

Proof-Theoretic Semantics for intensional transitive verbs

Nissim Francez*

May 25, 2014

1 Introduction

The purpose of this paper is to present a proof-theoretic semantics (PTS) of sentences headed by *intensional transitive verbs (ITV)* (briefly considered in [5]). Those verbs are also known as *opaque verbs*, and are traditionally characterized by the following three properties:

Admittance of non-specific (notional) objects

Resistance to substitutability of coextensives

Suspension of existential commitment

Let me state on the onset that I am not considering any specific *lexical* properties of members of this class of verbs transcending the above characteristics. I take those three properties as the *full* characterisation of the constructions studied. Whenever **seek**¹ is used in examples, no attempt is made to “really” understand what seeking is about, and any other ITV verb could serve the purpose of the example. So, approaches reducing **seek** to **try to find** are not appropriate on the onset. As not all transitive verbs admit notional arguments, I let $\hat{R}$ range over those which do, which are assumed to be a *given* class of words. This family includes² verbs like **seek**, **need**, **look for** and **want**, differing from the extensional transitive verbs like **kiss**, or **find**, the latter lacking the above characteristics.

One more thing that I like to mention right here is that I will confine my discussion to meaning of single sentences, not in the role ITVs play in discourse. So, phenomena like the absence of anaphora support, that are also mentioned as characterising ITVs (e.g., [7]), are outside the scope of this discussion. Still, I do believe that extending proof-theoretic semantics to discourse in general (and to dialogs) is a worthy task.

The study here is conducted as an extension of the proof-theoretic semantics for an extensional fragment of English, proposed in [5] and extended also in [4]. Rules from the basic proof-system for the extensional fragment are presented where needed in a simplified form, sufficient for the current concerns. The restriction to a fragment is a methodological choice, by which not all the issues related in the literature to ITVs are covered. However, it suffices for presenting the way proof-theoretic semantics can cope with ITVs and emphasising the advantages of this approach.

The motivation for PTS and more details on the whole approach can be found in [5].

*Computer Science dept., the Technion-IIT, Haifa, Israel (francez@cs.technion.ac.il)

¹All natural language expressions are displayed in San-Serif font and are mentioned, not used.

²In [9] there is a detailed presentation of the landscape of families of such verbs.

Before formalising the proof-theoretic characterizations of ITVs, the three characterizations are discussed in more detail.

1.1 Non-specific objects

In the fragment studied here, this is taken as admitting as objects of ITVs indefinite³ DPs with a non-specific reading. So, those verbs are ambiguous, having also a meaning allowing specific readings parallel to more ordinary extensional verbs. Other quantifiers can also stand in the object position of such verbs: one may look for *all/ most* sheep, still without looking for any specific one. I focus here on the existential quantifier embodied as an indefinite DP (but see also the treatment of existential determiners in page 11).

Consider, for example,

Jacob seeks a secretary. (1.1)

in contrast to its extensional counterpart

Jacob finds a secretary. (1.2)

While in (1.2) Jacob finds⁴ *some specific secretary*, there is no specific secretary Jacob necessarily seeks in one reading of (1.1), though there is also a reading of (1.1) where he does seek a specific secretary, analogously to finding one. Note that this ambiguity in the meaning of sentences like (1.1) is not a problem of quantifier scope ambiguity: the sentence contains just one quantifier. This issue has been noticed already in mediaeval times, where Buridan considered

Debeo tibi equum, namely, I owe you a horse (1.3)

The distinction between specific and non-specific meaning of indefinite DPs is reflected in the proof-theory in the difference in the way they are introduced into the object position by means of applying an *I*-rules of the meaning-conferring natural-deduction proof-system. This contrasts the model-theoretic semantics (MTS) rendering of this distinction. The dependence of meaning on derivations in the meaning-conferring proof-system is elaborated upon in Section 3.

1.2 Resistance to substitutability of coextensives

Here, the problem can be explained with reference to validity of the following argument.

John seeks Mary	
Mary is the daughter of the dean	

John seeks the daughter of the dean	(1.4)

In MTS, the issue is referred to as substitutivity to coextensive expressions. The copula *is* is interpreted as *Mary* and *the daughter of the dean* having the same extension (i.e. denotation) by referring to the same element in every model. In PTS, *is* is interpreted via the *I/E*-rules for identity (in Figure 1 below). The argument (1.4) is considered invalid in general, by an appeal to John's possible unawareness of the identity of *Mary* and *the daughter of the dean*. A major task of MTS is to reflect this awareness in models. Here, the proof-theoretic task here is to enforce the non-derivability of the conclusion from the premises by the rules.

³I use *a* and *some* interchangeably.

⁴Thus, the sentence can be paraphrased as *there is some secretary s.t. Jacobs finds her*, like in *de re* readings.

Let me note in passing, that there is no general consensus that resistance to substitutivity is a correct characteristic of ITVs (and other opaque sentences, most notably propositional attitudes). For example, Salmon [11] claims that resistance to substitutivity is only apparent, originating not from meaning, but rather from the pragmatics of use. Soames [12] also rejects failure of substitutivity, as well as others. I find myself in this camp, though for totally different reasons, not connected to “direct reference”, an MTS concept. Rather, I believe that when judging consequence, or validity of arguments, only literal meaning is to be taken into account, not incorporating details of the situation a premise is “asserted”, such as agent intentions/plans, or even the intentions/plans of the evaluator of the argument. Consequence is an objective relation once meaning has been defined.

However, since resistance to substitutivity has been taken as a characteristic of ITVs by many, I show here how it is enforceable proof-theoretically. Fortunately, as I show in Section 2.3, there is a simple “on/off switch” that can chose between enforcing resistance to substitutivity, or not.

1.3 Suspension of existential commitment

A sharpening of the intensionality problem occurs in sentences like

Jacob seeks a unicorn (1.5)

where unicorns need not exist at all in order to be sought (evading the debatable issue of non-existent objects). In MTS, existence is taking as reference to an entity in the domain of the model. Some means is needed to be introduced in order to interpret proof-theoretically existence (other than resorting to a free logic); in contrast to model-theoretic semantics, there is no access here to elements in models. This is explained in Section 2.4.

1.4 Model-theoretic semantics of ITVs

Defining the meaning of ITVs is considered a semantic problem notorious for its difficult specification of model-theoretic truth-conditions for sentences exhibiting it (e.g., [14], [7], [8] and [9] - draft on her home-page). There is no consensus regarding the truth-conditions of opaque sentences; not even regarding the semantic type of the object DP of the intensional verb, e.g., a quantifier ([14]), a property ([9]), or a minimal situation ([7]). There is also an indirect interpretation via *decomposition* of the intensional verb ([6]). Another approach ([1]) appeals to Davidsonian event semantics and its thematic roles. Recently, a *dynamic* semantics was proposed in [13], where an update via ITVs is proposed.

I want to stress again that according to the PTS programme, the rules proposed are meaning-conferring, a *definitional* tool (see Section 3). They should not be judged against any external yard-stick, in particular not one used in MTS. Completeness proofs as traditionally sought in logic are not a goal here, in spite of being an interesting topic. The rules need, though, meet some proof-theoretic criteria to qualify as meaning conferring. In [5], the rules for the extensional fragment are shown to meet the harmony criterion, one central such criterion.

2 A Proof-Theoretic Meaning for Opacity

2.1 Basic rules for the extensional fragment

The proof language L for the extensional fragment contains a (infinitely countable) set $\mathbf{P}$ of *individual parameters*, ranged over by bold-face meta-variables $\mathbf{j}$, $\mathbf{k}$. These individual parameters can occupy any DP position in a sentence, and are viewed syntactically as DPs. Their main purpose is to express *predication*, like in $\mathbf{j}$ *isa* girl, $\mathbf{j}$ *smiles* or $\mathbf{j}$ *loves every girl*. A sentence having an individual parameter in (at least) one of its DP-positions is called a pseudo-sentence; if all the sentence's DP-positions are occupied by parameters, its a *ground* pseudo-sentence, the analog of an atomic sentence in logic.

It is important to note that individual parameters are *artefacts* of the proof language. Their functioning is intermediate between constants and free variables, as they can be bound on the one hand, but can be thought also as *arbitrary* representatives, as will be clear from their use in the I/E -rules for restricted quantification. In particular, they play an important role in discharged assumptions in the I/E -rules. Note that individual parameters, being syntactic objects, artefacts of the proof language, do not carry any ontological burden, no ontological commitment, as do elements in models in the use of MTS as the underlying theory of meaning. Rules for the extensional fragment are delineated, in a simplified way, were needed, mainly for comparison with the rules of the extension of the fragment with ITVs.

2.2 The basic rules for non-specificity

The starting point is to augment the proof-language with another family of parameters, referred to as *notional parameters*, in addition to the individual parameters used for the extensional fragment. Also, the proof language is extended with a family of *notions*, a proof-theoretic counterpart of properties. Notions are formed from nouns (either lexical or compound by modification), from adjectives and from verb-phrases (the latter explained in detail in Section 2.2.1). Recall that such an extension, being another proof-language artefact, does not carry any ontological commitments, in contrast to the various entities assumed by MTS to populate models. The purpose of those additional notional parameters is to allow associating notions to them and the DP *Some X* (for a noun X) *to be introduced in a different way*, not assuming any individual parameter being an X , thus exhibiting also a reading escaping specificity. Note that I -rules were shown in [5] as explaining quantifier-scope ambiguity as a difference in the order of introduction of the subjects DP and the object DP. I -rules constitute a major tool available in PTS, with a strong explanatory power.

Let $\mathbf{B}$ be a countable set, disjoint from $\mathbf{P}$, of *notional parameters*, ranged over by (possibly indexed) meta-variables $\mathbf{n}$, $\mathbf{m}$. I need an analog of the predication $\mathbf{j}$ *isa* X , X a (possibly compound) noun, associating a notional parameter with the notion of being an X . I thus extend the proof-language L with ground pseudo-sentences of the following forms:

nominal: $\mathbf{n}$ is being a X , X a (possibly compound) noun. For example, $\mathbf{n}$ is being a secretary.

adjectival: $\mathbf{n}$ is being A , A an adjective. For example, $\mathbf{n}$ is being blonde.

verbal: $\mathbf{n}$ is V ing, V the gerundive form of a VP. For example, $\mathbf{n}$ is singing, $\mathbf{n}$ is loving $\mathbf{k}$ and $\mathbf{n}$ is $\mathbf{j}$ loving (the latter more colloquially expressed by passivisation, namely being loved by $\mathbf{j}$ (see Section 4)). I refer to such notions as *verbal*.

The rules for forming compound notions via modification by (intersective) adjectives and relativization, which use adjectival and verbal notions, are presented in Section 2.2.1. Note the difference between the two ways of connecting a parameter and a noun:

j isa X: expresses usual predication.

n is being a X: associates a name to a nominal notion.

Note that X can be a compound noun, so we may have notions like **being a secretary**, **being a blonde secretary**, **being a secretary who seeks a unicorn**, **being a girl who loves every boy**, etc.

For an opaque verb $\hat{R}$, we have now an additional ground pseudo-sentence of the form **j $\hat{R}$ n**. So, there are now two ways of $\hat{R}$ -relating an individual parameter **j** to an object:

- to another individual parameter, say **k**: **j $\hat{R}$ k**. For example, **j seeks k**.
- to a nominal notion or a notional parameter, say **n**: **j $\hat{R}$ n**. For example, **j seeks n**, **j seeks a secretary**.

It is important to observe that the following are *ill-formed*:

$$\mathbf{j} \text{ is being a } X, \quad \mathbf{n} \text{ isa } X$$

I now can formulate the following I -rule, introducing **some X** *notionally* into an object position of an ITV $\hat{R}$ with a non-specific reading.

$$\frac{\mathbf{n} \text{ is being a } X \quad \mathbf{j} \hat{R} \mathbf{n}}{\mathbf{j} \hat{R} \text{ some } X} (s_n I) \quad (2.6)$$

The non-specificity of **some X** is reflected by the absence of a predicative premise of the form **j isa X** . This is in contrast to the introduction of **some X** in its specific reading via the I -rule (sI), repeated below in a sufficiently simplified form, that has **k isa X** as a premise, expressing specificity.

$$\frac{\mathbf{k} \text{ isa } X \quad \mathbf{j} R \mathbf{k}}{\mathbf{j} R \text{ some } X} (sI) \quad (2.7)$$

Note that the ($s_n I$)-rule enforces the introduction of the notional object **some X** only at the lowest scope level. This again contrasts the introduction in the specific reading (sI), that can introduce **some X** at any scope level. To see the difference in scope relationship, consider, for X, Y nouns and R an extensional transitive verb, the following:

$$\text{every } X R \text{ some } Y \quad (2.8)$$

This sentence has two readings, where higher scope level is indicated by a higher scope index, indicated parenthetically. The full details of quantifier-scope ambiguity handling can be found in [5]. The (simplified) rules for introducing **every X** into subject position of an extensional transitive verb is as follows.

$$\frac{\frac{\mathbf{j} \text{ isa } X]_i}{\vdots} \quad \frac{\mathbf{j} \hat{R} \mathbf{k}}{\text{every } X R \mathbf{k}} (eI^i)}{\frac{\frac{\mathbf{j} \text{ isa } X]_i}{\vdots} \quad \frac{\mathbf{j} R \text{ some } Y}{\text{every } X R \text{ some } Y} (eI^i)} (eI^i) \quad \mathbf{j} \text{ fresh} \quad (2.9)$$

Following are the corresponding derivations. Note the correspondence between scope-level and order of introduction.

Subject wide-scope (sws):

$$\frac{\frac{\frac{\mathbf{r} \text{ isa } X]_i}{\mathcal{D}_1} \quad \frac{\mathbf{j} \text{ isa } Y}{\mathcal{D}_2}}{\frac{\mathbf{r} R \mathbf{j} \quad \mathbf{j} \text{ isa } Y}{\mathbf{r} R (\text{some } Y)_1} (sI)} \quad \frac{\mathbf{r} R (\text{some } Y)_1}{(\text{every } X)_2 R (\text{some } Y)_1} (eI^i) \quad (2.10)$$

Subject narrow-scope (sns):

$$\frac{\frac{\frac{[\mathbf{r} \text{ isa } X]_i}{\mathcal{D}_1}}{\mathbf{r} \hat{R} \mathbf{j}} (eI^i) \quad \frac{\mathcal{D}_2}{\mathbf{j} \text{ isa } Y}}{\frac{(\text{every } X)_1 \hat{R} \mathbf{j}}{(\text{every } X)_1 \hat{R} (\text{some } Y)_2} (sI)} (sI) \quad (2.11)$$

On the other hand,

$$\text{every } X \hat{R} \text{some } Y \quad (2.12)$$

has one reading (therefore, one derivation) only, with an object narrow scope.

$$\frac{\frac{\frac{[\mathbf{r} \text{ isa } X]_i}{\mathcal{D}_1}}{\mathbf{r} \hat{R} \mathbf{n}} \quad \frac{\mathcal{D}_2'}{\mathbf{n} \text{ is being a } Y}}{\frac{\mathbf{r} \hat{R} (\text{some } Y)_1}{(\text{every } X)_2 \hat{R} (\text{some } Y)_1} (sI)} (eI^i) \quad (2.13)$$

The harmonious⁵ *GE*-rule is

$$\frac{\mathbf{j} \hat{R} \text{some } X}{S} \quad \frac{\vdots}{\dot{S}} (s_n E^{i,j}) \quad \mathbf{n} \text{ fresh} \quad (2.14)$$

2.2.1 Monotonicity of Opacity

I introduce a partial ordering among notions, reflecting their *generality*. I will express the fact that being a *Y* is more general than being an *X* by

$$\text{being a } X \leq_n \text{being a } Y \quad (2.15)$$

The source of the partial order $\leq_n$ is from modification of notions by means of (intersective) adjectives and relative clauses⁶, as described below. Again, it is imported to realise that this partial order is internal to the proof-system, and does not reflect any ordering of something populating models⁷.

The meaning of sentences with opaque verbs admits *upward monotonicity* in the object argument ([15]), as exhibited by the following instance of the general inference involved.

$$\frac{\text{Jacob seeks a blonde secretary} \quad \text{every blonde secretary isa secretary}}{\text{Jacob seeks a secretary}} \quad (2.16)$$

The monotonicity is w.r.t. the partial order $\leq_n$. In the example above, the result will follow by introducing an appropriate rules and by showing being a blonde secretary $\leq_n$ being a secretary is provable.

First, I introduce the rules connecting notion with modification via (intersective) adjectives like **n** is being white. I assume here that the intersectivity of intersective adjectives holds also under the scope of opacity⁸.

⁵See [5] for the discussion of harmoniously-induced of NL *E*-rules, and the explanation of *GE*-rules.

⁶In a more extensive fragment, there may be more sources for this partial order.

⁷Contrast this partial order with the one in [15], based on extensions (in models) of properties.

⁸As far as I am aware of, this issue is not discussed in the literature. There are also opinions denying the monotonicity of opacity. A way to block it might be prohibiting intersectivity under the scope of opacity. I deal here only with the case where monotonicity *is* assumed to hold.

The I/E -rules are the following.

Adjectival modification: The I -rule is

$$\frac{\mathbf{n} \text{ is being a } X \quad \mathbf{n} \text{ is being } A}{\mathbf{n} \text{ is being a } A \ X} (a_n I) \quad (2.17)$$

A parameter associated with two (or more) notions is an analog to predicating two (or more) predicates on an individual parameter. For example, an instance of $(a_n I)$ is

$$\frac{\mathbf{n} \text{ is being a secretary} \quad \mathbf{n} \text{ is being blonde}}{\mathbf{n} \text{ is being a blonde secretary}} (a_n I)$$

The E -rule is

$$\frac{\mathbf{n} \text{ is being a } A \ X \quad \begin{array}{c} \vdots \\ \dot{S} \end{array}}{S} (a_n E^{i,j}) \quad (\mathbf{n} \text{ fresh}) \quad (2.18)$$

Relativization: Recall that the fragment allows three kinds of relativization. I will collapse all relative pronouns like *who*, *whom*, *which* etc. to *that*. I assume that all⁹ relativisation is carried out by means of verbal notions.

Intransitive subject: For example,

$$\text{John seeks a secretary that smiles} \quad (2.19)$$

Transitive subject: For example,

$$\text{John seeks a secretary that knows Mary} \quad (2.20)$$

Transitive object: For example,

$$\text{John seeks a secretary that Mary knows} \quad (2.21)$$

The rule for the compound noun has as premises being a notion corresponding to the unmodified noun (*secretary* in the examples), and a notion corresponding to a verbal notion. For the third case (transitive object), such a notion is usually expressed by using the passive gerundive form (see Section 4). In the examples, the three notions are *smiling*, *knowing Mary*, *being known by Mary*.

The notional relativization rules are the following.

$$\frac{\mathbf{n} \text{ is being a } X \quad \mathbf{n} \text{ is } Ving}{\mathbf{n} \text{ is being a } X \text{ that } V} (r_n I) \quad (2.22)$$

Here V is the verbal form from which the verbal notion is $Ving$ is formed. For example,

$$\frac{\mathbf{n} \text{ is being a secretary} \quad \mathbf{n} \text{ is knowing } \mathbf{k}}{\mathbf{n} \text{ is being a secretary that knows } \mathbf{k}} (r_n I)$$

and

$$\frac{\mathbf{n} \text{ is being a secretary} \quad \mathbf{n} \text{ is } \mathbf{k} \text{ knowing}}{\mathbf{n} \text{ is being a secretary that } \mathbf{k} \text{ knows}} (r_n I)$$

The E -rule is

$$\frac{\mathbf{n} \text{ is being a } X \text{ that } V \quad \begin{array}{c} \vdots \\ \dot{S} \end{array}}{S} (r_n E^{i,j}) \quad , \mathbf{n} \text{ fresh} \quad (2.23)$$

⁹This is just a simplification of presentation. Nothing prevents consideration, for example, of *j* seeks a secretary that is blonde, which is more colloquial expressed as *j* seeks a blonde secretary.

There is a special case of relativization worth noting. The verbal notion *Ving* may itself be based on an ITV, forming *nesting* of ITVs. Consider, for example, the following instance of $(r_n I)$:

$$\frac{\text{n is being a secretary} \quad \text{n is seeking a laptop}}{\text{n is being a secretary that seeks a laptop}} (r_n I)$$

Combined with $(s_n I)$, the following interpretation of a nested ITV results.

$$\frac{\text{j seeks n} \quad \frac{\text{n is being a secretary} \quad \text{n is seeking a laptop}}{\text{n is being a secretary that seeks a laptop}} (r_n I)}{\text{j seeks a secretary that seeks a laptop}} (s_n I) \quad (2.24)$$

We see that there is nothing special about this nesting. The above sentence is interpreted the same way as its following counterpart, that has an extensional TV instead of the ITV:

j seeks a secretary that has a laptop

In particular, the nested **seek** above is not applied in (2.24) to any individual parameter as a subject – no “unspecified seeker” arises ...

I now turn to the rules pertaining ‘ $\leq_n$ ’, the partial ordering of notions. The first source of $(\leq_n)$ is from an explicit sentence in the extensional fragment, relating two nouns, X and Y : **every** X **isa** Y . This induces the following I/E -rules for notional comparison (omitting Γ for brevity).

$$\frac{\text{every } X \text{ isa } Y}{\text{being a } X \leq_n \text{ being a } Y} (\leq_n I_1) \quad \frac{\text{being a } X \leq_n \text{ being a } Y}{\text{every } X \text{ isa } Y} (\leq_n E_1) \quad (2.25)$$

And similarly

$$\frac{\text{every } X \text{ V}}{\text{being a } X \leq_n \text{ Ving}} (\leq_n I_2) \quad \frac{\text{being a } X \leq_n \text{ Ving}}{\text{every } X \text{ V}} (\leq_n E_2) \quad (2.26)$$

Based on the $(a_n I)$ and $(r_n I)$ rules, the following additional I -rules for the notional partial order (with no premises) are introduced.

$$\overline{\text{being a } A \text{ } X \leq_n \text{ being a } X} (\leq_n I_3) \quad \overline{\text{being a } A \text{ } X \leq_n \text{ being a } A} (\leq_n I_4) \quad (2.27)$$

$$\overline{\text{being a } X \text{ that } V \leq_n \text{ being a } X} (\leq_n I_5) \quad \overline{\text{being a } X \text{ that } V \leq_n \text{ Ving}} (\leq_n I_6) \quad (2.28)$$

Let $\leq_n$ be the least partial order (over notions) extending the base cases above. The following lemma is a characterisation of $(\leq_n)$ (for the fragment under consideration).

Lemma 2.1 (notional ordering) *For any two notions being a X and being a Y , $\Gamma \vdash \text{being a } X \leq_n \text{ being a } Y$ iff one of the following cases applies.*

1. X, Y are (possibly compound) nouns and $\Gamma \vdash \text{every } X \text{ isa } Y$.
2. For some adjective A and (possibly compound) noun Z , X is $A \text{ } Z$ and Y is Z .
3. For some noun Z and some V , X is $a \text{ } Z \text{ that } V$ and Y is Z .

Similar clauses apply for the larger notion being adjectival or verbal, which I omit since they are not used in the sequel.

I now attend to some general issues related to monotonicity of opacity. The upward monotonicity¹⁰ exemplified by the following additional derived *I*-rule for non-specific objects is mostly accepted in the literature (in its model-theoretic guise); see, for example, [7] (p.7, fn. 4) and [15].

$$\frac{\mathbf{j} \hat{R} \text{ some } X \quad \text{being a } X \leq_n \text{ being a } Y}{\mathbf{j} \hat{R} \text{ some } Y} (s_n I_2) \quad (2.29)$$

The derivation of $(s_n I_2)$ follows the cases of the notional ordering lemma. I show one of the cases, the one based on $(\leq_n I_2)$ (omitting the context Γ). I have to show

$$\mathbf{j} \hat{R} \text{ some } A \quad X \vdash \mathbf{j} \hat{R} \text{ some } X \quad (2.30)$$

of which (2.16) is an instance. The derivation is

$$\frac{\mathbf{j} \hat{R} \text{ some } A \quad X \quad \frac{\mathbf{j} \hat{R} \mathbf{n}]_1 \quad \frac{\frac{[\mathbf{n} \text{ is being } A]_3 \quad [\mathbf{n} \text{ is being a } X]_4}{\mathbf{n} \text{ is being a } X} (a_n E^{3,4})}{\mathbf{n} \text{ is being a } X} (s_n I)}{\mathbf{j} \hat{R} \text{ some } X} (s_n E^{1,2}) \quad (2.31)$$

Similarly, one can derive

$$\mathbf{j} \hat{R} \text{ some } X \text{ that } V \vdash \mathbf{j} \hat{R} \text{ some } X \quad (2.32)$$

Note that accepting the $(s_n I)$ -rule detaches *seek*, for example, from anything like “the intentional state of the seeker”, occasionally appealed to in some model-theoretic treatments, e.g., [10] (not in connection with monotonicity). The second premise is an objective statement about the relationship between being an *X* and being a *Y*, independent of the seeker. I consider this right, as long as no more specific lexical semantics for *seek* is given. With the above rules, one can derive, for example, (2.16).

I assume the natural assumptions that the partial ordering of notions admits upper bounds. That is, for every *X, Y* there is some *Z* such that *being a X* $\leq_n$ *being a Z* and *being a Y* $\leq_n$ *being a Z*. Thus, a common upper bound of *being a sweater* and *being a pen* might be *being a material object*, or just *being a sweater or a pen* if coordination is added to the fragment. The issue of the existence *least* upper bound on notions is immaterial here. Note that I take the existence of upper bounds as an assumption, since the theory here does not include a lexical semantics theory, from which such an assumption might be derived as a fact.

Another issue related to monotonicity results from coordination. In the extensional fragment, there are two families of coordination rules related to determiners (besides the familiar logical rule for sentential coordination). For simplicity, I formulate only a simplified version pertaining to subjects of transitive verbs and an indefinite object. For two determiners δ_1, δ_2 and two (possibly compound) nouns *X, Y, Z*,

Determiner phrase coordination:

$$\frac{\delta_1 X \text{ } R \text{ some } Z \quad \delta_2 Y \text{ } R \text{ some } Z}{\delta_1 X \wedge \delta_2 Y \text{ } R \text{ some } Z} (DP \wedge) \quad (2.33)$$

Determiner coordination:

$$\frac{\delta_1 X \text{ } R \text{ some } Z \quad \delta_2 X \text{ } R \text{ some } Z}{\delta_1 \wedge \delta_2 X \text{ } R \text{ some } Z} (D \wedge) \quad (2.34)$$

¹⁰The downwards monotonicity is generally agreed not to hold.

There is an important issue resulting from accepting upward monotonicity, and in particular the existence of upper bounds, known as *the common objective* issue. It is exemplified in the following derivability claim. Let Z be an upper bound on X and Y , and suppose $\mathbf{j}$ and $\mathbf{k}$ are John and Mary, respectively.

$$\mathbf{j} \text{ seeks some } X, \mathbf{k} \text{ seeks some } Y, \text{being an } X \leq_n \text{ being a } Z, \text{being a } Y \leq_n \text{ being a } Z \vdash \mathbf{j} \text{ and } \mathbf{k} \text{ seek some } Z \quad (2.35)$$

The derivation of (2.35) is the following.

$$\frac{\frac{\mathbf{j} \text{ seeks some } X \quad \text{being a } X \leq_n \text{ being a } Z}{\mathbf{j} \text{ seeks some } Z} (s_n I_2) \quad \frac{\mathbf{k} \text{ seeks some } Y \quad \text{being a } Y \leq_n \text{ being a } Z}{\mathbf{k} \text{ seeks some } Z} (s_n I_2)}{\mathbf{j} \text{ and } \mathbf{k} \text{ seek some } Z} (\wedge_{DP} I) \quad (2.36)$$

An instance of (2.35) is, for example,

$$\mathbf{j} \text{ seeks a green sweater, } \mathbf{k} \text{ seeks a red sweater, being a green sweater } \leq_n \text{ being a sweater, being a red sweater } \leq_n \text{ being a sweater} \\ \vdash \mathbf{j} \text{ and } \mathbf{k} \text{ seek a sweater}$$

A similar inference obtains for more general subjects, e.g.,

$$\text{every girl seeks a green sweater, some boy seeks a red sweater,} \\ \text{being a green sweater } \leq_n \text{ being a sweater, being a red sweater } \leq_n \text{ being a sweater} \\ \vdash \text{every girl and some boy seek a sweater}$$

Also, using $(D\wedge)$ instead of $(DP\wedge)$,

$$\text{at least two girls seeks a green sweater, at most five girls seeks a green sweater, being a green sweater } \leq_n \text{ being a sweater,} \\ \vdash \text{at least two and at most five girls seek a sweater}$$

Note that

$$\mathbf{j} \hat{R} \text{ some } X, \mathbf{k} \hat{R} \text{ some } X \not\vdash \mathbf{j} \hat{R} \text{ some } X \text{ that } \mathbf{k} \hat{R} \quad (2.37)$$

For example,

$$\mathbf{j} \text{ seeks a secretary, } \mathbf{k} \text{ seeks a secretary} \not\vdash \mathbf{j} \text{ seeks a secretary that } \mathbf{k} \text{ seeks} \quad (2.38)$$

Another inference associated with monotonicity can be exemplified by

$$\frac{\mathbf{j} \text{ seeks a secretary}}{\mathbf{j} \text{ seeks something}} \quad (2.39)$$

The extensional fragment considered in [5] had only binary quantification, and *something* was not included in it. An easy extension including it is obtained by adding the following (simplified) rules for the transitive case.

$$\frac{\mathbf{j} R \mathbf{k}}{\mathbf{j} R \text{ something}} (stI) \quad \frac{\mathbf{j} R \text{ something}}{\mathbf{j} R \mathbf{k}} (stE), \quad \mathbf{k} \text{ fresh} \quad (2.40)$$

The difference between (2.40) and (2.7) is the absence in the former of any nominal qualification X of the object $\mathbf{k}$ of R . This extension gives rise to a notion *being something*. By considering this notion to be the greatest notion under the $\leq_n$ partial order, namely, for every X ,

$$\overline{\text{being a } X \leq_n \text{ being something}} (\leq_n I_{max}) \quad (2.41)$$

by which the inference in (2.39) results using monotonicity.

Consider next the notional interpretation of

$$\mathbf{j} \text{ and } \mathbf{k} \text{ seek something} \quad (2.42)$$

The sentence (2.42) is ambiguous between *two notional readings* (in addition to the obvious specific reading):

1. both **j** and **k** seek *the same* unspecific thing.
2. each of **j** and **j** seeks a possibly different unspecific thing.

The difference in meaning between the two readings is captured by means of the two ways the readings are derived.

1. The first reading results by

$$\frac{\text{j seeks something} \quad \text{k seeks something}}{\text{j} \wedge \text{k seek something}} (\wedge_{DPI}) \quad (2.43)$$

where both premises result from seeking the *same* notion **n**.

2. On the other hand, the second reading results by

$$\frac{\frac{\text{j seeks some } X \quad \text{being a } X \leq_n \text{ being something}}{\text{j seeks something}} (s_n I_2) \quad \frac{\text{k seeks some } Y \quad \text{being a } Y \leq_n \text{ being something}}{\text{k seeks something}} (s_n I_2)}{\text{j and k seek something}} (\wedge_{DPI}) \quad (2.44)$$

Here two generalisations to a common objective take place.

We saw already the effect of derivation method on proof-theoretic meaning in the discussion of quantifier scope ambiguity above. For a general discussion of the relationship between (reified) meaning and way of derivation, see [3].

Let me mention that in her model-theoretic semantics of ITVs [7], Moltmann accepts the argument to the common objective as valid. On the other hand, Zimmerman [15] rejects it (with the conclusion *John seeks something which Mary seeks*), based on considerations involving a principle he calls ‘exact matching’, a principle arising from the finer lexical semantics of *seek*. Since, as I mentioned above, I exclude more specific lexical meanings of ITVs from consideration here, I hold (2.35) valid.

I end the discussion of monotonicity of ITVs by a brief consideration of another source for monotonicity, arising when other determiners, besides *some*, are allowed as objects of $\hat{R}$. A fuller discussion involves a PTS for plurality, which I do not have at this stage.

Consider the family of positive¹¹ *existential* determiners of the form *at least n*, $n \geq 1$. The (simplified) *I/E*-rules for this family in the extensional fragment are the following. For simplicity, I formulate the rules only for the object position of an extensional transitive verb.

$$\frac{\{\mathbf{k}_i \text{ isa } X\}_{1 \leq i \leq m} \quad \{\mathbf{j} R \mathbf{k}_i\}_{1 \leq i \leq m}}{\mathbf{j} R \text{ at least } m X} (\geq mI) \quad \mathbf{k}_i \text{ pairwise distinct} \quad (2.45)$$

$$\frac{\mathbf{j} R \text{ at least } m X \quad \begin{array}{c} [\{\mathbf{k}_i \text{ isa } X\}_{1 \leq i \leq m}]_{l_1}, [\{\mathbf{j} R \mathbf{k}_i\}_{1 \leq i \leq m}]_{l_2} \\ \vdots \\ S \end{array}}{S} (\geq mE^{l_1, l_2}) \quad \mathbf{k}_i \text{ pairwise distinct and fresh} \quad (2.46)$$

For δ a determiner in this family, let $m(\delta)$ be the corresponding number of premises $\mathbf{k}_i \text{ isa } X$ in its δI -rule. Each such δ gives rise to a notion *being δX* . For example, *being at least three secretaries*. Those existential notions are ordered by

$$\overline{\text{being } \delta_1 X \leq_n \text{ being } \delta_2 X} (\leq I_\delta) \quad \text{iff } m(\delta_1) \geq m(\delta_2) \quad (2.47)$$

¹¹A discussion of the classification of determiners into positive and negative and a proof-theoretic treatment of the negative ones can be found in [4], using means outside the scope of the current paper.

$$\frac{\begin{array}{c} [S[\mathbf{j}]]_1 \\ \vdots \\ S[\mathbf{k}] \\ \mathbf{j} \text{ is } \mathbf{k} \end{array} (isI^1) \text{ where } S \text{ is fresh}}{\quad} \quad \frac{\begin{array}{c} [S[\mathbf{k}]]_1 \\ \vdots \\ S[\mathbf{j}] \\ S' \end{array} (isE^1) \quad \mathbf{j} \text{ is } \mathbf{k}}{S'} (isE^1)$$

Figure 1: Rules for the identity

(note the reversal of the ordering). Thus, the following is a correct inference:

$$\frac{\mathbf{j} \hat{R} \delta_1 \mathbf{X} \quad \text{being } \delta_1 X \leq_n \text{being } \delta_2}{\mathbf{j} \hat{R} \delta_2 X}$$

For example,

$$\text{being at least seven secretaries } \leq_n \text{being at least two secretaries}$$

Thus, the following is a correct inference¹².

$$\frac{\mathbf{j} \text{ seeks at least two secretaries}}{\mathbf{j} \text{ seeks a secretary}}$$

where both objects of **seek** have notional reading.

2.3 Rules for Resistance to substitutability

The argument in (1.4) is an instance of the following schematic basic argument **BA**, embodying substitutivity of the analogue of coextensives:

$$\frac{\begin{array}{c} \mathbf{j} \hat{R} \mathbf{k} \\ \mathbf{k} \text{ is } \mathbf{r} \\ \hline \mathbf{j} \hat{R} \mathbf{r} \end{array}}{\quad} \quad (2.48)$$

Recall that **is** is the expression of identity in the fragment. The *I/E*-rules for identity are presented in Figure 1. Here $S[-]$ is an arbitrary sentence in the fragment, where ‘-’ is some position within S that can be filled by a parameter. An immediately derived *E*-rule, obtained by choosing S' as $S[\mathbf{k}]$ itself, is

$$\frac{\mathbf{j} \text{ is } \mathbf{k} \quad S[\mathbf{j}]}{S[\mathbf{k}]} (is\hat{E})$$

which express the substitutivity of identicals.

A derivation corresponding to the argument in (2.48) would be a direct application of $(is\hat{E})$, the derived identity elimination rule, by taking S to be $\mathbf{j} \hat{R} \mathbf{k}$. We get

$$\frac{\mathbf{k} \text{ is } \mathbf{r} \quad \mathbf{j} \hat{R} \mathbf{k}}{\mathbf{j} \hat{R} \mathbf{r}} (is\hat{E})$$

All that is needed to block **BA** is to restrict (isE) (and hence $(is\hat{E})$ too) to apply only to sentences with extensional verbs! Thus, the rule (isE) can serve as a switch between accepting or rejecting the (failure of) substitutivity.

¹²I ignore here the plural morphology.

If we let notional equality to be defined in terms of the partial ordering

$$\text{being a } X \text{ is being a } Y \text{ iff } \text{being a } X \leq_n \text{being a } Y \wedge \text{being a } Y \leq_n \text{being a } X \quad (2.49)$$

substitutivity can be extended or blocked, as desired, also directly to notions.

$$\frac{\mathbf{j} \hat{R} \text{ some } X \quad \text{being a } X \text{ is being a } Y}{\mathbf{j} \hat{R} \text{ some } Y} \quad (2.50)$$

This can arise, for example, under a context Γ including both $\text{every } X \text{ isa } Y$ and $\text{every } Y \text{ isa } X$.

Note, in passing, that while resistance of ITVs to substitutivity of identicals in their object, notionally read, argument may be disputed, there is no dispute about such a substitutivity in the subject argument. Thus, the following is certainly a valid argument.

$$\frac{\mathbf{j} \text{ is } \mathbf{k} \quad \mathbf{j} \hat{R} \mathbf{r}}{\mathbf{k} \hat{R} \mathbf{r}} \quad (is\hat{E})$$

For example,

$$\frac{\text{John is the chairman} \quad \text{the chairman seeks a secretary}}{\text{John seeks a secretary}} \quad (is\hat{E})$$

Thus, any theory relying on “seekers intentions” (or the like) has to explain how identicals obtain “identical intentions”.

2.4 Existential Commitment

Existential commitment here is expressed via a use of an individual parameter. So, suspension of existence is manifested directly by the form of the $(s_n I)$ -rule introducing a non-specific indefinite DP into the object position, that does not have a premise using an individual parameter.

Suppose secretaries exist and unicorns do not exist. This would be embodied at the level of ground (i.e., atomic) sentences, so that $\mathbf{k}$ is a secretary could surface as a possible premise, while $\mathbf{k}$ is a unicorn will not. This difference between secretaries and unicorns has no bearing on the notions being a secretary and being a unicorn . At the atomic level, there is no difference in the status of $\mathbf{n}$ is being a secretary and $\mathbf{n}$ is being a unicorn as premises. Associating a name to a notion has no existential commitment. As a result, $\mathbf{j}$ seeking a (non-specific) secretary and $\mathbf{j}$ seeking a (non-specific) unicorn are derived in exactly the same way:

$$\frac{\mathbf{n} \text{ is being a secretary} \quad \mathbf{j} \text{ seeks } \mathbf{n}}{\mathbf{j} \text{ seek some secretary}} \quad (s_n I) \quad \frac{\mathbf{n} \text{ is being a unicorn} \quad \mathbf{j} \text{ seeks } \mathbf{n}}{\mathbf{j} \text{ seek some unicorn}} \quad (s_n I)$$

This can be contrasted with the derivation of $\mathbf{j}$ finds a unicorn, which needs a premise appealing to an individual parameter.

$$\frac{\mathbf{k} \text{ isa unicorn} \quad \mathbf{j} \text{ finds } \mathbf{k}}{\mathbf{j} \text{ finds some unicorn}} \quad (sI)$$

3 The reified proof-theoretic meaning of ITVs

According to the proof-theoretic semantics programme, meanings are determined by the rules of a meaning-conferring natural-deduction (ND) proof-system, provided it satisfies a certain balance between

its rules, known as *harmony*. The reader is referred to [5] or to [4] for a more comprehensive presentation of the PTS programme as applied to natural language semantics. According to most PTS theories, it is the *I*-rules that determine meanings. In [3], I proposed an *explicit* definition of the proof-theoretic meaning determined by the rules, called a *reified* proof-theoretic meaning. Below, I delineate the definitions, and consider the consequences of applying them to obtain the meanings of notional readings of sentences headed by ITVs.

Given some meaning-conferring *ND*, say $\mathcal{N}$, it induces in a standard way a definition of a $\mathcal{N}$ -*derivation*. There is a special family of $\mathcal{N}$ -derivations, called *canonical derivations*, that plays a major role in the definition of meanings. The rational of the following definition can be found in [2].

Definition 3.1 canonical derivation from open assumptions: A $\mathcal{N}$ -derivation $\mathcal{D}$ of ψ from assumptions Γ is canonical iff it satisfies one of the following two conditions.

- The last rule applied in $\mathcal{D}$ is an *I*-rule (for the main operator of ψ).
- The last rule applied in $\mathcal{D}$ is an assumption-discharging *E*-rule, the major premise of which is some φ in Γ , and its encompassed sub-derivations $\mathcal{D}_1, \dots, \mathcal{D}_n$ are all canonical derivations of ψ .

Denote by $\vdash_{\mathcal{N}}^c$ canonical derivability in $\mathcal{N}$ and by $\llbracket \psi \rrbracket_{\Gamma}^c$ the collection of all (if any) $\mathcal{N}$ -canonical derivations of ψ from Γ .

Next, the reified meaning of ψ conferred by $\mathcal{N}$, denoted by $\llbracket \psi \rrbracket^{\mathcal{N}}$, is defined as follows.

Definition 3.2 reified proof-theoretic meaning:

$$\llbracket \psi \rrbracket^{\mathcal{N}} =^{\text{df.}} \lambda \Gamma. \llbracket \psi \rrbracket_{\Gamma}^c \quad (3.51)$$

In words, the reified meaning of ψ is a mapping from collections of assumptions Γ to the canonical derivations of ψ from Γ (in $\mathcal{N}$ -usually omitted when clear from context). Whenever such derivations exists for some Γ , such a Γ is called *grounds for assertion* for ψ .

Before considering the meaning of ITVs, consider the following examples that shows the importance of canonicity in the definition of meaning. Consider the following derivation of a conjunction.

$$\frac{\alpha \quad (\alpha \rightarrow (\varphi \wedge \psi))}{\varphi \wedge \psi} (\rightarrow E) \quad (3.52)$$

This is a derivation of a conjunction – but not a canonical one, as it does not end with an application of $(\wedge I)$. Thus, the conjunction here was *not* derived according to its meaning! As far as this derivation is concerned, it could mean anything, e.g., disjunction. On the other hand, the following example derivation is according to the conjunction’s meaning, being canonical.

$$\frac{\frac{\alpha \quad \alpha \rightarrow \varphi}{\varphi} (\rightarrow E) \quad \frac{\beta \quad \beta \rightarrow \psi}{\psi} (\rightarrow E)}{\varphi \wedge \psi} (\wedge I) \quad (3.53)$$

Similar examples can be seen in natural language. The following rules (a simplification of the rules from [5]) are *I/E*-rules for intersective adjectives. The rules capture adjectival modification by such adjectives as a “silent” implicit conjunction. In the rule, X ranges over verbs and A ranges over (intersective) adjectives.

$$\frac{j \text{ isa } X \quad j \text{ is } A}{j \text{ isa } A X} (adj I) \quad \frac{j \text{ isa } A X}{j \text{ isa } X} (adj E_1) \quad \frac{j \text{ isa } A X}{j \text{ is } A} (adj E^{1,2}) \quad (3.54)$$

An instance of $(adjI)$ is

$$\frac{j \text{ isa girl} \quad j \text{ is beautiful}}{j \text{ isa beautiful girl}} (adjI)$$

The conclusion of $(adjI)$ can be obtained non-canonically by applying the E -rule for elimination of universal quantification below, not appealing at all to the conjunctive meaning of adjectival modification.

$$\frac{j \text{ isa girl} \quad \text{every girl isa beautiful girl}}{j \text{ isa beautiful girl}} (e\hat{E})$$

Let us now return to the proof-theoretical reified meanings of ITVs. The main observation is about the difference between the notional reading and the predicative reading of the following two generic sentences.

$$(*) \llbracket j \hat{R} \text{ some } X \rrbracket \quad (**) \llbracket j R \text{ some } X \rrbracket \quad (3.55)$$

Any canonical derivation of $(*)$ will end, essentially, with one of the rules (s_nI) and (s_nI_2) (the latter if monotonicity is appealed to). In both cases, there is a notional premise **being a** X involved. On the other hand, any canonical derivation of $(**)$ essentially end with an application of (sI) , with a predicative premise **isa** X .

Thus, the reified proof-theoretic meanings exhibit all the properties discussed above.

4 Passivisation

The incorporation of passivisation into the extensional fragment is a simple task, not raising any new proof-theoretic insights. However, for ITVs, there is an interesting point to note, so I present rules for determining its meaning. The emerging prediction from the rules is that the non-specific reading of indefinite DPs remains invariant under passivisation *but shifts to the DP in the subject position*.

I will use $\hat{R}^{-1}$ as the passive form of the ITV $\hat{R}$. I take passivisation to take place at the level of ground (atomic) sentences. The I/E -rules for passivisation are the following.

$$\frac{j \hat{R} n}{n \hat{R}^{-1} j} (passI) \quad \frac{n \hat{R}^{-1} j}{j \hat{R} n} (passE) \quad (4.56)$$

For example,

$$\frac{j \text{ seeks } n}{n \text{ is sought by } j} (passI)$$

The passive counterpart of (s_nI) is

$$\frac{n \hat{R}^{-1} j \quad n \text{ is being a } X}{\text{some } X \hat{R}^{-1} j} (s_npassI) \quad (4.57)$$

From $(passI)$, we easily derive the following derived rule $(pass\hat{I})$, lifting passivisation to the DP-level.

$$\frac{j \hat{R} \text{ some } X}{\text{some } X \hat{R}^{-1} j} (pass\hat{I}) \quad (4.58)$$

For example,

$$\frac{j \text{ seeks some secretary}}{\text{some secretary is sought by } j} (passI)$$

(with both premise and conclusion retaining the non-specific reading).

The derivation of $(s_n \hat{I})$ is (with Γ omitted)

$$\frac{\frac{\frac{\mathbf{j} \hat{R} \mathbf{n}]_1}{\mathbf{n} \hat{R}^{-1} \mathbf{j}} (passi) \quad [\mathbf{n} \text{ is being a } X]_2}{\mathbf{j} \hat{R} \text{ some } X \quad \text{some } X \hat{R}^{-1} \mathbf{j}} (s_n passI) \quad (s_n E^{1,2})$$

Here one can see that that passivisation inherits the non-specific reading of the indefinite DP, this time in the subject position, from the non-specific reading of the DP in the object position in the original active sentence.

Acknowledgments

I thank Bartosz Więckowski for lengthy discussions on the topic of the paper and for commenting on preliminary versions of it. I thank Itamar Francez for some clarifications regarding notional readings.

References

- [1] Graeme Forbes. *Attitude Problems*. Oxford University Press, 2000.
- [2] Nissim Francez. On the notion of canonical derivations from open assumptions and its role in proof-theoretic semantics. *Review of Symbolic Logic (under refereeing)*, 2014.
- [3] Nissim Francez. Views of proof-theoretic semantics: Reified proof-theoretic meanings. *Journal of Computational Logic*, 2014 (to appear). Special issue in honour of Roy Dyckhoff.
- [4] Nissim Francez and Gilad Ben-Avi. A proof-theoretic reconstruction of generalized quantifiers. *Journal of Semantics*, doi:10.1093/jos/ffu001, 2014.
- [5] Nissim Francez and Roy Dyckhoff. Proof-theoretic semantics for a natural language fragment. *Linguistics and Philosophy*, 33(6):447–477, 2010.
- [6] Richard K. Larson. The grammar of intensionality. In G. Preyer and G. Peter, editors, *Logical form and natural language*, pages 228–262. Oxford University Press, 2001.
- [7] Friederike Moltmann. Intensional verbs and quantifiers. *Natural Language Semantics*, 5(1):1–52, 1997.
- [8] Friederike Moltmann. Intensional verbs and their intentional objects. *Natural Language Semantics*, 16(3):239–270, 2008.
- [9] Friederike Moltmann. *Abstract Objects and the Semantics of Natural Language*. Oxford University Press, Forthcoming.
- [10] Mark Richard. Seeking a centaur, Adoring Adonis: Intensional transitives and empty terms. In P. French and H. Wettstein, editors, *Figurative Language*, pages 103–127. Blackwell, 2001. Midwest studies in philosophy 25.
- [11] Nathan Salmon, editor. *Frege’s puzzle*. Bradford Books, MIT Press, Cambridge, MA, 1986.
- [12] Scott Soames. Direct reference, propositional attitudes, and semantic content. *Philosophical Topics*, 15:47–87, 1987.
- [13] Marta Sznajder. *Dynamic Semantics for Intensional Transitive Verbs D a Case Study*. PhD thesis, University of Amsterdam, 2012.
- [14] Thomas Ede Zimmermann. On the proper treatment of opacity in certain verbs. *Natural Language Semantics*, 1:149–179, 1993.
- [15] Thomas Ede Zimmermann. Monotonicity in opaque verbs. *Linguistics and Philosophy*, 29:715–761, 2006. DOI 10.1007/s10988-006-9009-z.