

Signatures as open-ended types

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Abstract. Open-ended conceptions of time based on change and predication in natural language are analyzed through strings and records. A rudimentary system of types is provided through signatures within institutions.

1 Introduction

An essential characteristic of natural language, open-endedness is partly (if not wholly) responsible for the interest in what Parsons has dubbed

“sub-atomic semantics,” the study of those “formulas of English” that are treated as atomic formulas in most logical investigations of English

(Parsons 1990, page ix). A famous example of open-ended predicate-argument structure is

Jones did it slowly, deliberately, in the bathroom, with a knife, at midnight

(Davidson 1967, page 81). Open-endedness lurks also in

McTaggart’s dictum that ‘there could be no time if nothing changed’

(Prior 1967, page 85), what counts as change being open-ended. Variable adicity and time-as-change are analyzed below using the notion of an *institution*, introduced by Goguen and Burstall 1992 to cope with the proliferation of logical systems in computer science.¹ At the center of an institution are relations $\models_\Sigma$ of *satisfaction* between *models* and *sentences* of various *signatures* Σ . Signatures provide rudimentary types for subatomic semantics, or so we will argue. For variable adicity, we identify models with records, tracing open-endedness to the labels that may (or may not) appear in a record. For time-as-change, our models are strings, the alphabet of which is specified by the associated signature.

It is helpful to flesh out the examples a bit more for orientation.

¹ The theory of institutions has attracted considerable attention, and found numerous applications, including ontology design (e.g. Diaconescu 2012, Kutz et al 2010).

Example A We can represent a calendar year as the string

$$s_{mo} := \boxed{\text{Jan}} \boxed{\text{Feb}} \boxed{\text{Mar}} \cdots \boxed{\text{Dec}}$$

of length 12, or, were we interested also in days d1,d2...d31, the string

$$s_{mo,dy} := \boxed{\text{Jan,d1}} \boxed{\text{Jan,d2}} \cdots \boxed{\text{Jan,d31}} \boxed{\text{Feb,d1}} \cdots \boxed{\text{Dec,d31}}$$

of length 365 for a non-leap year.² In contrast to the points in the real line $\mathbb{R}$, a box can split, as $\boxed{\text{Jan}}$ in s_{mo} does (30 times) to

$$\boxed{\text{Jan,d1}} \boxed{\text{Jan,d2}} \cdots \boxed{\text{Jan,d31}}$$

in $s_{mo,dy}$, on introducing days d1, d2,..., d31 into the picture. Reversing direction and generalizing from

$$mo := \{\text{Jan,Feb,...Dec}\}$$

to any set A of temporal propopositions, we define the function ρ_A on strings (of sets) to componentwise intersect with A

$$\rho_A(\alpha_1 \cdots \alpha_n) := (\alpha_1 \cap A) \cdots (\alpha_n \cap A)$$

(throwing out non- A 's from each box) so that

$$\rho_{mo}(s_{mo,dy}) = \boxed{\text{Jan}}^{31} \boxed{\text{Feb}}^{28} \cdots \boxed{\text{Dec}}^{31}.$$

Time-as-change is then enforced by compressing all repeating blocks α^n (for $n \geq 1$) of a box α in a string s to α for its *block compression* $\mathcal{b}(s)$

$$\mathcal{b}(s) := \begin{cases} \mathcal{b}(\alpha s') & \text{if } s = \alpha \alpha s' \\ \alpha \mathcal{b}(\beta s') & \text{if } s = \alpha \beta s' \text{ with } \alpha \neq \beta \\ s & \text{otherwise} \end{cases}$$

so that if $\mathcal{b}(s) = \alpha_1 \cdots \alpha_n$ then $\alpha_i \neq \alpha_{i+1}$ for i from 1 to $n - 1$. In particular,

$$\mathcal{b}(\boxed{\text{Jan}}^{31} \boxed{\text{Feb}}^{28} \cdots \boxed{\text{Dec}}^{31}) = s_{mo}.$$

Writing $\mathcal{b}_A$ for the function mapping s to $\mathcal{b}(\rho_A(s))$, we have

$$\mathcal{b}_{mo}(s_{mo,dy}) = s_{mo}.$$

² We draw boxes (instead of the usual curly braces $\{$ and $\}$) around sets-as-symbols, stringing together “snapshots” much like a cartoon/film strip.

While the example of a calendar is (in itself) of marginal interest to subatomic semantics, it shows off clearly some tools to analyze, for instance, the pair

The soup cooled in an hour
The soup cooled for an hour

familiar from Dowty 1979 (Fernando 2013, 2014).

In general, we can refine a string s_A of granularity A to one $s_{A'}$ of granularity $A' \supseteq A$ with $\mathcal{L}_A(s_{A'}) = s_A$. Iterating over a chain

$$A \subseteq A' \subseteq A'' \subseteq \dots,$$

we can glue together strings $s_A, s_{A'}, s_{A''}, \dots$ such that

$$\mathcal{L}_X(s_{X'}) = s_X \text{ for } X \in \{A, A', A'', \dots\}.$$

That is, we can form the inverse limit of the maps $\mathcal{L}_X$. Details are supplied in section 2, where constraints can be added by arranging the sentence morphisms in an institution to be inclusions (leaving sentences intact).

We move away from sentence morphisms as inclusions to analyze records such as (a) for Davidson's aforementioned example over an untyped universe U .

$$(a) \quad \left[\begin{array}{l} \textit{who} = \text{JONES} \\ \textit{how} = \left[\begin{array}{c} \textbf{slow} \\ \textbf{deliberate} \end{array} \right] \\ \textit{where} = \left[\textbf{bathroom} \right] \\ \textit{when} = \left[\textbf{midnight} \right] \\ \textit{with-what} = \left[\textbf{knife} \right] \end{array} \right] \quad (b) \quad \left[\begin{array}{l} l_1 = u_1 \\ \vdots \\ l_n = u_n \end{array} \right]$$

Example B Given a large set Lab of labels (or attributes), we equip a set U with a partial function $F : U \times Lab \rightarrow U$ from $U \times Lab$ to U . The idea is that a record such as (b) is represented by an entity $u \in U$ such that the restriction $F \cap ((\{u\} \times Lab) \times U)$ of F to $\{u\} \times Lab$ is $\{((u, l_1), u_1), \dots, ((u, l_n), u_n)\}$. The partiality of F reflects the fact that not every element of Lab need be a label of every record, nor need every element of U be a record. As a Kripke frame (over accessibility relations labeled by Lab), F can be turned into a Kripke model over a set T of tests by adding a valuation $v \subseteq U \times T$. As we shall see in section 3, the notion of an institution extends v to a satisfaction relation $\models_\Sigma$ between Σ -models in U and Σ -sentences based on T and Lab ,³ subject to signature morphisms shaped by F . The signatures amount to types on sets of attribute-value pairs, where values may themselves be sets of attribute-value pairs. Types constitute the foundation of the proof-theoretic extension in Ranta 1994 of Discourse Representation Structures (Kamp and Reyle 1993). The focus here is not on discourse or anaphora but on the open-endedness of subatomic semantics, including the use of frames/records for predication (Fillmore 1985, Cooper 2012).

We adopt the usual notational conventions, writing **Set** for the category of sets and functions (which we take for granted). An *institution* is a 4-tuple $(\mathbf{Sig}, \mathbf{Mod}, \mathbf{Sen}, \{\models_\Sigma\}_{\Sigma \in |\mathbf{Sig}|})$ consisting of

³ The fonts on the record in (a) were chosen to suggest the following formalization: *italics* (e.g. *who*) for labels in Lab ; SMALL CAPS (JONES) for elements of U ; and **boldface** (e.g. **slow**) for tests in T .

- a category **Sig** with a set $|\mathbf{Sig}|$ of objects, called signatures
- functors $\mathbf{Mod} : \mathbf{Sig}^{op} \rightarrow \mathbf{Set}$ and $\mathbf{Sen} : \mathbf{Sig} \rightarrow \mathbf{Set}$ returning for each $\Sigma \in |\mathbf{Sig}|$, a set $\mathbf{Mod}(\Sigma)$ of Σ -models and a set $\mathbf{Sen}(\Sigma)$ of Σ -sentences, and
- for each $\Sigma \in |\mathbf{Sig}|$, a binary relation $\models_\Sigma$ between Σ -models and Σ -sentences such that
 - ($\dagger$) for all $\sigma \in \mathbf{Sig}(\Sigma, \Sigma')$, $M' \in \mathbf{Mod}(\Sigma')$ and $\varphi \in \mathbf{Sen}(\Sigma)$,

$$\mathbf{Mod}(\sigma)(M') \models_\Sigma \varphi \text{ iff } M' \models_{\Sigma'} \mathbf{Sen}(\sigma)(\varphi) .$$

The satisfaction condition ($\dagger$) is applied below to refine the inverse limit mentioned in Example A (section 2), and the typings on records in Example B (section 3).

2 Time-as-change and $\mathcal{bc}$ -systems

Henceforth, we shorten “temporal proposition” to *fluent*, and fix an infinite set Φ of fluents. Let us write $\mathit{Fin}(\Phi)$ for the set of finite subsets of Φ . Given a set X , the power set 2^X of X is the set of all subsets of X , while X^+ is the set of all non-empty strings over the alphabet X . A $\mathcal{bc}$ -system is a function f with domain $\mathit{Fin}(\Phi)$ such that for each $A \in \mathit{Fin}(\Phi)$, $f(A) \in (2^A)^+$ and

$$f(B) = \mathcal{bc}_B(f(A)) \text{ whenever } B \subseteq A.$$

In the terminology of institutions, the category **Sig** is $\mathit{Fin}(\Phi)$, partially ordered by $\subseteq$ — i.e. **Sig**-morphisms are pairs (B, A) such that $B \subseteq A \in \mathit{Fin}(\Phi)$. **Mod** maps a finite subset A of Φ to the set

$$\mathbf{Mod}(A) := \{\mathcal{bc}(s) \mid s \in (2^A)^+\}$$

and whenever $B \subseteq A$, $\mathbf{Mod}(A, B) : \mathbf{Mod}(A) \rightarrow \mathbf{Mod}(B)$ is the restriction of $\mathcal{bc}_B$ to $\mathbf{Mod}(A)$ — i.e., for $s \in \mathbf{Mod}(A)$,

$$\mathbf{Mod}(A, B)(s) = \mathcal{bc}_B(s) \in \mathbf{Mod}(B).$$

The set of $\mathcal{bc}$ -systems is the inverse limit, $\varprojlim \mathbf{Mod}$, of the inverse system defined by the contravariant functor **Mod**.

Returning to Example A, note that some $\mathcal{bc}$ -systems are better than others; for instance, we may wish to throw out functions f for which $f(mo) = \boxed{\text{Jan, May}}$. To refine $\varprojlim \mathbf{Mod}$, we can try refining **Mod**. Toward that end, we fix a $\mathit{Fin}(\Phi)$ -indexed family $\{\mathbf{Sn}(A)\}_{A \in \mathit{Fin}(\Phi)}$ of sets $\mathbf{Sn}(A)$ of sentences ψ with associated denotations $\llbracket \psi \rrbracket \subseteq (2^A)^+$.⁴ We let **Sen** map $B \in \mathit{Fin}(\Phi)$ to

$$\mathbf{Sen}(B) := \{\langle A \rangle \psi \mid A \subseteq B \text{ and } \psi \in \mathbf{Sn}(A)\}$$

⁴ For example, $\mathbf{Sn}(A)$ might consist of all regular expressions over the alphabet 2^A . We need not, however, insist here on regularity.

and whenever $B \subseteq C$, let $\mathbf{Sen}(B, C)$ be the inclusion $\mathbf{Sen}(B) \hookrightarrow \mathbf{Sen}(C)$ preserving $\langle A \rangle \psi$ (for $A \subseteq B$, $\psi \in \mathbf{Sn}(A)$), with

$$s \models_B \langle A \rangle \psi \quad \text{iff} \quad \mathcal{L}_A(s) \in \llbracket \psi \rrbracket$$

for all $s \in \mathbf{Mod}(B)$. One can verify that the satisfaction condition $(\dagger)$ holds. Furthermore, given a $\mathbf{Fin}(\Phi)$ -indexed family $\{\mathbf{Th}(A)\}_{A \in \mathbf{Fin}(\Phi)}$ of sets $\mathbf{Th}(A) \subseteq \mathbf{Sen}(A)$, we can refine $\mathbf{Mod}$ to the presheaf $\mathbf{Mod}_{\mathbf{Th}} : \mathbf{Sig}^{op} \rightarrow \mathbf{Set}$

$$\mathbf{Mod}_{\mathbf{Th}}(A) := \{s \in \mathbf{Mod}(A) \mid (\forall \varphi \in \bigcup_{B \subseteq A} \mathbf{Th}(B)) s \models_A \varphi\}$$

with $\mathbf{Mod}_{\mathbf{Th}}(A, C)$ as the restriction of $\mathbf{Mod}(A, C)$ to $\mathbf{Mod}_{\mathbf{Th}}(A)$ whenever $C \subseteq A$.

Can we view a $\mathcal{L}$ -system f as a completed whole without resorting to its finite approximations $f(A)$? To string f out fully, let us work with the set $\mathbb{Q}$ of rational numbers, and define a $\mathbb{Q}$ -string over Φ to be an expansion $\mathfrak{A} = \langle \mathbb{Q}, <, \{U_a\}_{a \in \Phi} \rangle$ of the linear order $\langle \mathbb{Q}, < \rangle$ on the rationals such that for each $a \in \Phi$, U_a is a subset of $\mathbb{Q}$ with finite boundary $\partial(U_a)$.⁵ The assumption that $\partial(U_a)$ is finite allows us to picture a $\mathbb{Q}$ -string $\mathfrak{A}$ over Φ by a $\mathcal{L}$ -system $f_{\mathfrak{A}}$. To see how, it is helpful to define a $\mathbb{Q}$ -segmentation to be a finite sequence $I_1 \cdots I_n$ of subsets of $\mathbb{Q}$ such that

$$\mathbb{Q} = \bigcup_{i=1}^n I_i \text{ and for } 1 \leq i < n, (\forall q \in I_i)(\forall q' \in I_{i+1}) q < q'.$$

That is, a $\mathbb{Q}$ -segmentation is an ordered finite partition of $\mathbb{Q}$ into intervals.

Proposition 1. *Given a subset U of $\mathbb{Q}$, the boundary of U is finite iff there is a $\mathbb{Q}$ -segmentation $I_1 \cdots I_n$ such that $U = \bigcup \mathbf{J}$ for some subset $\mathbf{J} \subseteq \{I_1, \dots, I_n\}$.*

Now for $f_{\mathfrak{A}}$. Put $f_{\mathfrak{A}}(\emptyset) := \square$, and for a finite non-empty subset A of Φ , we enumerate points in the boundary of U_a for $a \in A$ in increasing order

$$\bigcup_{a \in A} \partial(U_a) = \{q_1, q_2, \dots, q_k\} \text{ with } q_1 < q_2 < \dots < q_k$$

yielding a $\mathbb{Q}$ -segmentation $I_1 \cdots I_{2k+1}$ with $I_1 := (-\infty, q_1)$, and

$$I_{2i} := \{q_i\} \text{ and } I_{2i+1} := (q_i, q_{i+1}) \text{ for } 1 \leq i \leq k \text{ and } q_{k+1} := +\infty.$$

⁵ Recall that the *boundary* $\partial(U)$ of U is the closure of U minus the interior of U . We assume here the standard (order) topology on $\mathbb{Q}$ (given by unions of sets (q, q') of rationals strictly between q and q'). This makes the boundary of U the union

$$\begin{aligned} \partial(U) = & \{q \in U \mid \text{not } (\exists r < q)(\exists r' > q) (r, r') \subseteq U\} \cup \\ & \{q \in \mathbb{Q} - U \mid \text{not } (\exists r < q)(\exists r' > q) (r, r') \cap U = \emptyset\}. \end{aligned}$$

Next, for $1 \leq j \leq 2k + 1$, we collect each $a \in A$ such that U_a holds of every element of I_j in

$$\alpha_j := \{a \in A \mid I_j \subseteq U_a\}$$

and finally, block-compress for

$$f_{\mathfrak{A}}(A) := \mathcal{b}_A(\alpha_1 \alpha_2 \cdots \alpha_{2k+1}).$$

Recalling the strings s_{mo} and $s_{mo,dy}$ from Example A, we have

$$\begin{array}{l} \boxed{\text{Jan}} \boxed{\text{Feb}} \boxed{\text{Mar}} \cdots \boxed{\text{Dec}} = f_{\mathfrak{A}}(mo) \\ \boxed{\text{Jan,d1}} \boxed{\text{Jan,d2}} \cdots \boxed{\text{Dec,d31}} = f_{\mathfrak{A}}(mo \cup \{\text{d1, d2,} \dots \text{d31}\}) \end{array}$$

if $\mathfrak{A}$ is the $\mathbb{Q}$ -string over $\Phi \supseteq mo \cup \{\text{d1, d2,} \dots \text{d31}\}$ such that, for instance,

$$\begin{aligned} U_{\text{Jan}} &= (-\infty, 1], \quad U_{\text{Feb}} = (1, 2], \quad \dots, \quad U_{\text{Dec}} = (11, +\infty) \\ U_{\text{d1}} &= (-\infty, -31] \cup (1, 1\frac{1}{28}] \cup (2, 2\frac{1}{31}] \cup \dots \cup (11, 12] \\ U_{\text{d2}} &= (-31, -30] \cup (1\frac{1}{28}, 1\frac{2}{28}] \cup \dots \cup (12, 13] \\ &\vdots \\ U_{\text{d31}} &= (-1, 1] \cup (2\frac{30}{31}, 3] \cup \dots \cup (41, +\infty). \end{aligned}$$

Proposition 2. *Given $\mathbb{Q}$ -strings $\mathfrak{A}$ and $\mathfrak{A}'$ over Φ ,*

$$\mathfrak{A} \cong \mathfrak{A}' \text{ implies } f_{\mathfrak{A}} = f_{\mathfrak{A}}.$$

The converse of Proposition 2 fails without further assumptions. Already for the singleton $\Phi := \{a\}$, there are five pairwise non-isomorphic $\mathbb{Q}$ -strings $\mathfrak{A}$ with $f_{\mathfrak{A}}(\{a\}) = \boxed{\boxed{a}}$, given by the possibilities $(0, 1)$, $[0, 1)$, $(0, 1]$, $[0, 1]$ and $\{1\}$ for U_a . To cover all $\mathcal{b}$ -systems f over a countable set Φ , it is enough to consider $\mathbb{Q}$ -strings over Φ meeting an additional requirement. Keeping in mind that a $\mathbb{Q}$ -segmentation cannot consist of exclusively open or exclusively closed intervals, unless there is only one interval in the $\mathbb{Q}$ -segmentation (i.e., $\mathbb{Q}$), let us call a $\mathbb{Q}$ -string $\langle \mathbb{Q}, <, \{U_a\}_{a \in \Phi} \rangle$ over Φ *special* if for all $a \in \Phi$, each $q \in \partial(U_a)$ has a neighborhood (q', q'') such that

$$((q', q] \cap U_a = \emptyset \text{ and } (q, q'') \subseteq U_a) \text{ or } ((q', q] \subseteq U_a \text{ and } (q, q'') \cap U_a = \emptyset).$$

Of the five solutions to $f_{\mathfrak{A}}(\{a\}) = \boxed{\boxed{a}}$ above, only $(0, 1]$ is special (although we could have easily defined *special* to single out $[0, 1)$ instead).

Proposition 3. *If Φ is countable, then for every $\mathcal{L}$ -system f , there is a special $\mathbb{Q}$ -string $\mathfrak{A}$ over Φ such that $f_{\mathfrak{A}} = f$.*

We can prove Proposition 3 by adapting Cantor's back-and-forth construction of an isomorphism between $\langle \mathbb{Q}, < \rangle$ and a countable dense linear order without end points. A similar argument establishes

$$f_{\mathfrak{A}} = f_{\mathfrak{A}'} \text{ implies } \mathfrak{A} \cong \mathfrak{A}' \quad (1)$$

for special $\mathbb{Q}$ -strings $\mathfrak{A}$ and $\mathfrak{A}'$ and finite Φ . Somewhat disappointingly, however, (1) may fail for countable Φ . To see this, form arbitrarily small intervals around 0 or π so that the finite intersections are never empty but the entire intersection is in the case of π (as π is irrational).⁶ That is, there are special $\mathbb{Q}$ -strings $\mathfrak{A}$ and $\mathfrak{A}'$ over a countable Φ both satisfying every first-order sentence of the form

$$\exists x \bigwedge_{a \in A} \dot{U}_a(x) \quad \text{for } A \in \text{Fin}(\Phi)$$

but differing on the infinitary sentence $\exists x \bigwedge_{a \in \Phi} \dot{U}_a(x)$. The possibility that a number in a $\mathbb{Q}$ -string may belong to two or more sets U_a and $U_{a'}$ (with $a \neq a'$) deviates from the usual construal of strings as models of Monadic Second Order logic (e.g. Thomas 1997), under which a finite alphabet A partitions the string positions, validating

$$\forall x \bigvee_{a \in A} (U_a(x) \wedge \bigwedge_{a' \in A - \{a\}} \neg U_{a'}(x)).$$

The deviation is easily explained by a change in the string alphabet above from A to 2^A , smoothening the analysis of alphabet reductions, in accordance with the satisfaction condition on institutions (Fernando 2014).

3 Typing records institutionally

Given two sets Lab of *labels* and T of *tests* (as outlined in Example B), let us agree an institution $(\mathbf{Sig}, \mathbf{Mod}, \mathbf{Sen}, \{ \models_{\Sigma} \}_{\Sigma \in |\mathbf{Sig}|})$ is (Lab, T) -uniform if

(i) for all $\Sigma, \Sigma' \in |\mathbf{Sig}|$,

$$\mathbf{Sig}(\Sigma, \Sigma') \subseteq \{\Sigma\} \times Lab^* \times \{\Sigma'\}$$

with $(\Sigma, \epsilon, \Sigma)$ as the identity morphism on Σ (where ϵ is the null string) and writing $\Sigma \xrightarrow{l} \Sigma'$ for $(\Sigma, l, \Sigma') \in \mathbf{Sig}(\Sigma, \Sigma')$,

$$(\Sigma, l, \Sigma'); (\Sigma', l', \Sigma'') = (\Sigma, ll', \Sigma'') \quad \text{whenever } \Sigma \xrightarrow{l} \Sigma' \text{ and } \Sigma' \xrightarrow{l'} \Sigma''$$

⁶ More precisely, let Φ be the set of positive integers n , and $U_n := (-\frac{1}{n}, \frac{1}{n}]$. For the corresponding intervals $(q_n, q'_n] \subset \mathbb{Q}$ around π , it is enough that $\pi - \frac{1}{n} < q_n < \pi - \frac{1}{n+1}$ and $\pi + \frac{1}{n+1} < q'_n < \pi + \frac{1}{n}$.

(ii) for each $l \in Lab$, the union

$$l_{\mathbf{Mod}} := \bigcup \{ \mathbf{Mod}(\Sigma', l, \Sigma) \mid \Sigma \xrightarrow{l} \Sigma' \}$$

is a partial function on $\bigcup_{\Sigma \in |\mathbf{Sig}|} \mathbf{Mod}(\Sigma)$

(iii) $\mathbf{Sen} : \mathbf{Sig} \rightarrow \mathbf{Set}$ maps $\Sigma \in |\mathbf{Sig}|$ to a set $\mathbf{Sen}(\Sigma) = \bigcup_{n \geq 0} \mathbf{Sen}_n(\Sigma)$ where $\mathbf{Sen}_0(\Sigma) \subseteq T$ and

$$\mathbf{Sen}_{n+1}(\Sigma) := \mathbf{Sen}_n(\Sigma) \cup \{ \langle l \rangle \varphi \mid \varphi \in \mathbf{Sen}_n(\Sigma') \text{ and } \Sigma' \xrightarrow{l} \Sigma \}$$

and for all $\varphi \in \bigcup_{\Sigma \in |\mathbf{Sig}|} \mathbf{Sen}(\Sigma)$ and $l, l' \in Lab^*$,

$$\langle \epsilon \rangle \varphi = \varphi \quad \text{and} \quad \langle l \rangle \langle l' \rangle \varphi = \langle ll' \rangle \varphi$$

$$\mathbf{Sen}(\Sigma, l, \Sigma')(\varphi) = \langle l \rangle \varphi \quad \text{whenever } \Sigma \xrightarrow{l} \Sigma' \text{ and } \varphi \in \mathbf{Sen}(\Sigma)$$

(iv) for all $\Sigma, \Sigma' \in |\mathbf{Sig}|$, $t \in \mathbf{Sen}(\Sigma) \cap \mathbf{Sen}(\Sigma') \cap T$ and $u \in \mathbf{Mod}(\Sigma) \cap \mathbf{Mod}(\Sigma')$,

$$u \models_{\Sigma} t \quad \text{iff} \quad u \models_{\Sigma'} t.$$

Recalling the items U , F and v mentioned in Example B, we now derive these from a (Lab, T) -uniform institution $\mathcal{I} = (\mathbf{Sig}, \mathbf{Mod}, \mathbf{Sen}, \{\models_{\Sigma}\}_{\Sigma \in |\mathbf{Sig}|})$, setting

$$\begin{aligned} U_{\mathcal{I}} &:= \bigcup_{\Sigma \in |\mathbf{Sig}|} \mathbf{Mod}(\Sigma) \\ F_{\mathcal{I}} &:= \bigcup_{l \in Lab} \{ ((u, l), l_{\mathbf{Mod}}(u)) \mid u \in \text{domain}(l_{\mathbf{Mod}}) \} \\ v_{\mathcal{I}} &:= \bigcup_{\Sigma \in |\mathbf{Sig}|} \{ (u, t) \in \mathbf{Mod}(\Sigma) \times (\mathbf{Sen}(\Sigma) \cap T) \mid u \models_{\Sigma} t \}. \end{aligned}$$

Proposition 4. *Let $\mathcal{I} = (\mathbf{Sig}, \mathbf{Mod}, \mathbf{Sen}, \{\models_{\Sigma}\}_{\Sigma \in |\mathbf{Sig}|})$ be a (Lab, T) -uniform institution. Then $F_{\mathcal{I}} : U_{\mathcal{I}} \times Lab \rightarrow U_{\mathcal{I}}$ and whenever $\Sigma \xrightarrow{l} \Sigma'$,*

$$\mathbf{Mod}(\Sigma', l, \Sigma)(u') = F_{\mathcal{I}}(u', l) \quad \text{for all } u' \in \mathbf{Mod}(\Sigma')$$

and for all $\Sigma \in |\mathbf{Sig}|$, $u \in \mathbf{Mod}(\Sigma)$ and $t \in \mathbf{Sen}(\Sigma) \cap T$,

$$u \models_{\Sigma} t \quad \text{iff} \quad v_{\mathcal{I}}(u, t).$$

As the **Sig**-morphisms of a (Lab, T) -uniform institution are structured by the free monoid Lab^* , a partial function $F : U \times Lab \rightarrow U$ describes essentially one step, which we can extend to many, defining $F_+ : U \times Lab^* \rightarrow U$ by

$$\begin{aligned} F_+((u, \epsilon), u') &\quad \text{iff} \quad u = u' \\ F_+((u, ll'), u') &\quad \text{iff} \quad (\exists u'') \, F_+(u, l', u'') \text{ and } F(u'', l, u') \end{aligned}$$

for $l' \in Lab^*$ and $l \in Lab$. We reverse the string ll' in F_+ since **Mod** is contravariant, unlike **Sen**.

$$\frac{u \in \mathbf{Mod}(\Sigma)}{\mathbf{Mod}(\Sigma, l, \Sigma')(u) \in \mathbf{Mod}(\Sigma')} \quad \Sigma' \xrightarrow{l} \Sigma \qquad \frac{\varphi \in \mathbf{Sen}(\Sigma')}{\langle l \rangle \varphi \in \mathbf{Sen}(\Sigma)} \quad \Sigma' \xrightarrow{l} \Sigma$$

Next, we construct a category **Sig_F** of signatures from F , associating with $u \in U$ the language

$$\mathcal{L}_F(u) := \{l \in Lab^+ \mid (u, l) \in \text{domain}(F_+)\}$$

over Lab , and then taking a **Sig_F**-object to be a subset of such a set

$$|\mathbf{Sig}_F| := \{L \subseteq Lab^+ \mid (\exists u \in U) L \subseteq \mathcal{L}_F(u)\}.$$

As for **Sig_F**-morphisms, it is useful to define for $l \in Lab^*$ and $L \subseteq Lab^+$,

$$L_l := \{l' \in Lab^+ \mid ll' \in L\}$$

from which we define the hom-sets for $L, L' \in |\mathbf{Sig}_F|$ as

$$\mathbf{Sig}_F(L, L') := \{(L, l, L') \mid l \in Lab^* \text{ and } L' \subseteq L_l\}.$$

We can then form an institution around **Sig_F**, putting for $L \in |\mathbf{Sig}_F|$,

$$\mathbf{Mod}_F(L) := \{u \in U \mid L \subseteq \mathcal{L}_F(u)\}$$

$$\mathbf{Sen}_{F,T}(L) \cap T = T$$

and for $l \in L$, $u \in \mathbf{Mod}_F(L)$ and $t \in T$,

$$u \models_L^{F,v} \langle l \rangle t \text{ iff } v(F_+(u, l), t).$$

Proposition 5. *Fix a partial function $F : U \times Lab \rightarrow U$ and a relation $v \subseteq U \times T$. Then $\mathcal{I} = (\mathbf{Sig}_F, \mathbf{Mod}_F, \mathbf{Sen}_{F,T}, \{\models_L^{F,v}\}_{L \in |\mathbf{Sig}_F|})$ is a (Lab, T) -uniform institution with $U_{\mathcal{I}} = U$, $F_{\mathcal{I}} = F$ and $v_{\mathcal{I}} = v$.*

Observe that $\mathcal{L}_F(u)$ is the most specific signature in **Sig_F** of $u \in U$, its other signatures being proper subsets of $\mathcal{L}_F(u)$. But we can refine this typing of u further by incorporating v into F , setting

$$F^v := F \cup \{(u, t), u\} \mid (u, t) \in v\}$$

which remains a partial function, so long as $Lab \cap T = \emptyset$. It is well known that over deterministic transition systems (i.e. partial functions such as F), trace equivalence is a *bisimulation* (Park 1981). That is to say, assuming Lab and T are disjoint, the equivalence $\sim_{F,v}$ induced by F^v

$$u \sim_{F,v} u' \text{ iff } \mathcal{L}_{F^v}(u) = \mathcal{L}_{F^v}(u')$$

yields a useful notion of semantic equivalence that we can refine by adjoining suitable tests into v . Beyond institutions and strings, what section 3 has in common with section 2 is a conception of types as being prior to semantic entities, the latter amounting to maximally refined types.

To integrate the categories of signatures in sections 2 and 3, we can form their product. But whether this is how to build records into fluents and/or fluents into records (e.g., Cooper 2012), we leave for future work.

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