

MEREOLOGICAL COMPARATIVES IN DELINEATION SEMANTICS

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This paper makes a new contribution to the development of new logical systems for the analysis of natural language expression and reasoning. More precisely, I present a new logical analysis of nominal comparatives (i.e. sentences as in (1)) within the *Delineation Semantics* (DeIS) approach to gradability and comparison (Klein, 1980, among many others).

- (1) More linguists came to the party than stayed home to study.

Along with the *Degree Semantics* (DegS) framework (see Kennedy, 1997, for a summary), DeIS is one of the dominant logical frameworks for analyzing the meaning of gradable constituents of the adjectival syntactic category; however, there has been very little work done investigating the application of this framework to the analysis of gradability outside the adjectival domain. This state of affairs distinguishes the DeIS framework from its DegS counterpart, where such questions have been investigated in great deal since the beginning of the 21st century. Nevertheless, it has been observed (for example, by Doetjes et al. (2011)) that there is nothing inherently adjectival about the way that the interpretations of scalar predicates are calculated in DeIS, and therefore that there is enormous potential for this approach to shed light on the nature of gradability and comparison in the nominal and verbal domains. This paper gives the first attempt at realizing this potential within a Mereological extension of a (simplified) version of Klein (1980)'s system.

1. DELINEATION SEMANTICS FOR ADJECTIVAL COMPARATIVES

The proposal that there exists an analytical relationship between context-sensitivity and gradability lies at the heart of the Delineation approach to the semantics of scalar predicates; in particular, in this framework, the orderings associated with adjectival predicates (often called their *scales*) are derived from looking at how the denotation of these predicates vary according to a contextually given comparison class. Formally speaking, the semantics (for a language with constants and predicates of the type found in First Order Logic) is set up as follows :

Definition 0.1. Model. *A model is a tuple $M = \langle D, \llbracket \cdot \rrbracket \rangle$ where D is a non-empty domain of individuals, and $\llbracket \cdot \rrbracket$ is a function from pairs consisting of a member of the non-logical vocabulary and a comparison class (a subset of the domain) satisfying:*

- *For each individual constant a_1 , $\llbracket a_1 \rrbracket \in D$.*
- *For each $X \subseteq D$ and for each predicate P , $\llbracket P \rrbracket_X \subseteq X$.*

With respect to the linguistic analysis, let us assume furthermore that the sets of individuals that the predicates pick out are those individuals that clearly satisfy them (i.e. only non-borderline individuals).

In this framework, truth in a model is always given with respect to a distinguished comparison class. Here I suppose that if the subject of the sentence is not included in the distinguished

comparison class, the truth value of the sentence is undefined¹.

Definition 0.2. *Semantics of the positive form.* For all models M^2 , all comparison classes $X \subseteq D$, all predicates P and individuals $a_1 \in X$,

$$\llbracket P(a_1) \rrbracket_{X,M} = \begin{cases} 1 & \text{if } \llbracket a_1 \rrbracket_M \in \llbracket P \rrbracket_{X,M} \\ 0 & \text{if } \llbracket a_1 \rrbracket_M \in X - \llbracket P \rrbracket_{X,M} \\ i & \text{otherwise} \end{cases}$$

This very simple semantic analysis is now enriched with a set of extra constraints on the application of scalar predicates across classes. In this paper, we will adopt the constraint set proposed in van Rooij (2011a) as an analysis of the context-sensitivity and gradability properties of relative adjectives. Van Rooij proposes the following four constraints (set in my notation)³, where by $\bar{P}$ we pick out those individuals who clearly do not satisfy the predicate:

For all predicates P , all comparison classes $X \subseteq D$, and all $a_1, a_2 \in D$,

(2) **P*:** $\llbracket P \rrbracket_X \cap \llbracket \bar{P} \rrbracket_X = \emptyset$

(2) ensures that P and $\bar{P}$ behave as contraries.

(3) **No Reversal*:**

- 1.If $\llbracket P(a_1) \rrbracket_X = 1$ and $\llbracket P(a_2) \rrbracket_X = 0$, then there is no $X' \subseteq D$ such that $\llbracket P(a_2) \rrbracket_{X'} = 1$ and $\llbracket P(a_1) \rrbracket_{X'} = 0$.
- 2.If $\llbracket \bar{P}(a_1) \rrbracket_X = 1$ and $\llbracket \bar{P}(a_2) \rrbracket_X = 0$, then there is no $X' \subseteq D$ such that $\llbracket \bar{P}(a_2) \rrbracket_{X'} = 1$ and $\llbracket \bar{P}(a_1) \rrbracket_{X'} = 0$.

(3) ensures that, if in one comparison class, a_1 is categorized as (clearly) P and a_2 is categorized as (clearly) not P , then there are no comparison classes in which this categorization is reversed.

(4) **Upward Difference*:**

If $\llbracket P(a_1) \rrbracket_X = 1$ and $\llbracket \bar{P}(a_2) \rrbracket_X = 1$, then, for all $X' : X \subseteq X'$, there is some a_3, a_4 such that $\llbracket P(a_3) \rrbracket_{X'} = 1$ and $\llbracket \bar{P}(a_4) \rrbracket_{X'} = 1$.

(4) says that, if, in one comparison class, it is reasonable to make a $P/\bar{P}$ distinction between a_1 and a_2 , then, in all larger comparison classes that contain a_1 and a_2 , we must continue to make **some** distinction (although not necessarily the same one). This axiom can be thought of as a principle of contrast preservation in categorization.

¹As suggested by our judgements concerning sentences like (i).

(i) #Mary is tall for a boy in this class.

²For readability considerations, I will often omit the model notation, writing only $\llbracket \cdot \rrbracket_X$ for $\llbracket \cdot \rrbracket_{X,M}$.

³Van Rooij's constraints are modified versions of those in van Benthem (1982), which are called *No Reversal*, *Upward Difference* and *Downward Difference*, respectively. Hence the NR*, UD*, DD* labels.

(5) **Downward Difference*:**

If $\llbracket P(a_1) \rrbracket_X = 1$ and $\llbracket \bar{P}(a_2) \rrbracket_X = 1$, then, for all $X' \subseteq X$, if $a_1, a_2 \in X'$, then there is some a_3, a_4 such that $\llbracket P(a_3) \rrbracket_{X'} = 1$ and $\llbracket \bar{P}(a_4) \rrbracket_{X'} = 1$.

(5) is another principle of contrast preservation which states that, if we make a distinction between a_1 and a_2 based on $P/\bar{P}$ in one comparison class, then in all smaller comparison classes that include a_1 and a_2 , we must continue to make some distinction (although, again, not necessarily the same one).

With these constraints in place, we can define an ordering relation ($\succ_P$) associated with a scalar predicate P and a similarity relation ($\sim_P$) based on $\succ_P$ as follows, based on (van Rooij, 2011a, p.13):

Definition 0.3. Implicit Scale ($\succ$) and similarity ($\sim$). For all individuals $a_1, a_2 \in D$ and all predicates P ,

1. $a_1 \succ_P a_2$ iff $a_1 \in \llbracket P \rrbracket_{a_1, a_2}$ and $a_2 \in \llbracket \bar{P} \rrbracket_{a_1, a_2}$.
2. $a_1 \sim_P a_2$ iff $a_1 \not\succ_P a_2$ and $a_2 \not\succ_P a_1$.

Van Rooij shows that with the constraint set in (2)-(5), the $\succ_P$ relations are *semi-orders*: irreflexive, semi-transitive relations that satisfy the interval order property. A pleasant consequence of this is that, van Rooij argues, we immediately have an analysis of the properties of modes of comparison that do not involve special degree morphology, such as implicit comparative constructions like *Mary is tall, compared to John*. However, comparisons formed with the comparative morpheme *-er/more* (6) have different properties than comparisons that do not involve this morphology. Observe that, in contrast to the implicit comparative construction, for (6-a) to be true, Mary only needs to be slightly taller than John, not noticeably or significantly so.

- (6) a. Mary is **taller** than John.
b. This problem is **more difficult** than that one.

In order to reflect the difference in the order, van Rooij proposes that the orders that are relevant for evaluating the truth of sentences with explicit comparative morphology in English are stronger than the ones given by definition 0.3, namely, they are **strict weak orders**: irreflexive, transitive and almost connected relations. As shown by Luce (1956), for every semi-order, there is a unique most refined strict weak order, which we will notate $>_P$ (for the corresponding semi-order $\succ_P$), and this order can be constructed as follows: For all individuals $a_1, a_2 \in D$ and all predicates P , $a_1 >_P a_2$ iff there is some $a_3 \in D$ such that $a_1 \sim_P a_3$ and $a_3 \succ_P a_2$ **or** $a_2 \sim_P a_3$ and $a_1 \succ_P a_3$. This construction preserves the relations established by $\succ_P$, but also adds the individuals into the order that are found in the ‘gaps’ between a predicate’s clear extension (P) and its clear anti-extension ($\bar{P}$) in a comparison class. In addition, if we define an equivalence relation $\approx_P$ based on $>_P$ as $a_1 \approx_P a_2$ iff $a_1 \not>_P a_2$ and $a_2 \not>_P a_1$, then we can group individuals together into equivalence classes (notated as $[a_1]_P$) based on $\approx_P$ in the usual way⁴ and construct a linear ordering between equivalence classes as follows: for all $a_1, a_2 \in D$, $[a_1]_P >^*_P [a_2]_P$ iff $a_1 >_P a_2$.

⁴For all $a_2 \in D$, $a_2 \in [a_1]_P$ iff $a_2 \approx_P a_1$.

These structures are appropriate for the analysis of comparatives in which both the main clause and the *than* clause contains a single adjective; however, comparatives can also be formed from (a subset of possible) adjective-individual pairs (7), and indeed it is reasonable to think (as already suggested by Klein (1980)) that sentences such as (6) are simply special cases when the two adjectives happen to be the same.

- (7) a. This table is **longer** than that table is **wide**.
b. John is more **handsome** than he is **clever**.

Thus, the semantics for explicit comparative constructions needs to consider both the scales associated with the adjective in the main clause and the scales associated with the adjective in the *than* clause. Within the logical analysis, we add to the language a quaternary predicate *ER* that will combine with two individuals and two predicates to form well-formed formulas of the form $\text{MORE}(a_1, P_1, a_2, P_2)$ etc. Since formulas containing *MORE* can contain two context-sensitive predicates, they will be evaluated with respect to two (possibly distinct) comparison classes (one for each predicate)⁵, and their truth values will depend on whether the first individual occupies a higher rank on the first predicate's scale than the second individual on the second predicate's scale. To formalize this idea, the approach that I will pursue in this paper is that of Bale (2008) in which a 'universal scale of comparison' is constructed according to which individuals' rankings on different scales can be compared. The first step in the construction involves a normalization process: pairs consisting of an individual a_1 and a linear scale $>^*_P$ are mapped to what, following the terminology (and notation) in van Rooij (2011b), I will call their *Bale rank* (b_P , for linear scale $>^*_P$). Bale ranks are rational numbers between 0 and 1 whose values are calculated in the following way:

Definition 0.4. *Bale rank* (b_P). For all predicates P , all individuals a_1 ,

$$b_P(a_1) = \frac{|\{[a_2]_P : a_2 \in D \ \& \ [a_1]_P >^*_P [a_2]_P\}|}{|\{[a_3]_P : a_3 \in D\}|}$$

In other words, we map individuals according to their equivalence classes (or 'degrees') on a linear scale to the number of equivalence classes that lie below them on that scale divided by the total number of distinct degrees on the linear scale. Now that we can compare degrees on different scales by means of the universal scale, the truth of the explicit comparative construction is calculated with respect to comparing individuals belonging to equivalence classes that constitute these degrees:

Definition 0.5. *Semantics of the explicit comparative construction* For all comparison classes $X_1, X_2 \subseteq D$, predicates P_1, P_2 , and individuals a_1, a_2 ,

$$\llbracket \text{MORE}(a_1, P_1, a_2, P_2) \rrbracket_{X_1, X_2} = 1 \text{ iff } b_{P_1}(a_1) > b_{P_2}(a_2)$$

I now extend this analysis of the scales associated with adjectives and the meaning of the explicit comparative construction to gradable constituents and comparatives outside the

⁵As a convention, I will use numerals to associate predicates and comparison classes, i.e. in an expression like $\llbracket \text{MORE}(a_1, P_1, a_2, P_2) \rrbracket_{X_1, X_2}$, X_1 is the class that goes with P_1 and X_2 goes with P_2 .

adjectival domain.

2. DELINEATION SEMANTICS FOR NOMINAL COMPARATIVES

The heart of the proposal involves combining three (I hope) relatively uncontroversial points:

1. The proposal will be set within a mereological extension (see Simons, 1987; Hovda, 2008, among others) of the Delineation system presented above, in line with much of the work on the semantics of plural noun phrases and distributivity since Link (1983)⁶.
2. I assume that subject determiner phrases denote *generalized quantifiers* (Barwise and Cooper, 1981, among others): second order properties (i.e. properties of properties).
3. When the explicit comparative morpheme (ER) appears in the nominal domain, this morpheme combines with noun phrases containing the ‘Q-adjectives’ *many* or *few*, as proposed by Bresnan (1973), among many others.

In line with the work of Link (1983), I assume that plural predicates of the distributive class are constructed in the syntax through the concatenation of a pluralizing distributivity operator $*$: If P is a predicate, then P^* is a **plural predicate**. Furthermore, we also have introduced a nominal pluralization operator: $*$: If P is a singular predicate, then P^* is a plural NP.

Unlike in basic Delineation semantics, in which we supposed our domain to be an unordered set of individuals as in the models for first order logic, we now interpret the expressions of our language into a domain that encodes **mereological** (i.e. part-structure) relations between its individuals. More precisely, we define the our model structures as follows:

Definition 0.6. Model Structure. A model structure $\mathcal{M}$ is a tuple $\langle D, \preceq \rangle$, where D is a finite set of individuals, $\preceq$ is a binary relation on D .⁷

Furthermore, we stipulate that $\langle D, \preceq \rangle$ satisfies the axioms of classical extensional mereology (CEM).⁸

1. **Reflexivity.** For all $a_1 \in D$, $a_1 \preceq a_1$.
2. **Transitivity.** For all a_1, a_2, a_3 , if $a_1 \preceq a_2$ and $a_2 \preceq a_3$, then $a_1 \preceq a_3$.
3. **Anti-symmetry.** For all $a_1, a_2 \in D$, if $a_1 \preceq a_2$ and $a_2 \preceq a_1$, then $a_1 = a_2$.
4. **Strong Supplementation.**⁹ For all $a_1, a_2 \in D$, for all atoms a_3 , if, if $a_3 \preceq a_1$, then $a_3 \circ a_2$, then $a_1 \preceq a_2$.
5. **Fusion Existence.** For all $X \subseteq D$, if there is some $a_1 \in X$, then there is some $a_2 \in D$ such that $Fu(a_2, X)$ ¹⁰.

⁶For convenience, I limit my attention to distributive predicates like *come to the party* and *stay home to study*, i.e. instances of plural predication that allow for inferences to instances of singular predications.

⁷Note this $\preceq$ symbol should not be confused with the $\succ_P$ symbols that notate the semi-order ‘implicit scale’ relations. $\preceq$ notates invariant, predicate independent relations that are part of the model structure.

⁸This particular axiomatization is taken from Hovda (2008) (p.81). The version of *fusion* used here is what Hovda calls ‘type 1 fusion’.

⁹**Def. Overlap ($\circ$).** For all $a_1, a_2 \in D$, $a_1 \circ a_2$ iff $\exists a_3 \in D$ such that $a_3 \preceq a_1$ and $a_3 \preceq a_2$.

¹⁰**Def. Fusion (Fu).** For $a_1 \in D$ and $X \subseteq D$, $Fu(a_1, X)$ (a_1 fuses X) iff, for all $a_2 \in D$,

We can note that, in CEM, for every subset of D , not only does its fusion exist, but it is also unique (cf. Hovda (2008), p. 70), so we write $\bigvee X$ to notate the unique a_1 such that $Fu(a_1, X)$, for all $X \subseteq D$. Furthermore, since we stipulated that every domain D is finite, every structure $\langle D, \preceq \rangle$ is atomic. We define the notion of an *atom* as follows: $a_1 \in D$ is an atom iff there is no $a_2 \in D$ such that $a_2 \prec a_1$, and we will write $AT(X)$ for the set of atoms of a set $X \subseteq D$.

Above, it was proposed that the interpretation of predicates is relativized to comparison classes, i.e. distinguished subsets of the domain. Now, since our domain has more structure, we need to make a precision: while we will allow constants $(a_1, a_2, a_3 \dots)$ to denote in the entire domain (i.e. both singular and plural individuals), we stipulate that both the denotations of singular predicates and the comparison classes according to which they are interpreted are restricted to the set of atoms of D .

Definition 0.7. Model. A model is a tuple $\mathcal{M} = \langle D, \preceq, \llbracket \cdot \rrbracket \rangle$, where $\langle D, \preceq \rangle$ is a model structure and $\llbracket \cdot \rrbracket$ is a function satisfying:

1. If a is a constant, then $\llbracket a \rrbracket \in D$.
2. If P is a singular predicate and $X \subseteq AT(D)$, then $\llbracket P \rrbracket_X \subseteq X$.

The analysis of the semantics of distributive plural predicates will follow that of Link (1983). Link proposes that plural distributive predicates (ex. *linguists, have come to the party* etc.) are derived from singular predicates P (ex. *linguist, has come to the party*) through the addition of a $*$ or a * operator, which generates all the individual fusions of members of the extension of P ¹¹.

Definition 0.8. Interpretation of plurality ($^*/^*$). For all singular predicates P and $X \subseteq AT(D)$, $\llbracket P^{*/^*} \rrbracket_X = \{a_1 : Fu(a_1, A), \text{ for some } A \subseteq \llbracket P \rrbracket_X\}$.

Following standard assumptions in the linguistics literature since Bresnan (1973), I propose that the explicit comparative relations are built off of the scales associated with DPs containing the Q-adjectives *many* and *few*. These items can combine with count plural noun phrases to form DPs as shown in (8); therefore, we add to the language the expressions **MANY** and **FEW**, which combine with a plural NP P_1^* to form an expression of the category **DP** (i.e. **MANY** P_1^* or **FEW** P_1^*).

- (8) a. **Many** linguists came to the party.
b. **Few** philosophers came to the party.

As mentioned, I analyze DP subjects of the form *many linguists* and *few philosophers* as denoting generalized quantifiers, and, because these DPs are context-sensitive, I propose that they are evaluated with respect to a comparison class. In parallel to the comparison classes associated with singular predicates, the comparison classes associated with generalized

$a_2 \circ a_1$ iff there is some a_3 such that $a_3 \in X$ and $a_2 \circ a_3$.

¹¹I have chosen to separate the nominal and predicative pluralizing operators in the logic because it makes the syntax of our small language much simpler, but since we assign to them exactly the same interpretation, as shown in Definition 0.8, I suggest that they should be thought of as two separate occurrences of an interpretable [+plur] morphological marking.

quantifiers (i.e. DP denotations) will be made up of possible values for the quantifiers' argument: sets of properties. These sets of properties should be thought of constituting the range of relevant possible cases or situations that might influence whether or not the main predicate denotation would be categorized as satisfying the DP subject denotation.

Definition 0.9. Interpretations of Many/Few DPs. *If P is a predicate, $X \subseteq AT(D)$, $Z \subseteq \mathcal{P}(D)$, $\llbracket \text{MANY}P^* \rrbracket_{Z,X} \subseteq Z$, and $\llbracket \text{FEW}P^* \rrbracket_{Z,X} \subseteq Z$.*

Expressions of the category **DP** combine with plural predicates P^* to form formulas, which are interpreted in an exactly parallel manner to formulas containing the positive form of scalar adjectives (cf. Def. 0.2).

Definition 0.10. *For all predicates P_1, P_2 , $X_1, X_2 \subseteq AT(D)$ and $Z \subseteq \mathcal{P}(D)$,*

$$\llbracket \text{MANY}P_1^*(P_2^*) \rrbracket_{Z,X_1,X_2} = \begin{cases} 1 & \text{if } \llbracket P_2^* \rrbracket_{X_2} \in \llbracket \text{MANY}P_1^* \rrbracket_{Z,X_1} \\ 0 & \text{if } \llbracket P_2^* \rrbracket_{X_2} \in Z - \llbracket \text{MANY}P_1^* \rrbracket_{Z,X_1} \\ i & \text{otherwise} \end{cases}$$

$$\llbracket \text{FEW}P_1^*(P_2^*) \rrbracket_{Z,X_1,X_2} = \begin{cases} 1 & \text{if } \llbracket P_2^* \rrbracket_{X_2} \in \llbracket \text{FEW}P_1^* \rrbracket_{Z,X_1} \\ 0 & \text{if } \llbracket P_2^* \rrbracket_{X_2} \in Z - \llbracket \text{FEW}P_1^* \rrbracket_{Z,X_1} \\ i & \text{otherwise} \end{cases}$$

Of course, as in the adjectival domain, the analysis will be incomplete if we do nothing to restrict the application of *many/few* DPs across comparison classes. Thus, we impose higher-order versions of van Rooij (2011a)'s constraints that were adopted in the analysis of gradable adjectives above. However, in the nominal domain, instead of opposing positive extensions of predicates P and negative extensions $\bar{P}$, we oppose DPs containing *many* with those containing *few*. The first constraint (9) ensures that DPs containing *many* and DPs containing *few* are contraries.

(9) **P★:**

For all P , all $X \subseteq AT(D)$, and $Z \subseteq \mathcal{P}(D)$, $\llbracket \text{MANY}P^* \rrbracket_{Z,X} \cap \llbracket \text{FEW}P^* \rrbracket_{Z,X} = \emptyset$

Furthermore, we assume appropriate versions of the constraints that characterize the application of scalar predicates across comparison classes.

(10) **No Reversal★:**

- 1.If $\llbracket \text{MANY}P_1^*(P_2^*) \rrbracket_{Z,X_1,X_2} = 1$ and $\llbracket \text{MANY}P_1^*(P_3^*) \rrbracket_{Z,X_1,X_3} = 0$, then there is no $Z' \subseteq D$ such that $\llbracket \text{MANY}P_1^*(P_3^*) \rrbracket_{Z',X_1,X_3} = 1$ and $\llbracket \text{MANY}P_1^*(P_2^*) \rrbracket_{Z',X_1,X_2} = 0$.
- 2.If $\llbracket \text{FEW}P_1^*(P_2^*) \rrbracket_{Z,X_1,X_2} = 1$ and $\llbracket \text{FEW}P_1^*(P_3^*) \rrbracket_{Z,X_1,X_3} = 0$, then there is no $Z' \subseteq D$ such that $\llbracket \text{FEW}P_1^*(P_3^*) \rrbracket_{Z',X_1,X_3} = 1$ and $\llbracket \text{FEW}P_1^*(P_2^*) \rrbracket_{Z',X_1,X_2} = 0$.

(11) **Upward Difference★:**

If $\llbracket \text{MANY}P_1^*(P_2^*) \rrbracket_{Z,X_1,X_2} = 1$ and $\llbracket \text{FEW}P_1^*(P_3^*) \rrbracket_{Z,X_1,X_2} = 1$, then, for all $Z' : Z \subseteq Z'$, there are some P_4, P_5 such that $\llbracket \text{MANY}P_1^*(P_4^*) \rrbracket_{Z',X_1,X_4} = 1$ and $\llbracket \text{FEW}P_1^*(P_5^*) \rrbracket_{Z',X_1,X_5} = 1$.

(12) **Downward Difference*:**

If $\llbracket \text{MANY}P_1^*(P_2^*) \rrbracket_{Z, X_1, X_2} = 1$ and $\llbracket \text{FEW}P_1^*(P_3^*) \rrbracket_{Z, X_1, X_3} = 1$, then, for all $Z' \subseteq Z$, if $P_2^*, P_3^* \in Z'$, then there are some P_4, P_5 such that $\llbracket \text{MANY}P_1^*(P_4^*) \rrbracket_{Z', X_1, X_4} = 1$ and $\llbracket \text{FEW}P_1^*(P_5^*) \rrbracket_{Z', X_1, X_5} = 1$.

At the moment, the only thing that differentiates the Q-adjectives *many* and *few* from regular adjectives like *tall* or *beautiful* is their type: while *many/few* DPs denote sets of plural properties, singular predicates denote sets of atoms. However, in addition to being context-sensitive and attributing some kind of property to their argument, these DPs crucially describe aspects of **how much** of their argument satisfies their nominal restriction. In order to encode quantity into the denotation of the Q-adjectives, we need to impose some more constraints on the application of *many* and *few* across contexts. We start by defining a function that applies to two properties and yields the set of upper bounds of their intersection:

Definition 0.11. Upper bounds ($\uparrow$). For all properties $P_2, P_3 \subseteq D$,

$$\uparrow(P_2, P_3) = \{a_1 : a_1 \in P_2 \cap P_3 \text{ \& there is no } a_2 \in P_2 \cap P_3 : a_2 \prec a_1\}.$$

Extending our previous notation, we will write $AT(a_1)$ to indicate the set of atoms underneath a_1 (i.e. $\{a_2 : a_2 \in AT(D) \text{ \& } a_2 \preceq a_1\}$). Now, we impose the following constraints on the application of *many* DPs and *few* DPs within comparison classes Z :

(13) **Upward Many:**

For all properties $P_2 \subseteq D$, predicates P , $Z \subseteq \mathcal{P}(D)$ and $X \subseteq AT(D)$. Suppose $P_2 \in \llbracket \text{MANY}P^* \rrbracket_{Z, X}$. Then, for all $P_3 \in Z$,

$$\text{If } |AT(\bigvee(\uparrow(P_3, \llbracket P^* \rrbracket_X)))| \geq |AT(\bigvee(\uparrow(P_2, \llbracket P^* \rrbracket_X)))|, \text{ then } P_3 \in \llbracket \text{MANY}P^* \rrbracket_{Z, X}.$$

(13) takes the fusion (call it a_1) of the set of upper bounds of the intersection between a property of pluralities P_2 and the DP's nominal restriction (P^*), and makes sure that if P_2 is in the extension of $\text{MANY}P^*$, then all the other properties that contain a_1 in the same comparison class are also included in the extension of $\text{MANY}P^*$. With this architecture, we can now define the implicit scales associated with DPs containing *many* and *few* in an exactly parallel manner to the scales associated with adjectival predicates (see Def. 0.3)¹².

Definition 0.12. Implicit quantitative scale. ($\succ_{\text{MANY}/\text{FEW}}$) For all predicates P , properties $P_2, P_3 \subseteq D$, and $X \subseteq AT(D)$,

1. $P_2 \succ_{\llbracket \text{MANY}P^* \rrbracket_X} P_3$ iff $P_2 \in \llbracket \text{MANY}P^* \rrbracket_{\{P_2, P_3\}, X}$ and $P_3 \in \llbracket \text{FEW}P^* \rrbracket_{\{P_2, P_3\}, X}$.
2. $P_2 \succ_{\llbracket \text{FEW}P^* \rrbracket_X} P_3$ iff $P_2 \in \llbracket \text{FEW}P^* \rrbracket_{\{P_2, P_3\}, X}$ and $P_3 \in \llbracket \text{MANY}P^* \rrbracket_{\{P_2, P_3\}, X}$.

If we do this, we can show that, like with adjectival predicates, the $\succ_{\llbracket \text{MANY}P^* \rrbracket_X}$ and $\succ_{\llbracket \text{FEW}P^* \rrbracket_X}$ relations are semi-orders. Theorem 0.1 follows straightforwardly from van Rooij (2011a)'s

¹²Observe that since DPs containing MANY/FEW contain a context-sensitive predicate (P^*), the scales associated with the whole DP must be parametrized to a single interpretation of P^* at a comparison class X . Therefore, we write $\succ_{\llbracket \text{MANY}P^* \rrbracket_X}$ to reflect this.

corresponding result in the adjectival domain.

Theorem 0.1. Mereological Semi-Orders. *For all predicates P , all $X \subseteq AT(D)$, $\succ_{[MANY P^*]_X}$ is a semi-order. As is $\succ_{[FEW P^*]_X}$.*

Furthermore, thanks to the constraints proposed above, we can show that there exist relationships between the implicit scales associated with DPs containing MANY and the mereological relations in the model structure. In other words, in this analysis, the part-structure relations between the pluralities that properties affect (calculated as the fusion of the properties' upper bounds: $\bigvee(\uparrow(P_3, \llbracket P^* \rrbracket_X))$) are mirrored in the scale structure. Such monotonicity relationships have been argued to hold generally between nominal scales and mereological DP denotation structure by Schwarzschild (2006), among others, and this 'monotonicity requirement' it is a theorem of the system presented here.

Theorem 0.2. Monotonicity Requirement. *For all predicates P , all $X \subseteq AT(D)$ and all $P_2, P_3 \subseteq D$, if $\bigvee(\uparrow(P_3, \llbracket P^* \rrbracket_X)) \preceq \bigvee(\uparrow(P_2, \llbracket P^* \rrbracket_X))$, then $P_2 \succeq_{[MANY P^*]_X} P_3$ ¹³*

Now that we have associated semi-orders with DP constituents, we can construct strict weak orders from our $\succ_{[MANY P^*]_X}$ relations in the way suggested by Luce (1956), as well as the corresponding linear orders $>^*_{[MANY P^*]_X}$ ¹⁴. However, looking at the scales associated with a single DP would only give us the truth conditions for comparatives where the nominal restriction is the same in the main clause as in the *than* clause. To interpret comparatives where the DP restrictions are different (i.e. *More linguists came to the party than philosophers*), we need to make use of the constructions that we used to interpret indirect comparatives in the adjectival domain. In particular, using Bale's construction, we can assign to every property on a scale a bale rank, and give the semantics of formulas with the explicit comparative relation when it holds between two properties and two quantifiers, which is simply a higher-order version of the semantics that we gave to explicit comparatives in the adjectival domain¹⁵.

Definition 0.13. Explicit Comparative $\llbracket \text{MORE}(P_1, \text{MANY } P_2^*, P_3, \text{MANY } P_4^*) \rrbracket_{X_1, X_2, X_3, X_4} = 1$ iff

$$b_{[MANY P_2^*]_{X_2}}(\llbracket P_1 \rrbracket_{X_1}) > b_{[MANY P_4^*]_{X_4}}(\llbracket P_3 \rrbracket_{X_3})$$

¹³**Proof.** Suppose $\bigvee(\uparrow(P_3, \llbracket P^* \rrbracket_X)) \preceq \bigvee(\uparrow(P_2, \llbracket P^* \rrbracket_X))$, and suppose for a contradiction that $P_3 \not\succ_{[MANY P^*]_X} P_2$. Then, by Def. 0.12, $P_3 \in \llbracket \text{MANY } P^* \rrbracket_{\{P_2, P_3\}, X}$ and $P_2 \in \llbracket \text{FEW } P^* \rrbracket_{\{P_2, P_3\}, X}$. Since $\bigvee(\uparrow(P_3, \llbracket P^* \rrbracket_X)) \preceq \bigvee(\uparrow(P_2, \llbracket P^* \rrbracket_X))$, $|AT(\bigvee(\uparrow(P_3, \llbracket P^* \rrbracket_X)))| \leq |AT(\bigvee(\uparrow(P_2, \llbracket P^* \rrbracket_X)))|$. Since $P_3 \in \llbracket \text{MANY } P^* \rrbracket_{\{P_2, P_3\}, X}$, by (13), $P_2 \in \llbracket \text{MANY } P^* \rrbracket_{\{P_2, P_3\}, X}$. But by (9), since $P_2 \in \llbracket \text{FEW } P^* \rrbracket_{\{P_2, P_3\}, X}$, $P_2 \notin \llbracket \text{MANY } P^* \rrbracket_{\{P_2, P_3\}, X}$. $\perp$ So $P_2 \succeq_{[MANY P^*]_X} P_3$. $\square$

¹⁴**Def. Explicit** $(>_{[MANY P^*]_X})$ **and linear** $(>^*_{[MANY P^*]_X})$ **scales.** For all properties $P_1, P_2 \subseteq D$, all predicates P and all $X \subseteq AT(D)$,

- (i) $P_1 >_{[MANY P^*]_X} P_2$ iff there is some $P_3 \subseteq D$ such that:
 - 1. $P_1 \sim_{[MANY P^*]_X} P_3$ and $P_3 \succ_{[MANY P^*]_X} P_2$ **or**
 - 2. $P_2 \sim_{[MANY P^*]_X} P_3$ and $P_1 \succ_{[MANY P^*]_X} P_3$.
- (ii) $[P_1] \sim_{[MANY P^*]_X} [P_2]$ iff $P_1 >^*_{[MANY P^*]_X} P_2$.

The definitions for expressions with FEW are identical.

¹⁵**Def. DP Bale Rank.** For all predicates P , comparison classes $X \subseteq AT(D)$, and properties P_1 ,

$$b_{[MANY P^*]_X}(P_1) = \frac{|\{[P_2]_{[MANY P^*]_X} : [P_1]_{[MANY P^*]_X} >^*_{[MANY P^*]_X} [P_2]_{[MANY P^*]_X}\}|}{|\{[P_3]_{[MANY P^*]_X} : P_3 \subseteq D\}|}$$

Thus, in this system, (modulo consideration of type and syntactic complexity) we have a complete parallelism between comparison in the adjective and nominal domains.

(14) ADJECTIVAL DOMAIN

- a. MORE(a_1, P_1, a_2, P_1): This table is longer than that one (is).
- b. MORE(a_1, P_1, a_1, P_2): This table is longer than it is wide.
- c. MORE(a_1, P_1, a_2, P_2): This table is longer than that one is wide.

(15) NOMINAL DOMAIN

- a. MORE($P_1, \text{MANY}P_2^*, P_3, \text{MANY}P_2^*$):
More linguists came to the party than stayed home to study.
- b. MORE($P_1, \text{MANY}P_2^*, P_1, \text{MANY}P_4^*$):
More linguists came to the party than philosophers.
- c. MORE($P_1, \text{MANY}P_2^*, P_3, \text{MANY}P_4^*$):
More linguists came to the party than philosophers stayed home to study.

I therefore conclude that the Delineation approach to the semantics of gradable adjectives extends naturally to gradable constructions in the nominal domain.

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