

Towards an Understanding of Real-World Problem Structures – Scale-Free Constraint Networks

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Abstract

Many complex real-world systems can be modeled using a graphical representation such as a constraint network. If structure can be exploited, many challenging computational tasks can have good typical-case runtimes even if they are theoretically intractable in general. This paper reports on some early experiments in a PhD-level research agenda. We report on a novel constraint network generator for random constraint networks that have a scale-free macrostructure. This scale-free generator is based on the well known Barabasi-Albert preferential attachment model. We show that scale-free constraint networks exhibit interesting phase transition behaviours which have not been seen for other problem classes studied so far.

1 Introduction

Many complex real-world systems can be modeled using a graphical representation such as a constraint network. This approach is typical in domains such as diagnosis, hardware verification, planning, logistics, scheduling, software engineering and product configuration. It has been shown that many forms of problem structure can be exploited using surprisingly modest algorithmic modifications, leading to massive improvements in the typical run-time of standard decision procedures, e.g. finding models of very large industrial satisfiability problems. In the case of satisfiability, it has been shown that real-world problems often have a small number of variables that capture the complexity of a problem instance [Gomes *et al.*, 2000]. This kind of structure can be exploited using randomised backtracking algorithms, often leading to polynomial-like behaviour for systematic search. Recent results reported in the literature have shown that many systems can have surprisingly regular structure which can be rigorously characterised. For example, many real-world graphs exhibit small-world structure that can lead to heavy-tailed runtime distributions for systematic search procedures [Walsh, 1999], but such extreme behaviour can be easily eliminated using modest algorithmic changes. Therefore, exploiting such structure can yield surprising good typical-case runtimes. A large number of complex real world networks exhibit a scale-free structure [Song *et al.*, 2005;

Albert and Barabasi, 2002]. Power law degree distributions have been observed in graphs representing complex networks such as linked pages on the WWW, the social network of co-starring actors, protein-protein interaction networks, and co-authors and citations in academic papers.

2 Scale-Free Constraint Networks

The constraint network representations of many complex systems exhibit an interesting structure, namely that while most nodes have a small number of connections to other nodes, there are some that have a very large number of connections. More formally, scale-free graphs have the property that the degree distribution follows a power law. Such distributions are often referred to as being heavy-tailed. One method used to generate networks with power-law degree distribution is the Barabasi-Albert preferential attachment model [Barabasi and Albert, 1999]. We have studied scale-free binary constraint networks, whose macrostructure is defined by a scale-free graph. In a macrostructure graph representation of a CSP, each node represents a variable, and each edge represents a binary constraint between a pair of variables. The domain of possible values for each variable and the tightness of the individual constraints are not features of the macrostructure graph. In Figure 1 we present an example survival function (probability of a value exceeding x) of the degree distribution of a random scale-free constraint network.

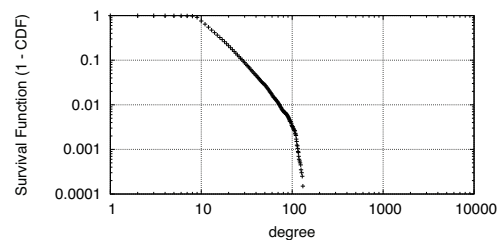


Figure 1: The survival function of the degree distribution of a binary scale-free constraint network ($n = 200$, $m = 10$).

We have implemented a random problem generator that is based on the Barabasi-Albert graph generator available as part of *NetworkX* [Hagberg *et al.*, 2008], a Python package for “the creation, manipulation, and study of the structure, dynamics, and functions of complex networks”. In this

generator, constraint graphs are parameterised in terms of n , the final number of nodes in the graph, m_0 the initial number of nodes, and m the expected number of neighbours of each node $m_0 + 1, \dots, n$ added to the initial set of unconnected nodes m_0 . Our model implements preferential attachment by maintaining a vector V of possible target nodes that each new node can be connected to, where each target node i is repeated k_i times in V , where k_i is the degree of i . At each timestep a random sample of size m is taken from V , and edges are added between the new node and each node in the sample with preference to each element of V based on its current degree. For each edge we construct a random binary constraint by randomly generating $\lceil t \times d^2 \rceil$ nogoods where d is the uniform domain size of variables in the problem and t is the tightness of each constraint.

3 Experiments

We performed a suite of experiments to study the phase transition behaviour of scale-free constraint networks. All experiments were performed on CSP instances with 200 variables (n), with a uniform domain size of 10 (d) and a random scale-free constraint graph macrostructure. To generate the scale-free constraint graph we set $m_0 = m$ for $1 \leq m \leq 199$ in steps of 2. We considered a variety of constraint tightnesses, $0.01 \leq t \leq 0.96$ in steps of 0.05. We generated 24 CSP instances for each combination of n, d, t , and m , reporting the average. Experiments were run on a cluster comprising six 2.4GHz Intel Xeon quad-core processors. Individual instances were first allowed to run for 30 seconds, and if unsolved after that time, were queued for solving later. Time-outs were progressively increased (5 minutes, 1 hour, 1 day, 2 weeks) until all problems were either solved or censored. The constraint solver used was `mistral`, which maintained arc consistency, selecting the next variable as the one that maximises the ratio of current domain size over future degree; values were selected lexicographically. Figure 2 presents a sample of results.

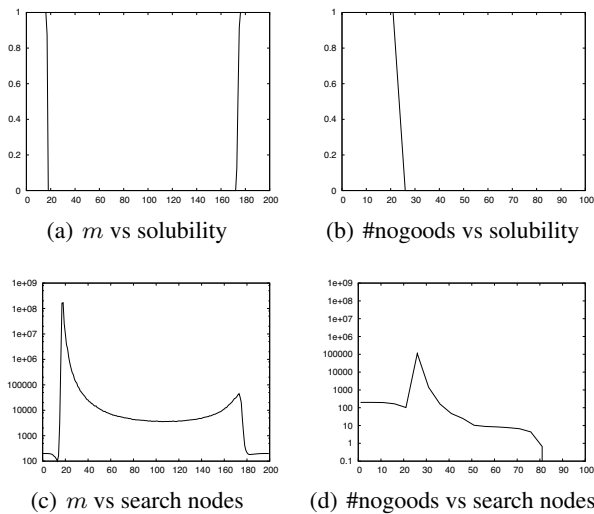


Figure 2: Phase transitions in scale-free CSPs

In Figures 2(a) and 2(c) we fix tightness at 0.15 (#nogoods = 15), and vary m . A unique characteristic of the model can be observed, namely that there is a satisfiable-unsatisfiable-satisfiable pattern in phase (Figure 2(a)), each transition corresponding to local peaks in computational difficulty (Figures 2(c)). In Figures 2(b) and 2(d) we fix m at 9 and vary tightness, and observe the standard satisfiable-unsatisfiable transition.

The phenomenon of multiple phase transitions in our scale-free constraint networks is due to the manner in which the network evolves a problem over m_0 variables to one over n variables. The number of constraints in the network peaks at approximately $m = \frac{n}{3}$ for all but small values of n , and falls gradually as m approaches n . Once the number of constraints is such that the network is no longer overconstrained, a second phase transition occurs from insoluble to soluble. Networks that start from fewer nodes, and each additional node is preferentially connected to a small number of nodes, tend to yield problems that are very challenging.

4 Conclusions

We report on a novel random constraint network generator for scale-free constraint networks which facilitates studies into the affects of this commonly occurring graph structure on reasoning about systems in the real-world. We observed a novel phase transition behaviour previously unseen in studies of combinatorial search.

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