

On cuts and cut-elimination in modal fixed point logics

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Syntactic cut-elimination is a notorious problem for modal fixed point logics such as temporal logics, the logic of common knowledge, or the modal μ -calculus. A result by Alberucci and Jäger [1] suggests that the cut-rule cannot be eliminated in a traditional Gentzen-style sequent calculus for the logic of common knowledge. The situation changes if one considers semi-formal systems that include an ω -rule. We know cut-free sequent systems for the logic of common knowledge [6] and also for the modal μ -calculus [7] that are based on ω -rules. However, the cut-rules cannot be eliminated syntactically in those systems since the usual cut-elimination procedures do not work.

In order to solve this problem, Brännler and Studer [3] develop a cut-elimination procedure for a semi-formal nested sequent calculus for the logic of common knowledge. Together with the embeddings of the traditional shallow system into the nested system and vice versa, they obtain a syntactic cut-elimination procedure for a traditional semi-formal system for common knowledge. Further, they are able to extend their method to the continuous fragment of the modal μ -calculus but they also show that it cannot be extended to the whole modal μ -calculus [4].

Mints and Studer [9] investigate a system for the modal μ -calculus that includes a Buchholz rule [5, 8]. Hence collapsing techniques can be used in this system, which yields a general syntactic cut-elimination procedure that works beyond the continuous fragment. Yet this method has only been applied to the one-variable fragment of the μ -calculus. It is open how to generalize it to the whole logic.

Another open question is whether there are natural finitary systems that feature syntactic cut-elimination. Making use of the finite model property, semi-formal systems can be finitized, which gives us finitary (though artificial) deductive systems for modal fixed point logics. Still the standard cut-elimination procedures for these systems do not work. A recent promising candidate for syntactic cut-elimination are annotated systems à la Brännler and Lange [2]. Currently, however, we only have partial results.

In our talk, we present these recent developments and discuss open questions concerning syntactic cut-elimination for modal fixed point logics.

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