

(Quantified) Horn Constraint Solving for Program Verification and Synthesis

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Abstract

We show how automatic tools for the verification of linear and branching time properties of procedural, multi-threaded, and functional programs as well as program synthesis can be naturally and uniformly seen as solvers of constraints in form of (quantified) Horn clauses over background logical theories. Such a perspective can offer various advantages, e. g., a logical separation of concerns between constraint generation (also known as generation of proof obligations) and constraint solving (also known as proof discovery), reuse of solvers across different verifications tasks, and liberation of proof designers from low level algorithmic concerns and vice versa.

KEYWORDS: Horn constraints, program verification, program synthesis

1 Introduction

A variety of interesting and important verification and synthesis questions about programs can be formulated as constraint satisfaction problems in form of implication/Horn constraints over a suitable background theory (Gupta et al. 2011a; Gupta et al. 2011c; Grebenshchikov et al. 2012; Popeea and Rybalchenko 2012; Bjørner et al. 2012; Beyene et al. 2014). Verification of temporal properties and program synthesis are particularly relevant examples of such questions. Then, given an efficient constraint solver we can obtain a program verifier or synthesizer by composing it with a constraint generator. As a result we achieve a separation of concerns such that deduction rules for reasoning about programs can be developed independently and interact compositionally with inference engines (that automate the deduction process).

2 Constraint generation

We illustrate the constraint based approach using the following example. Let a program be represented by an assertion $init(v)$ that describes a set of initial states, and an assertion $next(v, v')$ that describes a binary transition relation.

Safety To prove that each state that is reachable from an initial state by following the transition relation satisfies an assertion $safe(v)$ we generate the following constraint.

$$\begin{aligned} \exists inv : \\ & (\forall v : init(v) \rightarrow inv(v)) \\ & \wedge (\forall v \forall v' : inv(v) \wedge next(v, v') \rightarrow inv(v')) \\ & \wedge (\forall v : inv(v) \rightarrow safe(v)) \end{aligned}$$

Here we rely on second order existential quantifier to model the search for a safe (forward-)inductive invariant. Any model of inv is such an invariant.

Note that by using the following constraint system (but not the solver/inference engine) we can change the proof rule from forward invariance to backward invariance.

$$\begin{aligned} \exists binv : \\ & (\forall v : \neg safe(v) \rightarrow binv(v)) \\ & \wedge (\forall v \forall v' : binv(v') \wedge next(v, v') \rightarrow binv(v)) \\ & \wedge (\forall v : binv(v) \wedge safe(v) \rightarrow false) \end{aligned}$$

Furthermore, we can combine forward and backward reasoning in the same constraint system:

$$\begin{aligned} \exists inv \exists binv : \\ & (\forall v : init(v) \rightarrow inv(v)) \\ & \wedge (\forall v \forall v' : inv(v) \wedge next(v, v') \rightarrow inv(v')) \\ & \wedge (\forall v : \neg safe(v) \rightarrow binv(v)) \\ & \wedge (\forall v \forall v' : binv(v') \wedge next(v, v') \rightarrow binv(v)) \\ & \wedge (\forall v : inv(v) \wedge binv(v) \rightarrow false) \end{aligned}$$

All these reasoning approaches can be automated by the same solver, which can be made highly beneficial through consolidation of heuristics, optimizations, and improvements.

Termination To prove program termination, i.e., absence of infinite sequences of states that start in an initial state and follow the transition relation, we can resort to the following constraint.

$$\begin{aligned} \exists inv \exists round : \\ & (\forall v : init(v) \rightarrow inv(v)) \\ & \wedge (\forall v \forall v' : inv(v) \wedge next(v, v') \rightarrow inv(v')) \\ & \wedge (\forall v : \neg safe(v) \rightarrow binv(v)) \\ & \wedge (\forall v \forall v' : inv(v) \wedge next(v, v') \rightarrow round(v, v')) \\ & \wedge wf(round) \end{aligned}$$

Note that we rely on a second order predicate wf that holds for well-founded relations. Each solver can rely on a specific way of proving well-foundedness, e.g., using abstract interpretation (Cousot and Cousot 2012), ranking functions (Turing 1949) or transition invariants (Podelski and Rybalchenko 2004b; Rybalchenko 2005).

Instead of using forward invariance to keep track of reachable states, we can use alter-

native approaches, as exemplified by the constraint systems for reasoning about program safety.

Information flow property Security properties often require proving that any alternation of a (secret) program input is undetected by observing its (public) output. Let $final(v)$ be an assertion that describes the set of all states where computation can stop. The program satisfies the non-interference property if the following constraint is satisfiable.

$\exists io :$

$$\begin{aligned} & (\forall v : init(v) next(v, v') \rightarrow io(v, v')) \\ & \wedge (\forall v \forall v' \forall v'' : io(v, v') \wedge next(v', v'') \rightarrow io(v, v'')) \\ & \wedge (\forall v \forall v' \forall w \forall w' : v \neq w \wedge io(v, v') \wedge io(w, w') \wedge final(v) \wedge final(w') \rightarrow w = w') \end{aligned}$$

This constraint captures reasoning about pairs of computations through self-composition of input/output relation of the program, instead of performing a source-to-source transformation (Barthe et al. 2004) or a specialized inference procedure (Backes et al. 2009).

Temporal property For proving properties expressed in temporal logics we can rely on existing proof systems that reduce temporal reasoning to first-order reasoning (with auxiliary assertions), e.g., the proof system for CTL* (Kesten and Pnueli 2005). For example, to prove that there exists a computation that visits states satisfying an assertion $p(v)$ until finally it reaches a state satisfying an assertion $q(v)$, i.e., $(init(v), next(v, v')) \models E(p(v)Uq(v))$, we need to solve the following constraint.

$\exists inv \exists round :$

$$\begin{aligned} & (\forall v : init(v) \rightarrow inv(v)) \\ & \wedge (\forall v : inv(v) \wedge \neg q(v) \rightarrow \exists v' : next(v, v') \wedge inv(v') \wedge round(v, v')) \\ & \wedge (\forall v : inv(v) \rightarrow p(v)) \\ & \wedge wf(round) \end{aligned}$$

Note that we model the existence of a computation by relying on existential quantification in the constraints together with a recursive dependency (Beyene et al. 2013).

Reactive synthesis We formulate the reactive synthesis problem as constraint solving by turning this problem into a game solving problem (Beyene et al. 2014). For example, consider the synthesis of a system with an unknown transition relation $sys(v, v')$ that is executed in an adversarial environment with a given transition relation $env(v, v')$. The system's objective is to reach a state satisfying an assertion $goal(v)$ regardless of environment's behavior. The following constraint characterizes the synthesis problem.

$\exists inv \exists round :$

$$\begin{aligned} & (\forall v : init(v) \rightarrow inv(v)) \\ & \wedge (\forall v \forall v' : inv(v) \wedge \neg goal(v) \wedge env(v, v') \rightarrow \exists v'' : sys(v', v'') \wedge inv(v'') \\ & \quad \wedge round(v, v'')) \\ & \wedge wf(round) \end{aligned}$$

Other temporal objectives can be satisfied by relying on a temporal proof system, akin to temporal verification.

3 Constraint solving

In the previous section we illustrated how program verification and synthesis question can be formalized as second-order constraint solving problems in form of recursive implication constraints with well-foundedness conditions, and quantifier alternation.

Solving (various sub-classes of) such constraints is a thriving area of research. Solvers often take advantage of the fact that constraints have Horn-like structure, which enables iterative, abstraction based solving approaches.

Important classes include recursion-only case (Hoder et al. 2011; Grebenshchikov et al. 2012; McMillan and Rybalchenko 2012; Rümmer et al. 2013) that facilitates reasoning about safety properties, extension with well-foundedness for reasoning about liveness properties (Grebenshchikov et al. 2012), extension with universal quantification for inferring universally quantified invariants (Bjørner et al. 2013), and extension with existential quantification for dealing with synthesis and branching time questions (Beyene et al. 2013). Often such solvers rely on recursion-free fragments of implication constraints (McMillan 2005; Gupta et al. 2011b; Gurfinkel et al. 2013; Blanc et al. 2013) and ranking function synthesis (Podelski and Rybalchenko 2004a) as a basic inference components.

Constraint logic programming offers an effective tool for implementing solvers for quantified (Horn) implication constraints with well-foundedness, e.g., the HSF solver and its extensions (Grebenshchikov et al. 2012; Beyene et al. 2013; Bjørner et al. 2013) is implemented using ideas of blending meta-logic programming and constraint logic programming (Podelski and Rybalchenko 2007).

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