

Proof theory for ordered algebra: amalgamation and densification

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Abstract

Proof theory plays only a limited role in the context of superintuitionistic / substructural logics. We certainly have cut elimination, disjunction property, Herbrand's theorem, interpolation and density elimination. But is there anything else? Our purpose is to find further applications of proof theory by making it directly act on algebras (Heyting / FL algebras) rather than on syntactic formulas and proof systems.

It is well known that a superintuitionistic logic enjoys the interpolation property if and only if the corresponding variety of Heyting algebras enjoys the amalgamation property. This means that a proof theoretic method for interpolation (Maehara's Lemma) potentially has an effect of constructing an amalgam in the context of Heyting algebras. Likewise, a proof theoretic method for eliminating the density rule in the hypersequent calculus has an effect of densifying a given Heyting/FL chain.

In this talk, we will illustrate such direct applications of proof theory to Heyting / FL algebras. Specifically, we will explain how Maehara's lemma and the density elimination theorem can be construed as algebraic constructions for amalgamation and densification.