

Total space in resolution

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We show $\Omega(n^2)$ lower bounds on the total space used in resolution refutations of random k -CNFs over n variables, and of the graph pigeonhole principle and the bit pigeonhole principle for n holes. This answers the long-standing open problem of whether there are families of k -CNF formulas of size $O(n)$ requiring total space $\Omega(n^2)$ in resolution, and gives the first truly quadratic lower bounds on total space. The results follow from a more general theorem showing that, for formulas satisfying certain conditions, in every resolution refutation there is a memory configuration containing many clauses of large width.

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