

LINK FORMULAS IN IMPLICATION FRAGMENTS OF SUBSTRUCTURAL LOGICS

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Two well-known implication fragments of substructural logics are BCI (implication fragment of logic without contraction and weakening) and BCK (fragment of logic without contraction). The technique of *link formulas* was developed by the present author first for BCI (see [2], where they were called *split terms*), to deal with an open problem posed in [1]. Then, a modification of the technique turned out to work for BCK and delivered a proof of its structural incompleteness (see [3], where the terminology was changed to *split formulas*), solving an open problem posed in [5]. The technique can be used to obtain relatively easy proofs of admissibility of certain rules in BCK and BCI, yielding structural incompleteness proofs of several logics in the vicinity (see [4], where finally *link formulas* was the name of choice).

I will present the technique in a uniform way, applicable to BCI and BCK, as well as to the implication fragments of the relevant logic R and the intuitionistic logic. The crucial technical result is the following.

Theorem 1. *Let L be any of the four logics mentioned above. Let $\Gamma \Rightarrow v$ be a non-initial sequent, with Γ a multiset of formulas and v a variable. Then, $\Gamma \Rightarrow v$ is provable in L if and only if there exists a formula $\gamma = \gamma_n \gamma_{n-1} \cdots \gamma_1 \rightarrow \gamma_0 \in \Gamma$ and multisets $\{\Lambda_i\}_{i=1}^n$ of formulas, such that*

- (1) $\gamma_0 = v$,
- (2) for every $i \in \{1, \dots, n\}$ the sequent $\Lambda_i \Rightarrow \gamma_i$ is provable in L ,
- (3) $\Gamma \setminus \{\gamma\} \subseteq \bigcup_{i=1}^n \Lambda_i \subseteq n \cdot \Gamma$, where the unions are multiset unions and $n \cdot \Gamma$ stands for the n -fold multiset union of Γ with itself,
- (4) a condition $C(L)$ making (3) more precise is satisfied.

I will illustrate the technique by presenting a few applications: some old, some new. Time permitting, I will also make certain observations on extending the technique to logics without exchange.

REFERENCES

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