

Models Minimal Modulo Subset-Simulation for Expressive Propositional Modal Logics

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Abstract: Terminating procedures for the generation of models minimal modulo subset-simulation for normal modal logics have recently been presented. In this abstract we explain what are the challenges to generalise those procedures to more expressive modal logics, and discuss possible solutions.

1 Introduction

Model generation and minimal model generation are useful for computer science tasks such as fault analysis, model checking and debugging of logical specifications [8, 4]. For this reason, there have been several studies on minimal model generation for classical and non-classical logics [1, 5, 6, 7, 4, 2].

[7] presents terminating, minimal model sound and complete procedures for the generation of minimal models for all the normal modal logics in between **K** and **S5**, where a semantic notion of minimality, similar to the notions in [6, 4], is used. The minimality criterion is designed so that minimal models are semantically meaningful, more natural than models minimal with respect to other minimality criteria, and contain a minimal amount of information. The procedures in [7] are a combination of tableaux calculi and a minimality test to close branches.

In this abstract we discuss what are the main challenges that need to be faced to generalise the procedures in [7] to more expressive modal logics. Specifically, we are interested in extensions to multi-modal logics with universal modalities and inclusion axioms. Our ultimate aim is to obtain terminating, minimal model sound and complete procedures for all these generalisations.

2 Logics and Minimality Criterion

Syntax of modal formulae is defined as usual. Only one remark needs to be made to avoid confusion. That is, we use the notation $[U]$ and $\langle U \rangle$ for the universal modalities, and we use $[R_i]$ and $\langle R_i \rangle$ for the other modalities.

An inclusion axiom has the following form.

$$[R_i]\phi \rightarrow [R_1] \dots [R_n]\phi.$$

Such axioms represent frame properties of the form $R_1 \circ \dots \circ R_n \subseteq R_i$, where $\circ$ denotes relational composition.

We adopt the standard Kripke semantics of modal formulae. An *interpretation* $\mathcal{I}$ is a tuple $(W, \mathcal{R}, V)$, where W is a non-empty set of worlds, $\mathcal{R}$ is a set of accessibility relations $R_i \subseteq W \times W$ over W , and V is an interpretation function that assigns to each world $u \in W$ a set of propositional symbols, meaning that such propositional symbols hold in u . Given an interpretation $\mathcal{I}$, a world u and a modal formula ϕ , if $\mathcal{I}, u \models \phi$, then $\mathcal{I}$ is a *model* for ϕ .

Let $\mathcal{I} = (W, \mathcal{R}, V)$ and $\mathcal{I}' = (W', \mathcal{R}', V')$ be two models of a modal formula ϕ . A *subset-simulation* is a binary relation $S \subseteq W \times W'$ such that for any two worlds $u \in W$ and $u' \in W'$, if uSu' then the following hold.

- $V(u) \subseteq V'(u')$, and
- if uR_iv for some $R_i \in \mathcal{R}$, then there exist a $v' \in W'$ and $R_i \in \mathcal{R}'$ such that $u'R_iv'$ and vSv' .

If S is such that for all $u \in W$ there is at least one $u' \in W'$ such that uSu' , then we call S a *full subset-simulation* from $\mathcal{I}$ to $\mathcal{I}'$. Given two models $\mathcal{I}$ and $\mathcal{I}'$, if there is a full subset-simulation S from $\mathcal{I}$ to $\mathcal{I}'$, we say that $\mathcal{I}'$ *subset-simulates* $\mathcal{I}$, or $\mathcal{I}$ is *subset-simulated* by $\mathcal{I}'$. We write $\mathcal{I} \leq_{\subseteq} \mathcal{I}'$ if $\mathcal{I}$ is subset-simulated by $\mathcal{I}'$.

Subset-simulation is a preorder on models. That is, subset-simulation is a reflexive and transitive relation on models. For this reason it can be used to define the following minimality criterion. A model $\mathcal{I}$ of a modal formula ϕ is *minimal modulo subset-simulation* iff for any model $\mathcal{I}'$ of ϕ , if $\mathcal{I}' \leq_{\subseteq} \mathcal{I}$, then $\mathcal{I} \leq_{\subseteq} \mathcal{I}'$.

To have a visual idea of the minimality criterion, Figure 1 shows two possible models of a modal formula. The subset-simulation relationship is represented by directed dashed lines. Given the definition of the minimality criterion, the model on the left is considered minimal because it is subset-simulated by the model on the right.

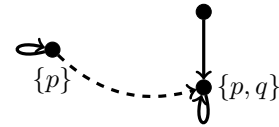


Figure 1: Example of minimality w.r.t. subset-simulation

As subset-simulation is not anti-symmetric, there can be models that subset-simulate each other, resulting in a symmetry class w.r.t. the ordering. As a result, many models minimal modulo subset-simulation can belong to the same symmetry class. To avoid the generation of all such models, and because they share a lot of positive information, we consider a procedure to be minimal model complete if it generates at least a minimal model for each symmetry class of minimal models.

3 Challenges and Possible Solutions

Due to lack of space, we do not explain the procedures in [7]. For this abstract it is enough to say that those procedures are based on tableaux calculi that are minimal model complete, minimal model soundness is achieved by using a minimality test called the subset-simulation test, and termination is guaranteed by the use of variations of ancestor equality blocking (an example of equality blocking can be found in [3]).

Three challenges need to be addressed to expand the procedures in [7]. First, some rules of the calculus need to be modified to the multi-modal case, and new rules need to be added. Second, minimal model completeness needs to be preserved. Finally, termination needs to be ensured. Minimal model soundness is not a challenge. This is because the subset-simulation test is logic independent enough to result in minimal model sound procedures for all logics we consider. This is true only if the procedures are minimal model complete.

Adopting the rules for the multi-modal case is easy. Also the introduction of rules for universal modalities and inclusion axioms does not pose particular problems. As an example, the following rule can be used for the universal modality $\langle \mathcal{U} \rangle$.

$$\frac{(\langle \mathcal{U} \rangle) \quad u : \langle \mathcal{U} \rangle \phi}{v : \phi} \quad \text{where } v \text{ is fresh.}$$

Regarding the inclusion axioms, it is enough to add a rule for each of them as in the following example. Suppose there is an inclusion axiom of the form $[R_1]\phi \rightarrow [R_2][R_3]\phi$. Then the following rule is added to the calculus.

$$\frac{(u, v) : R_2 \quad (v, w) : R_3}{(u, w) : R_1}.$$

Once the rules of the calculus are established, minimal model completeness needs to be proved. This can be proved by showing that for each minimal model there exists a branch of the tableau from which an equivalent model can be extracted. The proof we have in mind is a variation of the minimal model completeness proof in [6], where a similar minimality criterion and a similar calculus are used.

The last challenge is to preserve termination. This is the most complex problem to be solved. Decision procedures for reasoning in the logics under consideration already exist. What makes the task of minimal model generation harder is that termination techniques have a clear impact on models. It might happen, if a wrong termination strategy is used, that minimal model soundness and completeness are lost. What is clear from our previous studies is that some termination techniques, such as subset blocking, cannot be used because they conflict with the minimality criterion. This led us to think that only strategies based on equality blocking can be used, but we are still investigating which technique can be used to achieve termination while preserving minimal model completeness. Finding terminating procedures for the logics under consideration is important for theoretical and practical reasons. First, it would

imply that all the logics under consideration have a finite number of symmetry classes of minimal models. Second, termination is fundamental for an efficient and effective implementation of the procedures.

4 Conclusion

The procedures in [7] are an important contribution towards the generation of models minimal modulo subset-simulation for modal logics. In this abstract we focused on generalising such procedures to more expressive modal logics. We believe that it is possible to design minimal model sound and complete procedures for more expressive modal logics, and this can be done by modifying carefully the tableau calculus in [7].

The main remaining challenge is to ensure termination. We believe that some variation of equality blocking can help us to reach our goal, to prove that all the logics under consideration have a finite number of symmetry classes of minimal models, and to provide a basis for practical implementations of the procedures.

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