

A unified approach to Univalent Foundations and Homotopical Algebra

Nicola Gambino

May 15, 2014

Voevodsky's Univalent Foundations of Mathematics programme seeks to develop a new approach to the foundations of mathematics, based on dependent type theories extended with axioms inspired by homotopy theory. The most remarkable of these new axioms is the so-called Univalence Axiom, which allows us to treat isomorphic types as if there were equal.

Quillen's homotopical algebra, instead, provides a category-theoretic framework in which it is possible to develop an abstract version of homotopy theory, giving a homogeneous account of several situations where objects are identified up to a suitable notion of 'weak equivalence'. The central notion here is that of a model category, examples of which arise naturally in several different areas of mathematics.

The aim of this talk is to explain how the type theories considered in Univalent Foundations and the categorical structures considered in homotopical algebra are related, but different, and to describe categorically the common core of Univalent Foundations and homotopical algebra, which allows a simultaneous development of the two subjects. The axiomatisation will be based on work of several authors, including Awodey, van den Berg, Garner, Joyal, Lumsdaine, Shulman and Warren.

Acknowledgement. This material is based on research sponsored by the Air Force Research Laboratory, under agreement number FA8655-13-1-3038. The U.S. Government is authorized to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright notation thereon. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Research Laboratory or the U.S. Government.