

Connection matrices of finite rank instead of definability in Monadic Second Order Logic

N. LABAI ^{*} and J.A. MAKOWSKY ^{**}

Department of Computer Science
Technion - Israel Institute of Technology, Haifa, Israel
e-mail: {nadia,janos}@cs.technion.ac.il,

Abstract. Connection matrices for graph parameters with values in a field have been introduced by M. Freedman, L. Lovász and A. Schrijver (2007). Graph parameters with connection matrices of finite rank can be computed in polynomial time on graph classes of bounded tree-width. We introduce join matrices, a generalization of connection matrices, and allow graph parameters to take values in the tropical rings (max-plus algebras) over the real numbers. We show that rank-finiteness of join matrices implies that these graph parameters can be computed in polynomial time on graph classes of bounded clique-width. In the case of graph parameters with values in arbitrary commutative semirings, this remains true for graph classes of bounded linear clique-width. B. Godlin, T. Kotek and J.A. Makowsky (2008) showed that definability of a graph parameter in Monadic Second Order Logic implies rank finiteness. We also show that there are uncountably many integer valued graph parameters with connection matrices or join matrices of fixed finite rank. This shows that rank finiteness is a much weaker assumption than any definability assumption.

The full paper has been posted on arxiv as

Nadia Labai and Johann A. Makowsky
Finiteness conditions for graph algebras over tropical semirings

It was accepted for presentation at FPSAC 2014 (Chicago, June 29 -July 3, 2014)
and can be viewed at

<http://arxiv.org/abs/1405.2547>
<http://arxiv.org/pdf/1405.2547v1>

It will also appear in *Discrete Mathematics and Theoretical Computer Science*.

^{*} Partially supported by a grant of the graduate school of the Technion.

^{**} Partially supported by a grant of Technion Research Authority.