

Lyndon Interpolation for the Modal Mu-Calculus

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The propositional modal μ -calculus introduced by D. Kozen [2] is an extension of modal logic by least and greatest fixed point operators. It is a powerful system, which is widely used in the area of specification and verification of computer systems. In this talk we are interested in interpolation properties of this system. Recall that the Craig interpolation property for a propositional logic L says that if φ implies ψ , then there is an interpolant, that is, a formula θ containing only common atomic propositions of φ and ψ such that φ implies θ and θ implies ψ . The Lyndon interpolation property is a strengthening of the Craig one that also takes into consideration negative and positive occurrences of the shared atomic propositions; that is, the atomic propositions occurring in θ positively (negatively) must also occur both in φ and ψ positively (negatively). In [1], G. D'Agostino and M. Hollenberg proved that the modal μ -calculus enjoyed a major strengthening of the Craig property: the uniform interpolation. They also proved that the modal μ -calculus enjoyed an important consequence of the Lyndon interpolation called the Lyndon theorem: a formula $\varphi(p)$ is monotone in the interpretation of p if and only if it is equivalent to a formula where p occurs only positively. Despite these results, the question whether the modal μ -calculus enjoys the Lyndon interpolation seemingly remained open. In this talk we prove Lyndon interpolation for the modal μ -calculus by applying so-called circular proofs. We use a proof system for the modal μ -calculus, which is based on non-well-founded derivation trees with a global condition which roughly says that in each infinite branch, there must be an outermost greatest fixed point unfolded infinitely many often (see [3]). We analyse regular proof trees (so-called circular proofs) in this proof system and, in this way, obtain the Lyndon interpolation for the modal μ -calculus.

References

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- [3] T. Studer. On the proof theory of the modal mu-calculus. *Studia Logica*, 89:343–363, 2008.