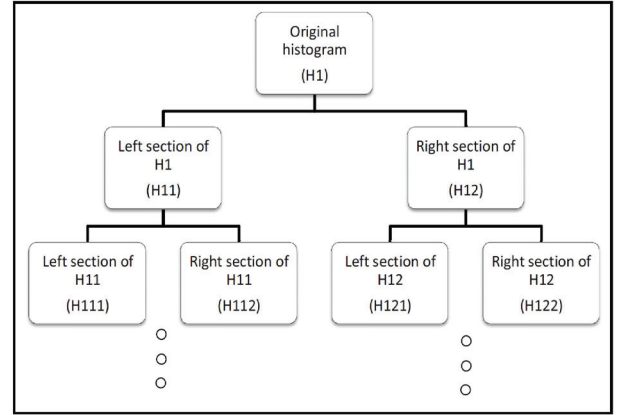


Fig.6. The numbering nomenclature that is used to denote different sections of the image histogram constructed as a tree.

Summary of termination criterion

Let H denotes the histogram of the input image I .

Step 1 Perform the first level segmentation with g_{opt} .

Step 2 Calculate τ as defined in (16) for the first level, based on entropy values.

Step 3 Create right and left histogram sections H_R and H_L about g_{opt} of image histogram H .

Step 4 Calculate τ_r and τ_l , where τ_r and τ_l are τ -values for right and left sections of histogram H .

Step 5 Compare τ_r and τ_l for the i^{th} level segmentation with τ -value of parent histogram H for the $(i-1)^{\text{th}}$ threshold.

Step 6 **IF** $\text{abs}(\tau_r - \tau) < \text{TOL}$ **THEN** Stop thresholding H_R
ELSE Assign H_R as H and return to Step 3
IF $\text{abs}(\tau_l - \tau) < \text{TOL}$ **THEN** Stop thresholding H_L
ELSE Assign H_L as H and return to Step 3

The above procedure automatically calculates the number of significant objects present in the input image. The tolerance value considered for our experiments is 0.01.

IV. EXPERIMENTAL RESULTS

In this section, experimental results are presented to illustrate the effectiveness of our proposed algorithm. We have

used Gaussian Membership function as our choice of parameterized function as described in Section II.B. The input images used in our experiments are shown in Fig.7.

The results of IT2 fuzzy based segmentation are presented in Fig. 8, 9, and 10. Segmented regions are shown with different gray levels. Performance comparison of type 1 and IT2 approaches have been performed by using ground truth image (mask). In general, to evaluate and compare image segmentation algorithms, masks are visually generated by different subjects as in [18] and [25]. Our masks follow this method and have already been used in [26]. A measure of performance η can be used to compare the results for a particular threshold stage as

$$\eta = 100 \times \frac{|M_1 \cap I_1| + \dots + |M_n \cap I_n|}{|I_1| + \dots + |I_n|}, \quad (17)$$

where, I_i and M_i denote the i^{th} class pixels in the original image and mask, respectively, after n multilevel threshold levels.

Fig. 8 shows that the g_{opt} obtained using the IT2 approach gives us better results than the T1 approach. In Table I optimal threshold g_{opt} for type 1 and IT2 fuzzy entropies for some images along with comparison of performances in percentage are listed. Table II shows multilevel threshold results in terms of g_{opt} . The number of threshold levels is determined by the proposed termination criterion. In Table V, the comparison of results obtained by using two different membership functions is shown.

It is to be noted that our proposed algorithm is different from Tizhoosh algorithm in respect to the MF generation method. We have compared our algorithm results with Tizhoosh results and concluded that our proposed MF gives satisfactory results. The second major contribution is automatic selection of number of thresholds based on the information of the IT2 fuzziness of the given image.

Results of Termination Criterion:

Experiments on a synthetic image

Fig.11 shows a synthetically generated image. The entropy curves are also shown along with the segmented images. We conducted experiments on the synthetic image to illustrate our termination criterion, which is based on the normalized range of entropy of the image. The normalization is performed with respect to the number of pixels present in the region of image that corresponds to the subinterval of interest in the histogram.

From Table III, we can observe that the τ -value decreases as the information in the subinterval decreases. We can utilize this information to determine the termination point of the segmentation process with the optimum threshold. We normalize with respect to the number of pixels since it performs better than normalizing with the gray levels. We define the images produced after 3 levels of segmentation in all possible ways, as shown in Fig.11. The experiments performed using synthetic image show that the segmentation steps that should be taken on the basis of above criterion are: 1, 2 and 5 as shown in Table VI.

The results of the termination criterion for the s2 real image are presented in Table IV and we can observe the validity of our termination criterion.

To calculate the efficiency of our multilevel proposed scheme we have calculated the recognition rate based on the masks shown in Fig.12. For this, Gaussian MF and Tizhoosh's MF using linguistic hedges are compared. The results are presented in Table V. The results show that using linguistic hedges from the histograms give desirable results.



Fig. 7. Input images used for experimental results- row1 from left to right block, table, s5; row2 left to right London, s2, s3; row3 from left to right cameraman, Lena and wolf; row4 is baseball.

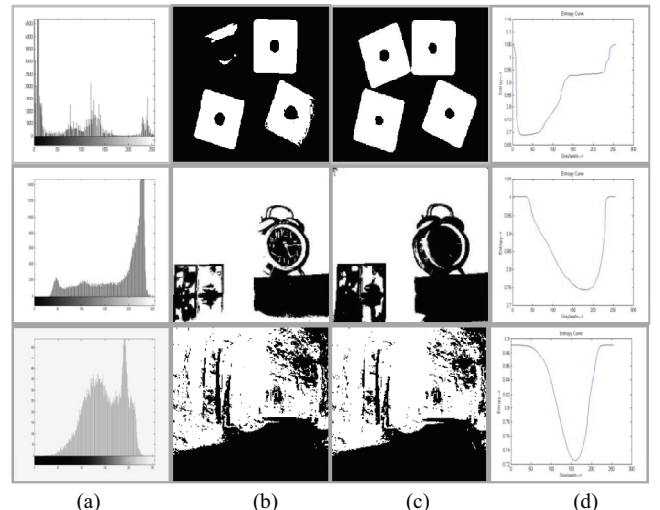


Fig. 8. Segmentation results of images block, table, and s5 showing row wise left to right column (a) shows histograms (b) shows T1 result (c) shows IT2 result and (d) shows IT2 entropy curve.

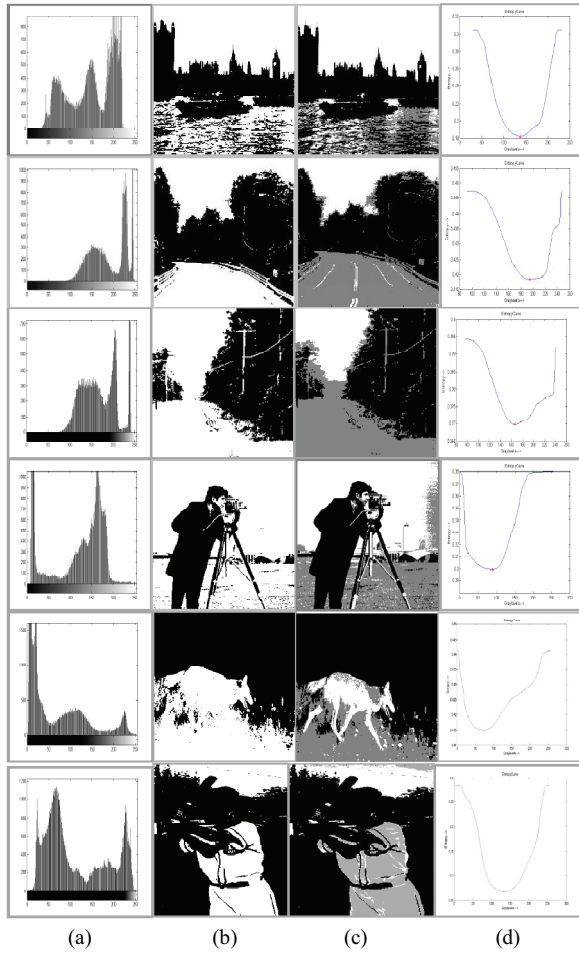


Fig. 9. IT2 based two stage thresholding of images London, s2, s3, cameraman, wolf, and baseball (top to bottom) columns showing (a) input image histogram, (b) stage1 thresholding, (c) stage2 thresholding, and (d) IT2 entropy curve.

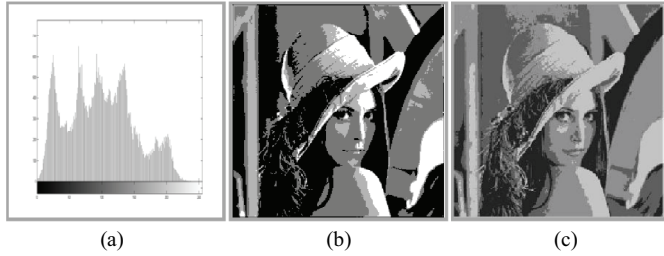


Fig. 10. IT2 based five stage thresholding for Lena from left to right column showing (a) input image (b) segmented image using proposed method and (c) segmented image by S. Arora et al. [27]

TABLE I
PERFORMANCE MEASURE COMPARISON OF T1 AND IT2 FUZZINESS

Image	T1		IT2	
	g_{opt}	% performance	g_{opt}	% performance
Block	110	97.97	50	99.42
Table	161	89.33	182	94.34
s5	154	79.80	159	82.76

TABLE II
OPTIMAL THRESHOLD IN MULTILEVEL IMAGE

Images	London	s2	s3	Camera man	Lena	Wolf	Baseball
level 1	141	194	167	59	112	71	137
level 2	158	235	214	147	158	162	197
level 3	-	-	-	-	186	-	-

TABLE III
 τ -VALUES NORMALIZED BY NUMBER OF PIXELS FOR SYNTHETIC IMAGE (TABLE FOLLOWS THE TREE DIAGRAM SHOWN IN FIG.6)

Levels							
Level 1	11.7						
Level 2	5.4			7.7			
Level 3	5.2	1.7		2.4	1.9		
Level 4	5.1	1	1.4	1.2	1.2	1.7	0.3

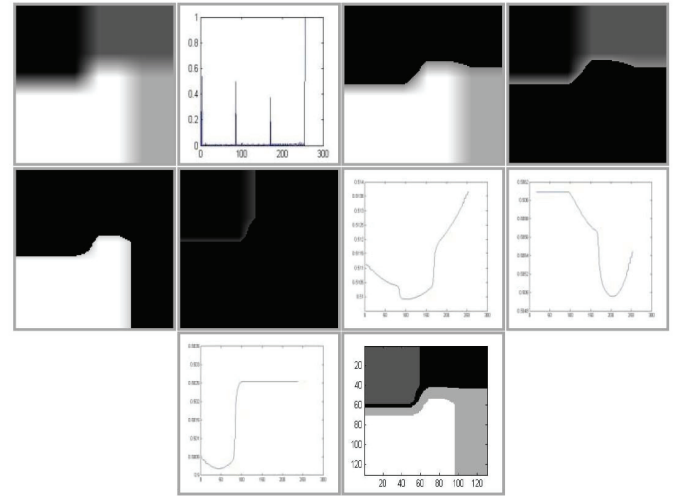


Fig. 11. IT2 based two stage thresholding for a synthetic image showing (a) input synthetic image (b) histogram of input image (c) stage1 thresholding (showing right histogram region) (d) stage1 thresholding (showing left histogram region) (e) stage 2 thresholding (right histogram region) (f) stage 2 thresholding (left histogram region) (g) entropy curve for stage 1 thresholding (h) entropy curve for stage 2 thresholding (right histogram) (i) entropy curve for stage 2 thresholding (left histogram) (j) overall segmented image.

TABLE IV
 τ -VALUES NORMALIZED BY NUMBER OF PIXELS FOR S2 REAL IMAGE WITH SEGMENTATION LEVELS

Level	H111	H11	H112	H1	H121	H12	H122
Level 1	12.23						
Level 2	11.64			1.06			
Level 3	5.96	1.59		1.05	0.92		

TABLE V
TABLE SHOWING THE CLASSIFICATION RATE COMPARISON OF PROPOSED GAUSSIAN MF AND TIZHOOSH MF

Method	S2 Image	S3 Image
Proposed Gaussian MF	93.5 %	92.7 %
Tizhoosh MF	93.0 %	89.6 %

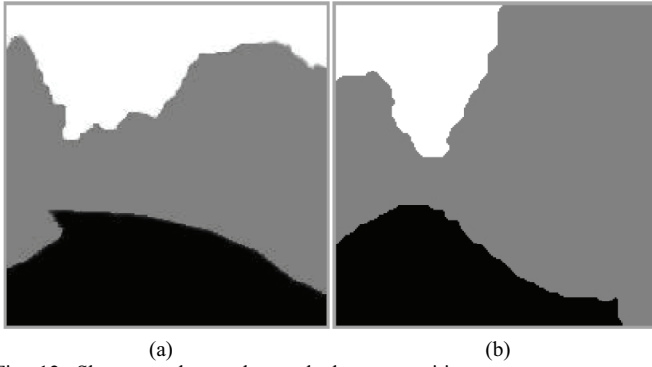


Fig. 12. Shows masks used to calculate recognition rate to compare our proposed hedge based MF with Tizhoosh MF. (a) Mask for input image s2 (b) mask for input image s3.

TABLE VI
DIFFERENT POSSIBLE COMBINATIONS OF SEGMENTED PARTS OF AN IMAGE

Step No.	Combinations
1	Original Image H1
2	H11 & H12
3	H111 & H112 & H12
4	H11 & H121 & H122
5	H111 & H112 & H121 & H122
6	H1111 & H1112 & H121 & H122
7	H111 & H1121 & H1122 & H121 & H122
8	H111 & H112 & H1211 & H1212 & H122
9	H111 & H112 & H121 & H1221 & H1222

V. CONCLUSION

We have successfully implemented IT2 fuzzy entropy based image segmentation by incorporating Gaussian membership functions. Our proposed algorithm performs multistage image segmentation successfully by using a termination criterion, which is based on the range of entropy values for a particular image and its thresholded sub-images. The experimental results show that the termination criterion along with Gaussian MF can be effectively used for thresholding of gray scale images with an improved recognition rate. Our future work is focused on a fast and more robust method. Moreover we are currently working on computing the value of linguistic hedges automatically based on the statistical properties of the image histogram.

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