

Observer-based Hybrid Fuzzy CMAC Controller for a Class of Uncertain Chaotic Systems

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Abstract—This paper presents an observer-based hybrid fuzzy cerebellar model articulation controller (CMAC) neural network control with a supervisory controller for a class of uncertain chaotic systems. This proposed control system is integrating sliding mode control (SMC) theory and CMAC neural network into fuzzy controller design. The total states of the chaotic system are not assumed to be available for measurement. A state observer is used to estimate unmeasured states of the systems. The supervised control is appended to assure that the fuzzy CMAC controller achieve a stable closed-loop system through Lyapunov stability theory. Finally, simulation results show that the effect of the approximation error on the tracking error can be attenuated efficiently by the proposed method.

Keywords- CMAC; fuzzy control; chaotic systems; observer; sliding mode control;

I. INTRODUCTION

Many different aspects of chaotic dynamics have attracted considerable attention in laser physics, digital communication, chemical reaction analysis and design, biological systems, power converters, circuit systems [1]-[5], etc. An interesting and challenging problem in the field is the control of chaotic systems. However, the chaotic systems have complex and unpredictable dynamical behavior. Inevitably, the constructed model comprises unmodeled nonlinearity and uncertain disturbance that conventional control strategies based on the accurate mathematical models of the controlled systems are difficult to derive.

The cerebellar model articulation controller (CMAC), developed by Albus in the 1970s [6]-[7], is a nonfully connected perceptron-like associative memory network with overlapping receptive-fields. The contents of these memory locations are referred as weights, and the output of this network is a linear combination of these weights in the memory addressed by the activated inputs [8]. Compared with the general multiplayer neural network with back-propagation algorithm, the CMAC has been applied to a wide variety of complex dynamical systems because of its simple computation, fast learning property and good generalization capability [9]-[10]. In the meantime, sliding mode control (SMC), or variable structure control, has successfully been developed as a popular robust strategy to a wide variety of systems having uncertainties and invariance to unknown disturbance [11]-[12]. Integrating CMAC neural network and SMC into fuzzy

controller design have acquired superior performance [13]. However, these control methods are based on the assumption that the system states are known or available for feedback. If system states are not available for measurement, the conventional adaptive laws would be difficult to be realized. To treat this problem, an observer needs to estimate the unmeasured states and the SPR-Lyapunov design approach has been used to design an adaptive scheme [14]-[15]. The goal of this paper is to design a hybrid observer-based fuzzy CMAC controller with supervisory control for a class of uncertain chaotic systems under the constraint that only the system output is available for measurement.

II. PROBLEM FORMULATION

Consider the n th-order nonlinear system described in the following form:

$$\begin{aligned} \dot{x}^{(n)} &= f(x) + bu + d \\ y &= x \end{aligned} \quad (1)$$

where $f(x)$ is unknown real continuous function, $u \in R$ and $y \in R$ are the control input and output of the system, respectively, b is a positive constant, and $x = [x, \dot{x}, \dots, x^{(n-1)}]^T = [x_1, x_2, \dots, x_n]^T$ is the state vector of system, d is the external disturbance with the properties of standard smoothness and it is assumed to have an upper bound D_N .

Rewriting (1) in the following form

$$\begin{aligned} \dot{x} &= Ax + B(f(x) + bu + d) \\ y &= C^T x \end{aligned} \quad (2)$$

$$\text{where } A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

The control objective is to design an observer-based fuzzy CMAC controller such that the system output y follows a given desired trajectory y_d . The desired trajectory vector y_d , the output tracking error vector e and the estimation error vector $\hat{e}$ are defined as, respectively,

$$y_d = [y_d \ \dot{y}_d \ \ddot{y}_d \ \cdots \ y_d^{(n-1)}]^T, \quad (3)$$

$$e = y_d - x = [e \ \dot{e} \ \ddot{e} \ \cdots \ e^{(n-1)}]^T = [e_1 \ e_2 \ \cdots \ e_n]^T \quad (4)$$

$$\hat{e} = y_d - \hat{x} = [\hat{e} \ \dot{\hat{e}} \ \ddot{\hat{e}} \ \cdots \ \hat{e}^{(n-1)}]^T \quad (5)$$

where $\hat{x}$ and $\hat{e}$ denote the estimates of x and e , respectively. Set the sliding surface as

$$S(e) = \lambda^T e \quad (6)$$

where $\lambda = [\lambda_1, \lambda_2, \dots, \lambda_n]^T$ is chosen such that $\lambda_n = 1$ and the polynomial $h(p)$

$$h(p) = \lambda_n p^{(n-1)} + \lambda_{n-1} p^{(n-2)} + \cdots + \lambda_1 p \quad (7)$$

is strictly Hurwitz [16].

Since (7) satisfies Hurwitz stability criterion, maintaining system states on surface $S(e, t)$ for all $t > 0$ is equivalent to the tracking problem $y = y_d$. The tracking control problem can be formulated by keeping the error vector e on the sliding surface defined as follows:

$$\dot{S} = y_d^{(n)} - f(x) - bu - d + \sum_{i=1}^{n-1} \lambda_i e^{(i)} \quad (8)$$

In the design of the sliding mode controller, an equivalent control is given first such that each state Lyapunov-like condition holds for system stability [12]:

$$\frac{1}{2} \frac{d}{dt} (S^2) \leq -\eta |S|, \quad \eta > 0 \quad (9)$$

Therefore the control problem is to obtain the optimal control input u^* that guarantees the sliding condition (9).

If function $f(x)$ is known and the system is free of external disturbance d , the control law can be designed as

$$u^* = b^{-1} [-f(x) + y_d^{(n)} + K_c^T e + \eta \text{sgn}(S)] \quad (10)$$

where $K_c = [k_{c1}, k_{c2}, \dots, k_{cn}]^T$ is the feedback gain vector, $\eta > 0$ and $\text{sgn}(S)$ is defined as

$$\text{sgn}(S) = \begin{cases} 1 & S > 0 \\ 0 & S = 0 \\ -1 & S < 0 \end{cases} \quad (11)$$

By plugging u^* into (8), we will have $\dot{S} = -\eta \text{sgn}(S)$. Thus, the sliding condition (9) can be easily verified. However, $f(x)$ is unknown, the control law (10) cannot be implement, and not all system state x_i are available. Therefore, to overcome such problems, we will design an observer to estimate the state x and the fuzzy CMAC systems are utilized to approximate unknown functions.

III. DESCRIPTION OF THE FUZZY CMAC SYSTEM

The CMAC is a simple and fast associative memory type neural network that is proposed as a well-established computational model based on the cerebellum in humans [6]-[7]. Fig. 1 depicts the structure of a two-dimensional CMAC. Each input variable, I_1 and I_2 , must be quantized and the problem space is divided into discrete regions (called an *element*) and 3 elements are accumulated as a block. For each

input variable, three kinds of segmentation are used, the variable I_1 is divided into blocks A , B and C , and the variable I_2 is divided into blocks a , b and c . Then, the areas formed by blocks, referred to as Aa , Ab , Ac , Ba , Bb , Bc , Ca , Cb and Cc are the addresses or the locations that store data which are called receptive-fields (or hypercubes). When each block is shifted by a small interval, different blocks will be obtained. For instance, blocks D , E and F in the second row for I_1 and d , e and f in the second column for I_2 are possible shifted regions. Then, Dd , De , Df , Ed , Ee , Ef , Fd , Fe and Ff are new hypercubes from the shifted regions. Similarly, hypercubes Gg , Gh , Gi , Hg , Hh , Hi , Ig , Ih and Ii are defined in the third layer. No hypercube is formed by the combination of different layers. Each activated element in each layer becomes a firing element, thus, the weight of each block can be obtained.

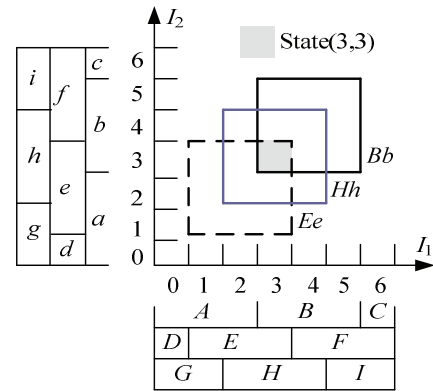


Figure 1. Block division of a 2-D CMAC.

CMAC uses a set of indices as an address in accordance with the current state to extract the stored data. Assume that N_n is the number of hypercubes; this is the same as the memory size in the case. Using the CMAC technique, a stored data y_i can be mathematically expressed as

$$y_i = a_i w = \sum_{j=1}^{N_n} a_{ij} w_j \quad (12)$$

where w indicates the column vector of memory contents and a_i is a memory element selection row vector that has N_e ones. Each hypercube is associated to an "arbitrary but deterministic" physical memory address. In Fig. 1, there are 27 hypercubes used to distinguish 49 different states in the 2-D CMAC. For example, let the hypercubes Bb , Ee , and Hh be addressed by the state $(v_1, v_2) = (3, 3)$. Since each state addresses exactly N_n hypercubes, only these three hypercubes are set to 1, and the others are set to 0. Note that after the way of block division (see Fig. 1) is determined, the vector a_i for a specific quantized state is fixed.

A fuzzy CMAC controller system can be controlled by the following linguistic rules

$$R^{(j)} : \text{IF } x_1 \text{ is } F_1^j \text{ and } \cdots \text{and } x_n \text{ is } F_n^j, \text{ THEN } \hat{f} = a_j w_l \quad (13)$$

and

$$R^{(k)} : \text{IF } x_1 \text{ is } F_1^k \text{ and } \cdots \text{and } x_n \text{ is } F_n^k, \text{ THEN } u_D = a_k w_D \quad (14)$$

where $\hat{f}$ is the approximation of f , $\mathbf{a}_j$ and $\mathbf{a}_k$ are the memory element selection row vectors, $\mathbf{w}_I$ and $\mathbf{w}_D$ are the weighting column vectors of the CMAC. The FCMAC system with center-average defuzzifier, product inference and singleton fuzzifier is defined as [16]

$$\hat{f} = \sum_{l=1}^N \mathbf{a}_j \mathbf{w}_I \mu^l(\mathbf{x}) / \sum_{l=1}^N \mu^l(\mathbf{x}) = (\mathbf{A}_I \mathbf{w}_I)^T \boldsymbol{\Phi}(\mathbf{x}) \quad (15)$$

$$u_D = \sum_{k=1}^N \mathbf{a}_k \mathbf{w}_D \mu^l(\mathbf{x}) / \sum_{j=1}^N \mu^l(\mathbf{x}) = (\mathbf{A}_D \mathbf{w}_D)^T \boldsymbol{\Phi}(\mathbf{x}) \quad (16)$$

where $\mu^l(\mathbf{x}) = \prod_{i=1}^n \mu_{F_i^l}(x_i)$ is the matching degree of the l th, N is the number of the total IF-THEN rules, $\mathbf{A}_I$ and $\mathbf{A}_D$ are matrix constituted from the memory element selection vector $\mathbf{a}_j$ and $\mathbf{a}_k$, respectively, and $\boldsymbol{\Phi} = (\phi_1, \phi_2, \dots, \phi_N)^T$ is a column vector whose proper dimension depends on the number of fuzzy rules with ϕ_j is defined as

$$\phi_j(\mathbf{x}) = \left(\prod_{i=1}^n \mu_{F_i^j}(x_i) \right) / \sum_{l=1}^N \left(\prod_{i=1}^n \mu_{F_i^l}(x_i) \right) \quad (17)$$

IV. STATE OBSERVER SCHEME

In the previous section, due to assumption that the state vector $\mathbf{x}$ is immeasurable, $f(\mathbf{x})$ are unavailable, by replacing the function $f(\mathbf{x})$, system state $\mathbf{x}$, error vector $\mathbf{e}$ and sliding surface S in (10) with their estimates $\hat{f}(\hat{\mathbf{x}})$, $\hat{\mathbf{x}}$, $\hat{\mathbf{e}}$ and $\hat{S}(\hat{\mathbf{e}})$. Since the fuzzy CMAC systems have the property such that they can approximate any continuous functions on a compact set, utilize fuzzy CMAC systems $\hat{f}(\hat{\mathbf{x}})$ be of the form

$$\hat{f}(\hat{\mathbf{x}}) = \mathbf{w}_I^T \mathbf{A}_I^T \boldsymbol{\Phi}(\hat{\mathbf{x}}) \quad (18)$$

where $\boldsymbol{\Phi}(\hat{\mathbf{x}}) = [\phi_1(\hat{\mathbf{x}}), \dots, \phi_N(\hat{\mathbf{x}})]^T$. An indirect controller is comprised of a fuzzy CMAC based certainty equivalent controller and is given by

$$u_I = b^{-1}[-\hat{f}(\hat{\mathbf{x}}) + y_d^{(n)} + K_c^T \hat{\mathbf{e}} + \eta \text{sgn}(\hat{S})] \quad (19)$$

where $\hat{S} = \lambda^T \hat{\mathbf{e}}$. The direct fuzzy CMAC controller is chosen as

$$u_D = \mathbf{w}_D^T \mathbf{A}_D^T \boldsymbol{\Phi}(\hat{\mathbf{x}}) \quad (20)$$

where u_D is the fuzzy CMAC system to directly approximate the control law (10). Thus, the resulting control law is

$$u = u_H + u_s = \alpha u_I + (1 - \alpha) u_D + u_s \quad (21)$$

where $u_H = \alpha u_I + (1 - \alpha) u_D$ is the hybrid fuzzy CMAC controller, u_I is the indirect fuzzy CMAC controller as (19), u_D is the direct fuzzy CMAC controller as (20), u_s is utilized to compensate the modeling error and external disturbance, and $\alpha \in [0, 1]$. Substituting (21) into (2), after some manipulation, one has

$$\begin{aligned} \dot{\mathbf{e}} &= \mathbf{A}\mathbf{e} - \mathbf{B}K_c^T \hat{\mathbf{e}} + \mathbf{B}[\alpha(\hat{f}(\hat{\mathbf{x}}) - f(\mathbf{x})) \\ &\quad - (1 - \alpha)b(u_D(\hat{\mathbf{x}}) - u^*(\mathbf{x}))] - \mathbf{B}[bu_s + \eta \text{sgn}(\hat{S}) + d] \end{aligned} \quad (22)$$

$$e_1 = C^T \mathbf{e}$$

where $u^*(\mathbf{x}) = b^{-1}[-f(\mathbf{x}) + y_d^{(n)} + K_c^T \hat{\mathbf{e}} + \eta \text{sgn}(\hat{S})]$. Thus, the tracking problem has been converted to the regulation problem of designing a state observer for estimating the state vector $\mathbf{e}$ in (22).

Design the following observer to estimate the error vector $\mathbf{e}$ in (22)

$$\begin{aligned} \dot{\hat{\mathbf{e}}} &= \mathbf{A}\hat{\mathbf{e}} - \mathbf{B}[K_c^T \hat{\mathbf{e}} + \eta \text{sgn}(\hat{S})] + K_o(e_1 - \hat{e}_1) \\ \hat{e}_1 &= C^T \hat{\mathbf{e}} \end{aligned} \quad (23)$$

where $K_o = [k_{o1}, k_{o2}, \dots, k_{on}]^T$ is the observer gain vector, which is selected such that the characteristic polynomial of $\mathbf{A} - K_o C^T$ is strictly Hurwitz because $(C, \mathbf{A})$ is observable. Define the observation errors $\tilde{\mathbf{e}} = \mathbf{e} - \hat{\mathbf{e}}$ and $\tilde{e}_1 = e_1 - \hat{e}_1$. Subtracting (23) from (22) yields

$$\begin{aligned} \dot{\tilde{\mathbf{e}}} &= (\mathbf{A} - K_o C^T) \tilde{\mathbf{e}} + \mathbf{B}[\alpha(\hat{f}(\hat{\mathbf{x}}) - f(\mathbf{x})) \\ &\quad - (1 - \alpha)b(u_D(\hat{\mathbf{x}}) - u^*(\mathbf{x})) - bu_s - d] \\ \tilde{e}_1 &= C^T \tilde{\mathbf{e}} \end{aligned} \quad (24)$$

Since $(C, \mathbf{A})$ is observable, there exist a positive-definite symmetric matrix P which satisfies the Lyapunov equation

$$\begin{aligned} (\mathbf{A} - \mathbf{B}K_o^T)^T P + P(\mathbf{A} - \mathbf{B}K_o^T) &= -Q \\ PB &= C \end{aligned} \quad (25)$$

where Q is a given positive-definite matrix.

V. ADAPTIVE LAW AND STABILITY ANALYSIS

This section is to use fuzzy CMAC system to approximate the nonlinear functions $f(\mathbf{x})$, $u_D(\mathbf{x})$ and derive the adaptive laws to adjust the weighting factors of the fuzzy CMAC system for the purpose of forcing the estimation error $\tilde{\mathbf{e}}$ to converge to zero.

First, replace the estimation $\hat{f}(\hat{\mathbf{x}})$ of the indirect control effort and the direct control effort $u_D(\hat{\mathbf{x}})$ in specific fuzzy CMAC systems as (13) and (14), i.e.

$$\hat{f}(\hat{\mathbf{x}}) = \hat{f}(\hat{\mathbf{x}} | \mathbf{w}_I) = \mathbf{w}_I^T \mathbf{A}_I^T \boldsymbol{\Phi}(\hat{\mathbf{x}}) \quad (26)$$

$$u_D(\hat{\mathbf{x}}) = u_D(\hat{\mathbf{x}} | \mathbf{w}_D) = \mathbf{w}_D^T \mathbf{A}_D^T \boldsymbol{\Phi}(\hat{\mathbf{x}}) \quad (27)$$

Suppose there exists constant optimal parameter vectors $\mathbf{w}_I^*$, $\mathbf{w}_D^*$

$$\mathbf{w}_I^* = \arg \min_{\mathbf{w}_I \in \Omega_f} \left[\sup_{\mathbf{x} \in U_1, \hat{\mathbf{x}} \in U_2} |f(\mathbf{x}) - \hat{f}(\hat{\mathbf{x}} | \mathbf{w}_I)| \right] \quad (28)$$

$$\mathbf{w}_D^* = \arg \min_{\mathbf{w}_D \in \Omega_D} \left[\sup_{\mathbf{x} \in U_1, \hat{\mathbf{x}} \in U_2} |u^*(\mathbf{x}) - u_D(\hat{\mathbf{x}} | \mathbf{w}_D)| \right] \quad (29)$$

where $\mathbf{x}$, $\hat{\mathbf{x}}$, $\mathbf{w}_I$ and $\mathbf{w}_D$ belong to compact sets U_1 , U_2 , Ω_f and Ω_D , respectively, which are defined as $U_1 = \{\mathbf{x} \in R^n : \|\mathbf{x}\| \leq M_1\}$, $U_2 = \{\hat{\mathbf{x}} \in R^n : \|\hat{\mathbf{x}}\| \leq M_2\}$, $\Omega_f = \{\mathbf{w}_I \in R^N : \|\mathbf{w}_I\| \leq M_f\}$ and $\Omega_D = \{\theta_D \in R^N : \|\theta_D\| \leq M_D\}$, where M_1 , M_2 , M_f and M_D are the designed parameters. Let the minimum approximation error be defined as

$$w = \alpha[\hat{f}(\hat{\mathbf{x}} | \mathbf{w}_I^*) - f(\mathbf{x})] - (1 - \alpha)b[(u_D(\hat{\mathbf{x}} | \mathbf{w}_D^*) - u^*(\mathbf{x}))] \quad (30)$$

Applying (28) and (29), the error dynamics (24) of the observation can be formulated as

$$\begin{aligned} \dot{\tilde{\mathbf{e}}} &= (A - K_o C^T) \tilde{\mathbf{e}} + B[\alpha \tilde{\mathbf{w}}_I^T A_I^T \Phi(\hat{\mathbf{x}}) \\ &\quad - (1 - \alpha)b \tilde{\mathbf{w}}_D^T A_D^T \Phi(\hat{\mathbf{x}}) + w - bu_s - d] \\ \tilde{\mathbf{e}}_1 &= C^T \tilde{\mathbf{e}} \end{aligned} \quad (31)$$

where $\tilde{\mathbf{w}}_I = \mathbf{w}_I - \mathbf{w}_I^*$, $\tilde{\mathbf{w}}_D = \mathbf{w}_D - \mathbf{w}_D^*$.

Assumption 1. We can choose K_c to guarantee $\mathbf{x} \approx \hat{\mathbf{x}}$. The bound function $f^U(\mathbf{x})$ is known such that

$$|f(\mathbf{x})| \leq f^U(\mathbf{x}) \approx f^U(\hat{\mathbf{x}}) < \infty,$$

Considering the error dynamical equation (24), let the Lyapunov function candidate be

$$V_{\tilde{\mathbf{e}}} = \frac{1}{2} \tilde{\mathbf{e}}^T P \tilde{\mathbf{e}} \quad (32)$$

Differentiating (32) with respect to time yields

$$\begin{aligned} \dot{V}_{\tilde{\mathbf{e}}} &= \frac{1}{2} \dot{\tilde{\mathbf{e}}}^T P \tilde{\mathbf{e}} + \frac{1}{2} \tilde{\mathbf{e}}^T P \dot{\tilde{\mathbf{e}}} \\ &= \frac{1}{2} \tilde{\mathbf{e}}^T \{(A - K_o C^T)^T P + P(A - K_o C^T)\} \tilde{\mathbf{e}} + \tilde{\mathbf{e}}^T PB[\alpha \hat{f}(\hat{\mathbf{x}}) \\ &\quad - f(\mathbf{x})] - (1 - \alpha)b(u_D(\hat{\mathbf{x}}) - u^*(\mathbf{x})) - bu_s - d] \\ &\leq -\frac{1}{2} \tilde{\mathbf{e}}^T Q \tilde{\mathbf{e}} - \tilde{\mathbf{e}}^T PB bu_s + \|\tilde{\mathbf{e}}^T PB\|(|\hat{f}(\hat{\mathbf{x}})| + |f(\mathbf{x})| + \\ &\quad |y_d^{(n)} + K_c^T \hat{\mathbf{e}}| + |bu_D(\hat{\mathbf{x}})| + \eta + |d|) \end{aligned} \quad (33)$$

Based on Assumption 1 and by observing (33), the supervisory control u_s is chosen as

$$\begin{aligned} u_s &= I^* \text{sgn}(\tilde{\mathbf{e}}^T PB) b^{-1} (|\hat{f}(\hat{\mathbf{x}})| + f^U + |y_d^{(n)} + K_c^T \hat{\mathbf{e}}| \\ &\quad + |bu_D(\hat{\mathbf{x}})| + \eta + D_N) \end{aligned} \quad (34)$$

where $\hat{f}(\hat{\mathbf{x}})$ and $\hat{u}_D(\hat{\mathbf{x}})$ can be obtained from the output of the fuzzy CMAC system, $\text{sgn}()$ is a sign function, I^* is defined as

$$I^* = \begin{cases} 1 & V_{\tilde{\mathbf{e}}} > \bar{V} \\ 0 & V_{\tilde{\mathbf{e}}} \leq \bar{V} \end{cases} \quad (35)$$

where $\bar{V}$ is a pre-specific constant by designer. Consider the case $V_{\tilde{\mathbf{e}}} > \bar{V}$, substituting (34) into (33) yields

$$\begin{aligned} \dot{V}_{\tilde{\mathbf{e}}} &\leq -\frac{1}{2} \tilde{\mathbf{e}}^T Q \tilde{\mathbf{e}} + |\tilde{\mathbf{e}}^T PB| (|\hat{f}(\hat{\mathbf{x}})| + |f(\mathbf{x})| \\ &\quad + |y_d^{(n)} + K_c^T \hat{\mathbf{e}}| + |bu_D(\hat{\mathbf{x}})| + \eta + |d|) \\ &\quad - (|\hat{f}(\hat{\mathbf{x}})| + f^U + |y_d^{(n)} + K_c^T \hat{\mathbf{e}}| + |bu_D(\hat{\mathbf{x}})| + \eta + D_N) \\ &\leq -\frac{1}{2} \tilde{\mathbf{e}}^T Q \tilde{\mathbf{e}} \leq 0 \end{aligned} \quad (36)$$

Theorem 1: Consider the nonlinear system (1) with control law (19), (21) and (34), $\hat{f}(\hat{\mathbf{x}} | \mathbf{w}_I)$ and $u_D(\hat{\mathbf{x}} | \mathbf{w}_D)$ are given by (26) and (27), and the parameter adaptation law are chosen as

$$\dot{\theta}_f = -r_1 \tilde{\mathbf{e}}^T P B A_I^T \Phi(\hat{\mathbf{x}}) \quad (37)$$

$$\dot{\theta}_D = r_2 b \tilde{\mathbf{e}}^T P B A_D^T \Phi(\hat{\mathbf{x}}) \quad (38)$$

where $r_1 > 0$ and $r_2 > 0$ are positive adaptive gain constants to be designed. If Assumption 1 is satisfied, the proposed control scheme can ensure that all the closed-loop signals are bounded, and the tracking errors converge to zero.

Proof:

Choose the Lyapunov-like function candidate as

$$V = \frac{1}{2} \tilde{\mathbf{e}}^T P \tilde{\mathbf{e}} + \frac{\alpha}{2r_1} \tilde{\mathbf{w}}_I^T \tilde{\mathbf{w}}_I + \frac{1 - \alpha}{2r_2} \tilde{\mathbf{w}}_D^T \tilde{\mathbf{w}}_D \quad (39)$$

with $P = P^T > 0$. Taking the time derivative of V yields

$$\dot{V} = \frac{1}{2} (\dot{\tilde{\mathbf{e}}}^T P \tilde{\mathbf{e}} + \tilde{\mathbf{e}}^T P \dot{\tilde{\mathbf{e}}}) + \frac{\alpha}{r_1} \tilde{\mathbf{w}}_I^T \dot{\tilde{\mathbf{w}}}_I + \frac{1 - \alpha}{r_2} \tilde{\mathbf{w}}_D^T \dot{\tilde{\mathbf{w}}}_D \quad (40)$$

Substituting (24) into (40) and the fact $\dot{\tilde{\mathbf{w}}}_I = \dot{\mathbf{w}}_I$, $\dot{\tilde{\mathbf{w}}}_D = \dot{\mathbf{w}}_D$, the above equation becomes

$$\begin{aligned} \dot{V} &= \frac{1}{2} [\tilde{\mathbf{e}}^T (A - K_o C^T)^T P \tilde{\mathbf{e}} + \tilde{\mathbf{e}}^T P (A - K_o C^T) \tilde{\mathbf{e}}] \\ &\quad + \tilde{\mathbf{e}}^T PB(w - bu_s - d) + \frac{\alpha}{r_1} \tilde{\mathbf{w}}_I^T (r_1 \tilde{\mathbf{e}}^T P B A_I^T \Phi(\hat{\mathbf{x}}) + \dot{\mathbf{w}}_I) \\ &\quad - \frac{1 - \alpha}{r_2} \tilde{\mathbf{w}}_D^T (r_2 b \tilde{\mathbf{e}}^T P B A_D^T \Phi(\hat{\mathbf{x}}) - \dot{\mathbf{w}}_D) \end{aligned} \quad (41)$$

Using parameter adaptation law (37)-(38) and (25), and the definition of u_s in (34), then (41) can be formulated as

$$\dot{V} = -\frac{1}{2} \tilde{\mathbf{e}}^T Q \tilde{\mathbf{e}} + \tilde{\mathbf{e}}^T PB(w - bu_s - d) \leq -\frac{1}{2} \tilde{\mathbf{e}}^T Q \tilde{\mathbf{e}} \leq 0 \quad (42)$$

This means that the inequality $\dot{V}_i < 0$ is obtained and the whole closed-loop system is stable.

VI. SIMULATION RESULTS

Chaotic system is a nonlinear deterministic system that displays complex, noisy-like and unpredictable dynamical behavior. To illustrate the performance and efficiency of the proposed fuzzy CMAC control system, it is applied to control a Duffing forced oscillation system [16]. The oscillation system has chaotic phenomena if the control input equals zero. The dynamic equation of a Duffing forced oscillation system can be described as follows:

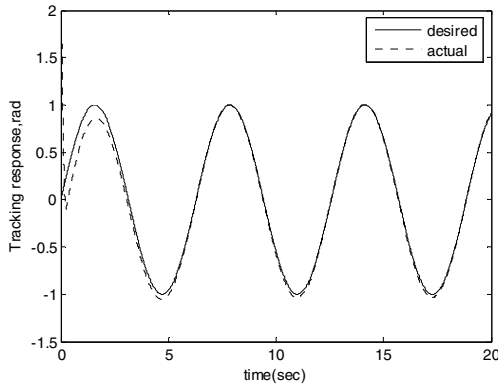
$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (f + bu + d), \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \end{aligned} \quad (43)$$

where $f(x_1, x_2) = -0.1x_2 - x_1^3 + 12\cos(t)$, y is the system output, u is the applied force (the control input) and d is the external disturbance. Also the external disturbance $d(t)$ is assumed to be a square wave with an amplitude ± 1 and a period of 2π .

The control objective is to maintain the output x_1 of the system to track the desired trajectory $y_d = \sin(t)$ under the constraint that only the system output is available for measurement. In the implementation, the fuzzy sets of states $\hat{x}_1$ and $\hat{x}_2$ are defined as $\{NB, NM, NS, ZE, PS, PM, PB\}$ and their membership functions are

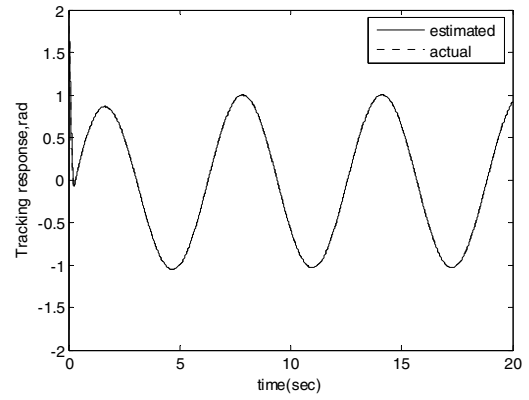
$$\begin{aligned} \mu_{NB}(\hat{x}_i) &= 1/(1 + \exp(5(\hat{x}_i + 3))), \quad \mu_{NM}(\hat{x}_i) = \exp(-(\hat{x}_i + 2)^2), \\ \mu_{NS}(\hat{x}_i) &= \exp(-(\hat{x}_i + 1)^2), \quad \mu_{ZE}(\hat{x}_i) = \exp(-(\hat{x}_i)^2), \\ \mu_{PS}(\hat{x}_i) &= \exp(-(\hat{x}_i - 1)^2), \quad \mu_{PM}(\hat{x}_i) = \exp((\hat{x}_i - 2)^2), \\ \mu_{PB}(\hat{x}_i) &= 1/(1 + \exp(-5(\hat{x}_i - 3)^2)). \end{aligned}$$

Select the positive matrices $Q = \text{Diag}[10 \ 10]$ and choose the feedback and observer gain vector as $K_c^T = [30, 50]$ and $K_o^T = [60, 140]$. The sliding surface is selected as $\hat{S} = \Lambda^T \tilde{e}$, where $\Lambda = [5, 1]^T$. The initial states are chosen as $x(0) = [0.2, 0.2]^T$, $\hat{x}(0) = [1.5, 1.5]^T$ and let adaptation gains as $r_1 = r_2 = 100$. Consider three cases of prescribed adaptation levels $\alpha = 1, 0, 0.5$, respectively, the simulation results are shown in Fig. 2 to Fig. 4. The simulation results show that the tracking error has been attenuated efficiently. Thus the proposed adaptive control scheme can control the system to follow the desired trajectory and achieve desired performance.

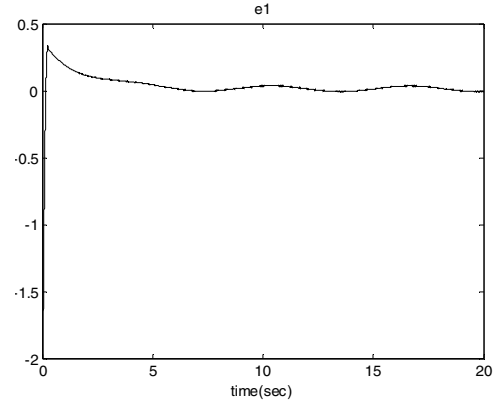


(a) Trajectories of the state x_1 (dash line) and desired y_d (solid line)

Figure 2 The responses of Duffing forced oscillation system for $\alpha = 1$.

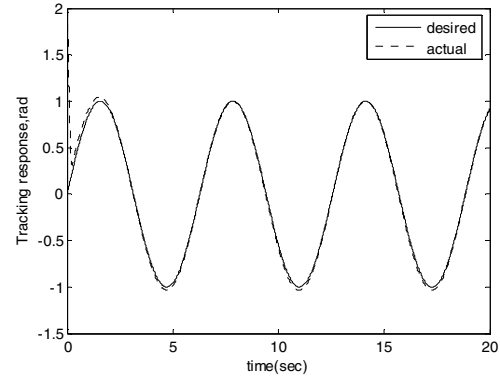


(b) Trajectories of the state x_1 (solid line) and estimated state $\hat{x}_1$ (dash line)

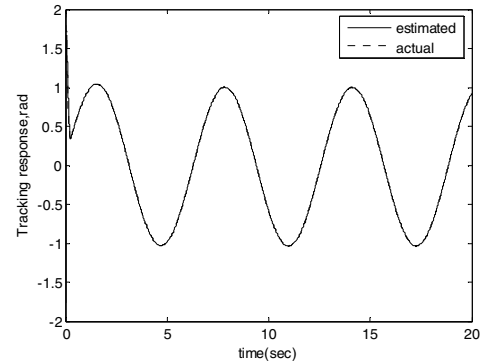


(c) The trajectory of the actual tracking error.

Figure 2 The responses of Duffing forced oscillation system for $\alpha = 1$.

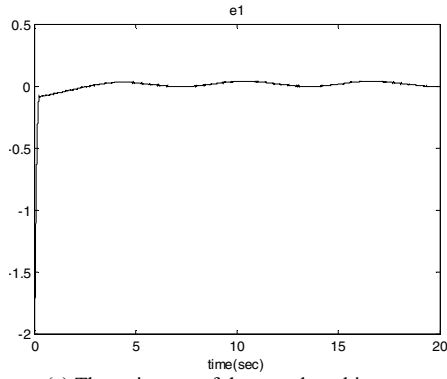


(a) Trajectories of the state x_1 (dash line) and desired y_d (solid line)



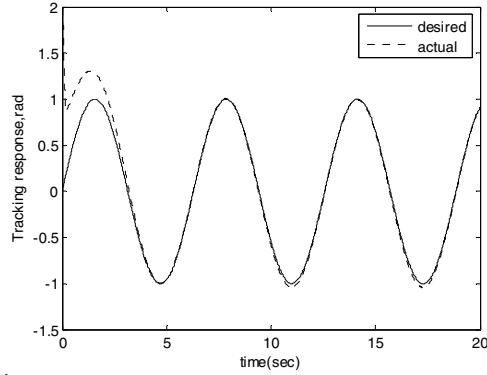
(b) Trajectories of the state x_1 (solid line) and estimated state $\hat{x}_1$ (dash line)

Figure 3 The responses of Duffing forced oscillation system for $\alpha = 0.5$

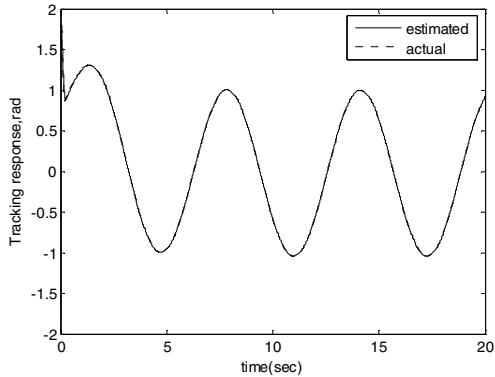


(c) The trajectory of the actual tracking error.

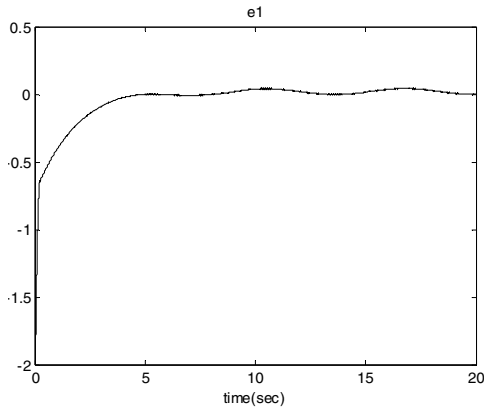
Figure 3 The responses of Duffing forced oscillation system for $\alpha = 0.5$



(a) Trajectories of the state x_1 (dash line) and desired y_d (solid line)



(b) Trajectories of the state x_1 (solid line) and estimated state $\hat{x}_1$ (dash line)



(c) The trajectory of the actual tracking error.

Figure 4 The responses of Duffing forced oscillation system for $\alpha = 0$.

VII. SIMULATION RESULTS

The goal of this work is the development and implementation of a hybrid fuzzy CMAC control based sliding mode control for the robust trajectory tracking of uncertain chaotic systems. This design obtains robustness in the sense that the self-tuning mechanism can automatically adapt the fuzzy CMAC controller by using a learning algorithm and the global asymptotic stability of the algorithm is established via Lyapunov stability criterion. The simulation presented in the Duffing forced oscillation system control indicates that the proposed approach is capable of achieving a good trajectory following performance without the knowledge of plant parameters. Although only the Duffing forced oscillation system has been studied in this paper, the proposed control scheme can also be used to address the other class of chaotic systems.

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