

Resolution-Based Uniform Interpolation for Expressive Description Logics

Patrick Koopmann and Renate A. Schmidt

University of Manchester, UK

1 Introduction

Ontologies represented using description logics model domains of interest in terms of concepts and binary relations, and are used in a range of areas including medical science, bio-informatics, the semantic web or artificial intelligence. Often, ontologies are large and complex and cover 10,000s of concepts. Uniform interpolants are restricted views of ontologies, that only use a specified set of symbols, while preserving all entailments that can be expressed using these symbols. Uniform interpolation can be used for analysing hidden relations in an ontology, removing confidential concepts from an ontology, computing logical differences between ontologies or extracting specialised ontologies for ontology reuse and has many more applications.

We follow a resolution-based approach to make the computation of uniform interpolants of larger ontologies feasible. Uniform interpolants cannot always be represented finitely in the language of the input ontology, for which situation we offer three solutions: extending the signature with additional concepts, approximating the uniform interpolant, or using fixpoint operators.

2 Approach

We use a resolution-based approach to eliminate symbols outside of the desired signature one by one. Due to the form of the classical resolution rule, $C_1 \vee A, C_2 \vee \neg A \implies C_1 \vee C_2$, resolution allows to derive consequences on a specific symbol A in a goal-oriented manner, simply by resolving on this symbol A . Saturating a set of axioms for a specific symbol A makes axioms containing this symbol superfluous for entailments not using A , so that these axioms can be removed. Depending on the description logic under consideration, additional rules are needed to ensure that all resolvents sufficient for the uniform interpolant are inferred.

Since we compute uniform interpolants for a specific target language, we only infer axioms that can be expressed in that language. We further use a specific, flattened normal form based on structural transformation, which is preserved by the introduction of new symbols by the rules of the calculus. This keeps the inference rules simple, and since we can ensure that only a finite number of new symbols is introduced, also gives a finite bound on the set of clauses that can be derived.

New symbols are introduced in such a way that they can always be eliminated in the last step by simple replacement rules. If the definitions of the new symbols are cyclic, we can either keep them, eliminate them by incremental approximation or eliminate them using fixpoint operators.

3 Results

So far, we have introduced methods for computing uniform interpolants of $\mathcal{ALCH}$ ontologies with and without ABoxes [1, 4], and of $\mathcal{SHQ}$ TBoxes [3]. A prototypical implementation in

Scala is available at <http://www.cs.man.ac.uk/~koopmanp>, and can be used both from the command line and as a Java library. Evaluations on real life ontologies show that we can compute uniform interpolants for smaller signatures (up to 150 symbols) even of ontologies as large as the National Cancer Institute Thesaurus containing over 135,000 axioms in a few minutes [2].

4 Future Work

We are currently investigating how our method can be extended to deal with $\mathcal{SHQ}$ ontologies with ABoxes and description logics with inverse roles, motivated by related second-order quantifier elimination methods for modal logics [5]. Apart from that, we want to develop some heuristics to predict where the limits of our implementation are, i.e., for which signatures it is hard or impossible to compute a uniform interpolant in reasonable time, for example, because they exclude symbols with too many occurrences in the ontology.

References

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