

Fixed-Parameter Algorithms for Reasoning Problems in NP and Beyond

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Many computational problems that arise in the context of AI and reasoning are NP-hard or even hard for the second level of the Polynomial Hierarchy. Practical instances of these problems often contain some “hidden structure” that can be utilized algorithmically. There are various ways of capturing the hidden structure in terms of parameters which in turn can be used by *fixed-parameter algorithms* [2]. In order to achieve fixed-parameter tractability for problems that are hard for the second level of the Polynomial Hierarchy, the parameter usually needs to be quite restrictive. For such problems, it therefore seems to be an even more compelling approach to utilize the hidden structure captured by the parameter not to *solve* the problem, but to *reduce* it to a different problem of lower complexity, say, to a problem in NP. The parameter can thus be less restrictive and so reasonably small for larger classes of inputs. Clearly such a reduction cannot be computed in polynomial time, unless the Polynomial Hierarchy collapses. However, the reduction can be fixed-parameter tractable and thus *break complexity barriers* between the levels of the Polynomial Hierarchy. SAT, the propositional satisfiability problem, is ideally suited as a target problem since by means of fixed-parameter tractable reductions to SAT one can make the spectacular solving power of today’s SAT solvers [7] applicable to problems at higher levels of the Polynomial Hierarchy. In fact, the extremely successful practical technique of Bounded Model Checking [1] can, in retrospect, be seen as a fixed-parameter tractable reduction to SAT. Recently developed fixed-parameter tractable reductions to SAT [3–6] break complexity barriers for several problems, including the main reasoning problems of disjunctive answer-set programming, problems arising in propositional abductive reasoning, and problems arising in agenda safety. There are already basic concepts and results for a *hardness theory* [3, 5] that can be used to provide evidence that certain parameterized problems do not admit fixed-parameter tractable reductions to SAT.

References

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