Lehrstuhl für Automatentheorie
Institut für Theoretische Informatik, TU Dresden
Prof. Dr. F. Baader

Hans-Grundig-Str. 25
Dresden Johannstadt
Tel.: 0351/463–38237
Tel.: 0351/463–39171

5. Exercises for the Course „Logic-based Knowledge Representation“

Exercise 19:

Let K be a set of constructors to generate complex concepts.

- A constructor $k \in K$ is *redundant in* K, if, for each concept C using constructors in K, there exists an equivalent concept C' using only constructors in $K \setminus \{k\}$.
- Two sets K and K' of constructors are *equivalent* if, for each concept C using only constructors in K, there exists an equivalent concept C' using only constructors in K' and, vice versa, for each concept C using only constructors in K', there exists an equivalent concept C' using only constructors in K.

Define, for each of the following two sets of constructors K, a minimal set K' such that K' and K are equivalent and K' does not contain any redundant constructor.

(a) $K := \{\neg, \sqcup, \sqcap, \forall, \exists\}$,

(b) $K := \{\leq n, \geq n, \neg, \sqcup, \sqcap, \forall, \exists\}$.

Exercise 20:

Consider the following two extensions of the Description Logic $\mathcal{ALCN}$ discussed in the lecture:

(i) Addition of a new concept constructor $(r \sqsubseteq s)$, where r and s are role names, with the following semantics:

$$(r \sqsubseteq s)^{\mathcal{I}} = \{a \in \Delta^{\mathcal{I}} \mid \text{for all } b \in \Delta^{\mathcal{I}}, (a, b) \in r^{\mathcal{I}} \text{ implies } (a, b) \in s^{\mathcal{I}}\}$$

(ii) Addition of transitive roles: the set of role names N_R is now partitioned into two sets N_{rR} and N_{tR} (i.e., $N_R = N_{rR} \uplus N_{tR}$). Elements of N_{tR} are interpreted as transitive relations, i.e., for all interpretations $\mathcal{I}$ and all $r \in N_{tR}$, $r^{\mathcal{I}}$ must be transitive.

Then do the following:

(a) Devise new tableau rules for dealing with the above two extensions of $\mathcal{ALCN}$.

(b) For every new rule R, prove that, if the application of R to an ABox A yields new ABoxes $A_1, \ldots, A_n$, then A is consistent iff, for some $1 \leq i \leq n$, A_i is consistent (c.f. Exercise 17).

(c) Sketch the modifications that need to be made to the $\mathcal{ALCN}$-ABox consistency algorithm and its soundness, completeness, and termination proofs when adding the new rules for extensions (i) and (ii), respectively.

Exercise 21:

In the lecture, it has been proved that the tableau algorithm uses space polynomial in the length of the input ABox—provided that numbers in number restrictions are coded unarily.
Modify the tableau algorithm such that this is also true for binary coding of numbers in number restrictions (and such that it still decides consistency of ABoxes).

Exercise 22:

Let $\varphi := \forall a_1 \exists a_2 \; a_1 \rightarrow a_2$ be a quantified Boolean formula.

(a) Show that φ is valid.

(b) Define an $\mathcal{ALC}$-concept C_φ such that φ is valid if and only if C_φ is satisfiable. Describe a canonical model of C_φ.