

A Parameterized Complexity Analysis of Generalized CP-Nets^{*}

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Abstract

Generalized CP-nets (GCP-nets) allow a succinct representation of preferences over multi-attribute domains. As a consequence of their succinct representation, many GCP-net related tasks are computationally hard. Even finding the more preferable of two outcomes is **PSPACE**-complete. In this work, we employ the framework of (parameterized) complexity theory to achieve two goals: First, we want to gain a deeper understanding of the complexity of GCP-nets. Second, we search for efficient fixed-parameter tractable algorithms.

Each GCP-net has a corresponding preference graph. As a first step, we analyse the classical and parameterized complexity of computing the diameter of a given GCP-net. For the general case, we establish **PSPACE**-completeness of this problem. We show that the complexity drops to $\Pi_2\text{P}$ -completeness for a variant of the Diameter problem, where the diameter has to be below some value k which itself is polynomially bounded with respect to the number of variables. After that, we analyse the Diameter problem from a parameterized point of view by considering the diameter as parameter. It turns out that the problem is in the class **XP** (and hence admits an efficient computation for small diameters). However, by showing also **co-W[1]**-hardness, we rule out fixed-parameter tractability. In our analysis, we identify the unbounded size of the diameter as the main source of complexity. We thus carry out experiments to get an idea how likely an arbitrarily (i.e., exponentially) big diameter is. It turns out that for randomly generated GCP-nets, the diameter is typically in the order of magnitude of the number of variables (i.e., attributes) of a GCP-net.

We then study the parameterized complexity of the Dominance problem of GCP-nets for various combinations of parameters including the diameter k , number of variables $|V|$, number of rules $|R|$, maximum size of the conditions c , and the maximum number of occurrences of effects e . We obtain three kinds of results: For some parameter combinations (such as k and c), the Dominance problem is in **FPT**. For other parametrizations (such as k alone), we establish **W[1]**-completeness. Finally, we also identify parameter combinations (such as e and c), for which the Dominance problem is **paraNP**-hard. On the one hand, this

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parameterized complexity analysis gives us a better understanding of the actual source of complexity of the Dominance problem. On the other hand, it also underlines the importance of considering *combinations* of parameters, since there are cases where single parameters do not help – only combinations of parameters yield fixed-parameter tractability.