

Program extraction in Church’s simple theory of types (Abstract)

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In [Chu40] Church introduced a system of higher-order logic formulated as a simply typed lambda-calculus with base types ι for individuals and o for propositions. In this system logical connectives and quantifiers appear as constants (e.g. $\wedge : o \rightarrow o \rightarrow o$ and $\forall_\iota : (\iota \rightarrow o) \rightarrow o$). One of the attractions of this system is the fact that it is on the one hand rather expressive, allowing the formalization of large parts of mathematics, and on the other hand very simple and convenient to handle formally. A variant of Church’s theory is the logical basis for the system Isabelle/HOL [NPW02] and it also plays a role in the development of Martin-Löf Type Theory and the latest developments in Homotopy Type Theory [Uni13].

Church’s theory was originally formulated with classical logic and a choice operator. If one uses intuitionistic logic instead, drops choice and adds monotone (co)inductive definitions, one obtains a system that, though impredicative, has a clear computational meaning. One can make this precise by a realizability interpretation combined with a theorem for witness-extraction. I will present this system, demonstrate its usefulness by means of examples and discuss its status from a constructive respectively classical point of view.

References

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