

Kurth's Criterion H Revisited*

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Abstract

Criterion H is one of the criteria for termination of string rewriting that Winfried Kurth implemented in his thesis. Criterion H can be slightly improved. The improved version can be translated into a termination proof by Semantic Labelling.

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1 Criterion H

In his dissertation [4], Winfried Kurth implemented a suite of criteria for the termination of single-rule string rewriting systems (SRSs). In the meantime, each of his Criteria A to G is covered by new termination criteria [5, 6, 7, 1]. Senizergues [9], Kobayashi et al. [3, 10], Moczydlowski and Geser [8], and Hofbauer, Waldmann, and Zantema [2, 12] have introduced further criteria. Criterion H has not yet been covered. We give a criterion that is slightly more powerful than Kurth's Criterion H and show that it can be translated into a proof by Semantic Labelling.

Let $p(k) := \max\{0, k - 1\}$ denote the total predecessor on $\mathbb{N}_0$.

► **Definition 1** ([4, Definition 4.39]). Functions $\mu : \mathbb{N}_0^* \rightarrow \mathbb{Z}^*$ and $\hat{\mu} : \mathbb{N}_0^* \rightarrow \mathbb{N}_0^*$ are defined by

$$\mu((k_1, \dots, k_n)) = (k_1 - 1, k_1 + k_2 - 2, k_1 + k_2 + k_3 - 3, \dots, k_1 + \dots + k_n - n)$$

and by $\hat{\mu}((k_1, \dots, k_n)) = (m_1, m_2, \dots, m_n)$ where the m_i are defined recursively by $m_1 = p(k_1)$ and $m_i = p(m_{i-1} + k_i)$ for all $2 \leq i \leq n$.

If $\mu(\tau) \in \mathbb{N}_0^*$ then $\hat{\mu}(\tau) = \mu(\tau)$.

Let the alphabet $\{a, b\}$ be given. Let $|s|_c$ denote the number of occurrences of the letter $c \in \{a, b\}$ in the string s . The tuple representation $T(w)$ of a string w is defined by $T(w) = (k_0, k_1, k_2, \dots, k_n)$ if $w = a^{k_0}ba^{k_1}ba^{k_2} \dots ba^{k_n}$, $k_0, \dots, k_n \in \mathbb{N}_0$.

For $\tau, \tau' \in \mathbb{N}_0^*$, let $\tau >_{\text{mult}} \tau'$ denote the multiset extension of the order on $\mathbb{N}_0$, applied to the multiset of τ and the multiset of τ' . Let $\geq_{\text{mult}}$ denote $>_{\text{mult}} \cup =$.

► **Theorem 2** (Criterion H [4, Satz 4.40]). *The SRS $(R, \{a, b\})$ terminates if the following conditions hold for every rule $(\ell \rightarrow r) \in R$:*

1. $|r|_a > 0$ and $|r|_b > 0$;
2. $|\ell|_a - |\ell|_b \geq |r|_a - |r|_b$;

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3. $\mu(T(\ell)) \in \mathbb{N}_0^*$ and $\mu(T(r)) \in \mathbb{N}_0^*$;
4. $\mu(T(\ell)) >_{\text{mult}} \mu(T(r))$.

For instance, the SRS $aaba \rightarrow abaab$ satisfies Criterion H and hence terminates [4, Satz 4.44]. Kurth states Theorem 2 only for the single-rule case, but his proof works in the general case: he shows $\hat{\mu}(T(s\ell t)) >_{\text{mult}} \hat{\mu}(T(srt))$ for all $s, t \in \{a, b\}^*$.

For $s = a_1 a_2 \dots a_n$ let $\bar{s} = a_n \dots a_2 a_1$. For an SRS R , let $\bar{R} = \{\bar{\ell} \rightarrow \bar{r} \mid (\ell \rightarrow r) \in R\}$. We refer to the map $s \mapsto \bar{s}$ as string reflection. Given an SRS (R, Σ) where Σ may differ from $\{a, b\}$, and two non-empty sets $X, Y \subseteq \Sigma$ such that $X \cap Y = \emptyset$, let the projection $h : \Sigma^* \rightarrow \{a, b\}^*$ be defined by $h(c) = a$ if $c \in X$, $h(c) = b$ if $c \in Y$, and $h(c) = \varepsilon$ if $c \in \Sigma \setminus (X \cup Y)$. Let $h(R) = \{h(\ell) \rightarrow h(r) \mid (\ell \rightarrow r) \in R\}$. Kurth uses string reflection and projection as preprocessing steps in combination with Criterion H. For the purpose of a simple presentation, we drop these preprocessing steps for the time being.

2 Technical Stuff

We give a few definitions and basic results in order to state a stronger version of Theorem 2.

► **Definition 3.** For each $x \in \mathbb{N}_0$, let $\hat{\mu}_x : \mathbb{N}_0^* \rightarrow \mathbb{N}_0^*$ be defined by $\hat{\mu}_x((k_1, \dots, k_n)) = (m_1, m_2, \dots, m_n)$ where the m_i are defined recursively by $m_1 = p(x + k_1)$ and $m_i = p(m_{i-1} + k_i)$ for all $2 \leq i \leq n$.

By definition, $\hat{\mu}_0 = \hat{\mu}$.

Let $\text{last}((k_1, \dots, k_n)) = k_n$. For $n \in \mathbb{N}_0$, $p_1, \dots, p_n, x \in \mathbb{Z}$ let $(p_1, \dots, p_n) + x$ denote $(p_1 + x, \dots, p_n + x)$. Let $\text{xcrit}(\tau) := \max\{0, -m_1, -m_2, \dots, -m_n\}$ where $(m_1, m_2, \dots, m_n) = \mu(\tau)$. By definition, $\text{xcrit}(\tau)$ is the least $x \in \mathbb{N}_0$ such that $\mu(\tau) + x \in \mathbb{N}_0^*$. Let $\geq^*$ denote the pointwise order on $\mathbb{Z}^*$, i.e. $(p_1, \dots, p_m) \geq^* (q_1, \dots, q_n)$ iff $m = n$ and $p_i \geq q_i$ for all $1 \leq i \leq n$.

► **Lemma 4.** For all $\sigma, \tau \in \mathbb{N}_0^*$ and $x, y \in \mathbb{N}_0$:

1. if $y \geq x$ then $\hat{\mu}_y(\tau) \geq^* \hat{\mu}_x(\tau)$;
2. if $y \geq x$ then $\hat{\mu}_x(\tau) + (y - x) \geq^* \hat{\mu}_y(\tau)$;
3. if $x \geq \text{xcrit}(\tau)$ then $\hat{\mu}_x(\tau) = \mu(\tau) + x$;
4. $\hat{\mu}_x(\tau) \geq^* \mu(\tau) + x$;
5. if $\hat{\mu}_{\text{xcrit}(\sigma)}(\sigma) >_{\text{mult}} \hat{\mu}_{\text{xcrit}(\sigma)}(\tau)$ then $\hat{\mu}_x(\sigma) >_{\text{mult}} \hat{\mu}_x(\tau)$ for all $x \geq \text{xcrit}(\sigma)$;
6. if $\text{last}(\hat{\mu}_{\text{xcrit}(\sigma)}(\sigma)) \geq \text{last}(\hat{\mu}_{\text{xcrit}(\sigma)}(\tau))$ then $\text{last}(\hat{\mu}_x(\sigma)) \geq \text{last}(\hat{\mu}_x(\tau))$ for all $x \geq \text{xcrit}(\sigma)$.

Proof. Claim 1: Let $y \geq x$, $\tau = (k_1, \dots, k_n)$, $\hat{\mu}_y(\tau) = (q_1, \dots, q_n)$, $\hat{\mu}_x(\tau) = (p_1, \dots, p_n)$. Then $q_1 = p(y + k_1)$, $q_i = p(q_{i-1} + k_i)$ for all $2 \leq i \leq n$, $p_1 = p(x + k_1)$, $p_i = p(p_{i-1} + k_i)$ for all $2 \leq i \leq n$. We show that $q_i \geq p_i$ for all $1 \leq i \leq n$ by induction on i . Case $i = 1$: Then $q_1 = p(y + k_1) \geq p(x + k_1) = p_1$. Case $i > 1$: By inductive hypothesis, $q_{i-1} \geq p_{i-1}$. Then $q_i = p(q_{i-1} + k_i) \geq p(p_{i-1} + k_i) = p_i$.

Claim 2 and Claim 3 are proven in a similar way.

Claim 4: $\hat{\mu}_x(\tau) \geq^* \hat{\mu}_y(\tau) + (x - y) = (\mu(\tau) + y) + (x - y) = \mu(\tau) + x$ where $y := \text{xcrit}(\tau)$.

Claim 5: Let $y = \text{xcrit}(\sigma)$. By Claim 3, the premise, and Claim 2 we get $\hat{\mu}_x(\sigma) = \mu(\sigma) + x = \hat{\mu}_y(\sigma) + (x - y) >_{\text{mult}} \hat{\mu}_y(\tau) + (x - y) \geq^* \hat{\mu}_x(\tau)$. Note here that $\sigma \geq^* \tau$ implies $\sigma \geq_{\text{mult}} \tau$ for all $\sigma, \tau \in \mathbb{N}_0^*$.

Claim 6: By Claim 3, the premise, and Claim 2 we get $\text{last}(\hat{\mu}_x(\sigma)) = \text{last}(\mu(\sigma)) + x = \text{last}(\hat{\mu}_y(\sigma)) + (x - y) \geq \text{last}(\hat{\mu}_y(\tau)) + (x - y) \geq \text{last}(\hat{\mu}_x(\tau))$. ◀

Let $\&$ denote concatenation on $\mathbb{N}_0^*$.

- **Lemma 5.** 1. $\hat{\mu}_x(T(as)) = \hat{\mu}_{x+1}(T(s))$ for all $s \in \{a, b\}^*$;
 2. $\hat{\mu}_x(T(bs)) = (p(x)) \& \hat{\mu}_{p(x)}(T(s))$ for all $s \in \{a, b\}^*$;
 3. For all $s \in \{a, b\}^*$, if $\hat{\mu}_x(T(s)) = (p_1, \dots, p_{n-1}, p_n)$ then $\hat{\mu}_x(T(sa)) = (p_1, \dots, p_{n-1}, p_{n-1} + \text{last}(T(s)))$;
 4. For all $s \in \{a, b\}^*$, if $\hat{\mu}_x(T(s)) = (p_1, \dots, p_n)$ then $\hat{\mu}_x(T(sb)) = (p_1, \dots, p_n, p(p_n))$.

Proof. Claim 1: Let $T(s) = (k_1, k_2, \dots, k_n)$, $\hat{\mu}_x(T(as)) = (q_1, \dots, q_n)$, and $\hat{\mu}_{x+1}(T(s)) = (p_1, \dots, p_n)$. We have $T(as) = (k_1 + 1, k_2, \dots, k_n)$, $q_1 = p(x + k_1 + 1)$, $q_i = p(q_{i-1} + k_i)$ for $2 \leq i \leq n$, $p_1 = p(x + k_1 + 1)$, $p_i = p(p_{i-1} + k_i)$ for $2 \leq i \leq n$. By induction on i , one proves $q_i = p_i$ for all $1 \leq i \leq n$.

Claims 2, 3, and 4 are proven in a similar way. ◀

3 An Improvement

For $s, t \in \{a, b\}^*$ let $s >_G t$ if

$$\text{last}(\hat{\mu}_x(T(sa))) \geq \text{last}(\hat{\mu}_x(T(ta))) \text{ and } \hat{\mu}_x(T(s)) >_{\text{mult}} \hat{\mu}_x(T(t)) \text{ for all } x \in \mathbb{N}_0.$$

Let a reduction order be a well-founded and transitive binary relation on strings that is closed under left and right contexts.

► **Theorem 6.** $>_G$ is a reduction order on $\{a, b\}^*$.

Proof. Transitivity of $>_G$ follows from transitivity of both $\geq$ and $>_{\text{mult}}$. Well-foundedness of $>_G$ follows from well-foundedness of $>_{\text{mult}}$. Closure under left and right contexts remains to be proved. To this end, let $s >_G t$, which is $\text{last}(\hat{\mu}_x(T(sa))) \geq \text{last}(\hat{\mu}_x(T(ta)))$ and $\hat{\mu}_x(T(s)) >_{\text{mult}} \hat{\mu}_x(T(t))$ for all $x \in \mathbb{N}_0$. We claim $as >_G at$. We have $\text{last}(\hat{\mu}_x(T(asa))) = \text{last}(\hat{\mu}_{x+1}(T(sa))) \geq \text{last}(\hat{\mu}_{x+1}(T(ta))) = \text{last}(\hat{\mu}_x(T(ata)))$ and $\hat{\mu}_x(T(as)) = \hat{\mu}_{x+1}(T(s)) >_{\text{mult}} \hat{\mu}_{x+1}(T(t)) = \hat{\mu}_x(T(at))$ by Lemma 5(1) and the premise. Next we claim $bs >_G bt$. Then $\text{last}(\hat{\mu}_x(T(bsa))) = \text{last}(\hat{\mu}_{p(x)}(T(sa))) \geq \text{last}(\hat{\mu}_{p(x)}(T(ta))) = \text{last}(\hat{\mu}_x(T(bta)))$. Next, $\hat{\mu}_x(T(bs)) = (p(x)) \& \hat{\mu}_{p(x)}(T(s)) >_{\text{mult}} (p(x)) \& \hat{\mu}_{p(x)}(T(t)) = \hat{\mu}_x(T(bt))$ by Lemma 5(2) and the premise. For the proof of claim $sa >_G ta$, let $\hat{\mu}_x(T(sa)) = (p_1, \dots, p_m)$, let $\hat{\mu}_x(T(ta)) = (q_1, \dots, q_n)$, and let $k = \text{last}(T(sa)) \geq 1$ and $k' = \text{last}(T(ta)) \geq 1$. We have $\text{last}(\hat{\mu}_x(T(sa))) = p_m + 1 \geq q_n + 1 = \text{last}(\hat{\mu}_x(T(ta)))$, by Lemma 5(3) and the premise. By premise, $\hat{\mu}_x(T(s)) = (p_1, \dots, p_{m-1}, p_{m-1} + k - 1) >_{\text{mult}} (q_1, \dots, q_{n-1}, q_{n-1} + k' - 1) = \hat{\mu}_x(T(t))$. With that, $\hat{\mu}_x(T(sa)) = (p_1, \dots, p_{m-1}, p_{m-1} + k) >_{\text{mult}} (q_1, \dots, q_{n-1}, q_{n-1} + k') = \hat{\mu}_x(T(ta))$. Finally, we claim $sb >_G tb$. Let $\hat{\mu}_{x+1}(T(sa)) = (p_1, \dots, p_m)$ and $\hat{\mu}_x(T(ta)) = (q_1, \dots, q_n)$. Then $\text{last}(\hat{\mu}_x(T(sba))) = p(p(p_m)) + 1 \geq p(p(q_n)) + 1 = \text{last}(\hat{\mu}_x(T(tba)))$ and $\hat{\mu}_x(T(sb)) = (p_1, \dots, p_m, p(p(p_m))) >_{\text{mult}} (q_1, \dots, q_n, p(p(q_n))) \geq_{\text{mult}} (q_1, \dots, q_n, p(p(q_n))) = \hat{\mu}_x(T(tb))$. ◀

Lemmas 4(4), 4(5) and Theorem 6 prove:

► **Theorem 7.** The SRS $(R, \{a, b\})$ terminates if the following conditions hold for every rule $(\ell \rightarrow r) \in R$:

1. $\text{last}(\hat{\mu}_x(T(\ell a))) \geq \text{last}(\hat{\mu}_x(T(ra)))$ for all $x \leq \text{xcrit}(T(\ell))$;
2. $\hat{\mu}_x(T(\ell)) >_{\text{mult}} \hat{\mu}_x(T(r))$ for all $x \leq \text{xcrit}(T(\ell))$.

► **Theorem 8.** The premises of Theorem 7 hold whenever the premises of Theorem 2 hold.

Proof. Let $(\ell \rightarrow r) \in R$ where $\ell, r \in \{a, b\}^*$, and let (1) $|r|_a > 0$ and $|r|_b > 0$; (2) $|\ell|_X - |\ell|_Y \geq |r|_X - |r|_Y$; (3) $\mu(T(\ell)) \in \mathbb{N}_0^*$ and $\mu(T(r)) \in \mathbb{N}_0^*$; (4) $\mu(T(\ell)) >_{\text{mult}} \mu(T(r))$. By (3) we have $\text{xcrit}(T(\ell a)) = 0$. By (2) then, $\text{last}(\hat{\mu}_0(T(\ell a))) = \text{last}(\mu(T(\ell a))) = |\ell|_a - |\ell|_b \geq |r|_a - |r|_b = \text{last}(\mu(T(ra))) = \text{last}(\hat{\mu}_0(T(ra)))$ which proves Premise 1. By (4), $\hat{\mu}_0(T(\ell)) = \mu(T(\ell)) >_{\text{mult}} \mu(T(r)) = \hat{\mu}_0(T(r))$, which proves Premise 2. $\blacktriangleleft$

The following examples show that the improvement is strict, even if we allow preprocessing steps reflection and projection.

► **Example 9.** The SRS $baababaab \rightarrow abaabbaabab$, after preprocessing, satisfies the premises of Theorem 7 but not those of Theorem 2. Check that $T(\bar{\ell}) = (0, 2, 1, 2, 0)$, $T(\bar{r}) = (0, 1, 2, 0, 2, 1)$. We get

$$\begin{aligned}\hat{\mu}_0(T(\bar{\ell})) &= (0, 1, 1, 2, 1) \neq (-1, 0, 0, 1, 0) = \mu(T(\bar{\ell})) \text{ and} \\ \hat{\mu}_0(T(\bar{r})) &= (0, 0, 1, 0, 1, 1) \neq (-1, -1, 0, -1, 0, 0) = \mu(T(\bar{r})); \\ \hat{\mu}_1(T(\bar{\ell})) &= (0, 1, 1, 2, 1) = \mu(T(\bar{\ell})) + 1 \text{ and} \\ \hat{\mu}_1(T(\bar{r})) &= (0, 0, 1, 0, 1, 1) = \mu(T(\bar{r})) + 1.\end{aligned}$$

► **Example 10.** The SRS $abaaabba \rightarrow abbababaaab$ satisfies, after preprocessing, the premises of Theorem 7 but not those of Theorem 2. Check that $T(\ell) = (1, 3, 0, 1)$ and $T(r) = (1, 0, 1, 1, 3, 0)$. We get

$$\begin{aligned}\hat{\mu}_0(T(\ell)) &= (0, 2, 1, 1) = \mu(T(\ell)), \\ \hat{\mu}_0(T(r)) &= (0, 0, 0, 0, 2, 1) \neq (0, -1, -1, -1, 1, 0) = \mu(T(r)).\end{aligned}$$

These two examples and the SRS $abccabca \rightarrow abcacabccab$ are the only SRSs $\ell \rightarrow r$ with $|r| \leq 11$ that satisfy the premises of Theorem 7 after preprocessing, but satisfy neither Theorem 2 after preprocessing, nor any of the above-mentioned improvements of Kurth's Criteria A to G. In contrast, 16 SRSs with $|r| = 11$ satisfy Theorem 2, too.

4 Semantic Labelling

There is an interesting connection between Theorem 7 and Semantic Labelling [11]. For $s \in \{a, b\}^*$, let the interpretation $[s] : \mathbb{N}_0^* \rightarrow \mathbb{N}_0^*$ be defined by $[a](x) = x + 1$ and $[b](x) = p(x)$. The reflected strings are semantically labelled. Symbol a receives no label; symbol b receives $p(x)$ as its label where x is the interpretation value. For convenience, let $\overline{\text{lab}}_x(s)$ denote $\overline{\text{lab}}_x(\bar{s})$, which is s reflected, labelled, and reflected again. The tuple of labels of b symbols is extracted by a string homomorphism $h : (\{a, b\} \times \mathbb{N}_0)^* \rightarrow \mathbb{N}_0^*$ that is defined by $h(a_x) = \varepsilon$ and $h(b_x) = (x)$.

► **Lemma 11.** For all $s \in \{a, b\}^*$, $x \in \mathbb{N}_0$:

1. $[\bar{s}](x) = \text{last}(\hat{\mu}_x(T(sa)))$;
2. $h(\overline{\text{lab}}_x(sb)) = \hat{\mu}_x(T(s))$.

Proof. The proof of Claim 1 is a straightforward induction on $|s|$. Claim 2 is shown by induction on $|s|$. Case $s = \varepsilon$. Then $h(\overline{\text{lab}}_x(b)) = h(b_{p(x)}) = (p(x)) = \hat{\mu}_x((0)) = \hat{\mu}_x(T(\varepsilon))$. Case $s = as'$. Then $h(\overline{\text{lab}}_x(as'b)) = h(a \overline{\text{lab}}_{[a](x)}(s'b)) = h(\overline{\text{lab}}_{x+1}(s'b)) = \hat{\mu}_{x+1}(T(s'b)) = \hat{\mu}_x(T(as'b))$ by Lemma 5(1). Case $s = as'$. Then $h(\overline{\text{lab}}_x(bs'b)) = h(b_{p(x)} \overline{\text{lab}}_{[b](x)}(s'b)) = (p(x)) \& h(\overline{\text{lab}}_{p(x)}(s'b)) = (p(x)) \& \hat{\mu}_x(T(bs'b)) = \hat{\mu}_x(T(bs'b))$ by the inductive hypothesis and Lemma 5(2). $\blacktriangleleft$

► **Example 12.** $h(\overline{\text{lab}}_x(aabab)) = h(a_x a_{x+1} b_{x+2} a_{x+1} b_{x+2}) = (x+2, x+2)$ and $h(\overline{\text{lab}}_x(abaabb)) = h(a_x b_{x+1} a_x a_{x+1} b_{x+2} b_{x+1}) = (x+1, x+2, x+1)$.

A proof of termination of R by Theorem 7 can be turned into a proof by Semantic Labelling in the following way:

- **Theorem 13.** For a SRS $(R, \{a, b\})$ let $R' = \{\ell a^i b \rightarrow r a^i b \mid (\ell \rightarrow r) \in R, i \in \mathbb{N}_0\}$.
1. If Condition 1 of Theorem 7 holds then $\square$ is a quasi-model of R' .
 2. If Condition 2 of Theorem 7 holds then $\overline{\text{lab}}_x(R')$ is ordered by the reduction order $>_{\text{mult}}$ on $\mathbb{N}_0^*$.

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